Institutt for fysikk

Eksamensoppgave i

TFY4170 - Fysikk II

Faglig kontakt under eksamen: Justin Wells

Tlf.: 45 16 36 97

Eksamensdato: 06.06.2017

Eksamenstid (fra-til): 0900-1300

Hjelpemiddelkode/Tillatte hjelpemidler: ***C***

Annen informasjon:

Hvert delspørsmål gir 2 poeng for en fullstendig og korrekt svar. Ufullstendige eller delvis riktige svar får mellom 0 og 2 poeng.

Eksamen teller 100% på sluttkarakteren.

Du kan svare på Engelsk, Bokmål eller Nynorsk.

Målform/språk: English, Bokmål, Nynorsk.

Antall sider med oppgaver: 5

Antall sider med formler og uttrykk: 2

Kontrollert av:

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Dato Sign

Question 1: Interactions with barriers

In this question, we will investigate how waves and particles behave at barriers.

Consider a particle (for example, a football) with kinetic energy E_k, travelling towards a very large, tall and hard wall (i.e. an infinite barrier) at an arbitrary angle.

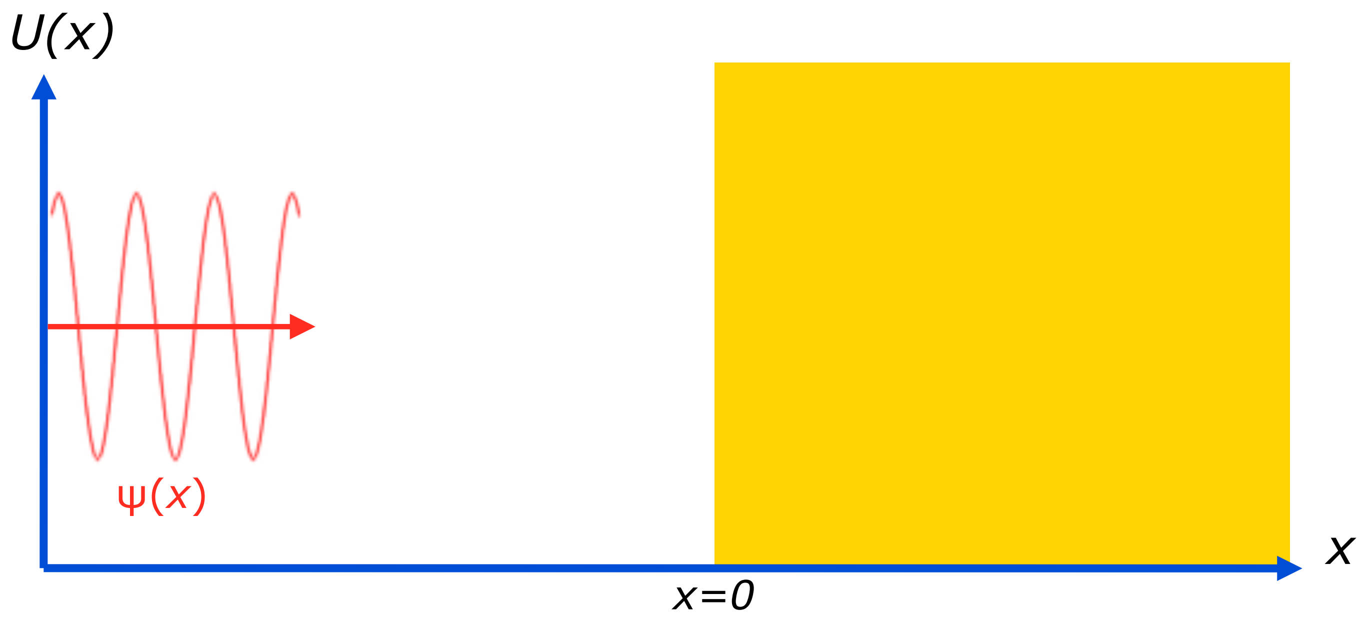
a) What will happen to the football? What is the velocity (magnitude and direction) of the ball just before and just after the collision?

[note: you may assume ideal conditions, i.e. no energy losses, no wind, etc]

Consider a plane wave (wavelength λ, frequency f) approaching the same wall at the same angle.

b) What will happen to the wave? What is the velocity (magnitude and direction) of the incoming and reflected waves?

We will now investigate the same situation using quantum mechanics. As a simplification, we will only consider the x dimension.



Look at the figure. Assume that the barrier is of infinite height and thickness.

c) Solve the Schrødinger equation for $x < 0$ (i.e. the left side of the barrier).

d) Describe the wavefunction at $x > 0$.

We will now repeat this question for the case that the barrier is large, but not infinite height. i.e. $U(x) > E$

e) Solve the Schrødinger equation for both $x < 0$ and $x > 0$.

[hint remember that the wave function must be smooth and continuous at x = 0]

f) Describe how is your solution is different to the previous case with an infinite barrier?

Imagine that the incoming wavefunction is an electron.

g) what is the probability of the electron penetrating into the barrier by a distance $x > a$?

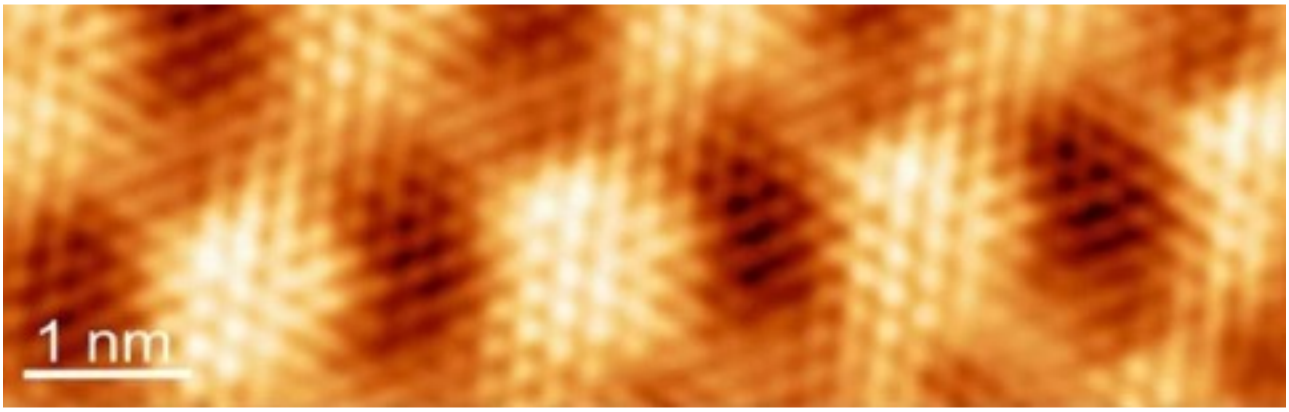
h) If the incoming number of particles is increased (i.e. an electron beam corresponding to a current of I = 1 A), estimate how thick the barrier needs to be such that 1 electron per second is able to pass through (Assume that E-U = 1 eV).

i) In Q1a-h, you have described how particles and waves interact with barriers using both classical and quantum approaches. Briefly describe (or give examples) of situations where each of these approaches is valid or invalid.

Question 2: The *Scanning Tunnelling Microscope* (STM)

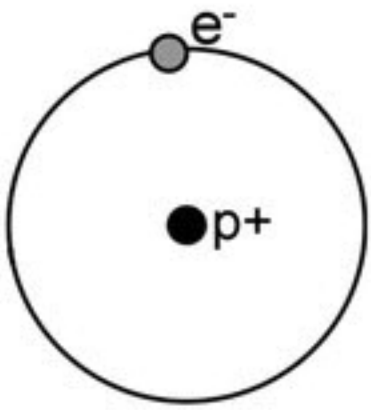
In Q1g,h you have described the principle behind the STM: A type of microscope which is commonly used to measure the atomic scale structure of materials. In the case of an STM, the gap between a conductive "tip" and a conductive sample creates a barrier through which electrons can only penetrate by quantum mechanical "tunnelling".

a) Make a sketch to show how an STM works, and how it generates an image.



b) Take a look at the STM image of graphene (above). Using your answers from Q1g,h can you comment on what may be causing the variations in intensity? [hint, there are two different variables in your equation which are both visible in the image]

Question 3: Understanding atoms



Consider an electron in orbit around a proton. This is known as the "planetary model" of an atom (in this case, the atom is hydrogen). A planet is able to form a stable orbits around the sun because the gravitational attraction is equal to the centripetal force, i.e.

$$F_G = \frac{Gm_1m_2}{r^2} = \frac{mv^2}{r}$$

a) Which force is responsible for the orbit of the electron in an atom? Write an equation for the electron orbit in a hydrogen atom (i.e. the equivalent to the gravitational force equation, above)

b) Do you expect this orbit to be stable? i.e. describe any relevant energy-loss mechanisms, and the implications of energy loss from the electron-proton orbit

Niels Bohr postulated that "special orbits exist" and that these special orbits have "orbital angular momentum L as an integer multiple of $h/2\pi$".

c) What is special about these orbits? What implication does $L = nh/2\pi$ have for the wave properties of an electron in an orbit?

The quantisation of angular momentum creates a corresponding quantisation of radius and energy;

d) Use the quantisation of angular momentum (L), together with your answer to Q2a to derive an expression for the allowed energy levels in the atom.

Look at the attached photo of the *aurora borealis* (nordlys). Notice that it is a particular shade of green (i.e. there is no variation in the colour). *Aurora borealis* is caused when energetic particles from the sun excite electrons in the atoms of our atmosphere, which then relax and emit a photon.

e) Is this consistent with your answer from Q2d? Assume that the relevant atomic relaxation is from *n*=15 to *n*=14 in an oxygen atom (*Z*=8).

f) Other atomic transitions are also possible — for example *n*=16 to *n*=15 in an oxygen atom. How would this look?

If we wish to understand the atomic orbitals of an electron, we need to solve the 3D Schrødinger equation (given in the "additional information" in radial coordinates). This is difficult because the wavefunction (ψ) is a function of r, θ and φ.

g) If you should need to solve the the Schrødinger equation for an atom, we would use "separation of variables": What does this mean? What assumption are you introducing, and is it reasonable?

In the Schrødinger equation, *U* is the potential energy. Generally, we assume that U is the Coulomb potential of the positively charge nucleus, and that it is purely radial.

h) Under which circumstances is this a good assumption, and under which circumstances does it fail?

After solving the Schrødinger equation, we find that each electron can be described by a unique set of quantum numbers (so far in Q2, we have been considering only involves the *principal quantum number* "*n*").

i) What are the other quantum numbers? What physical property are they describing?

Question 4: Nanoscience and Material Science.

During the lectures and exercises, we have solved the "1D square well", describing a particle trapped in a box. We have shown that the Schrødinger equation reduces to the wave equation:

$$\frac{d^2\Psi(x)}{dx^2} + k^2\Psi(x) = 0$$

With solutions in the form: $\Psi(x) = A\cos kx + B\sin kx$

The energy of the solutions is given by: $$E = \frac{n^2\hbar^2\pi^2}{2ma}$$

where *a* is the size of the box

Consider a solid sample of the alkali metal lithium (Li). Li has atomic number Z=3 and atomic radius = 0.15 nm. Imagine that all of the electrons are localised to their parent Li atom (i.e. 3 electrons per atom, all of which are trapped by the potential of the nearest nucleus).

a) What is the minimum energy required to promote an electron from the lowest energy configuration to the next available empty state? Do you think that this is also consistent with Li being a metal?

Being an alkali metal, it is more reasonable to assume that one electron per atom is "free" to travel within the sample (i.e. one electron per atom is delocalised and contributes to the metallic bonding and the electrical conductivity).

b) Imagine that we have a chain of Li atoms of length 1 cm. How many atoms (and how many delocalised electrons) does it contain?

c) Assuming that T = 0 K, what is the energy of the highest occupied state (i.e. the Fermi energy)?

d) What is the energy separation of the states at the Fermi energy? Is this consistent with it being a metal?

One of the most powerful applications of nanoscience is to be able to make structures such that the material properties are exactly what we want for a certain application.

e) Imagine that I want to make an object which is metallic at 100°C, but poorly conducting at room temperature, if I should make it from lithium, what size should it be?

Electrons have spin +1/2 or -1/2. This means that electrons are Fermions. It also means that electrons which share the same space must have a unique set of quantum numbers.

Imagine that we can force electrons to always travel together in pairs. We could consider each electron pair to be a new "particle". Instead of solving Q4a-c for electrons, we could solve it again for our new electron pair "particles". Now, the situation would be different: the spin of the new quasiparticle would be -1, 0 or +1 (i.e. all of the combinations of +1/2 and -1/2 electrons).

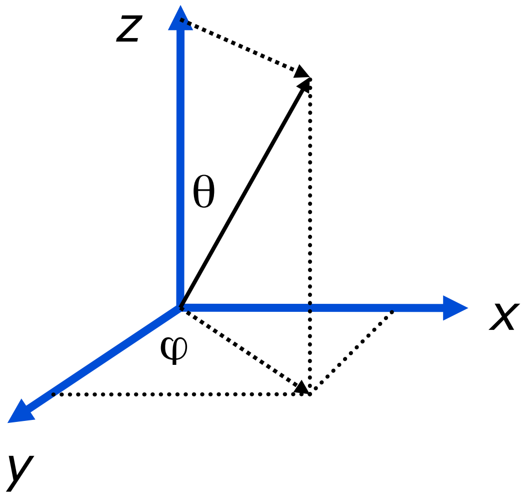
f) Briefly discuss this scenario: Are these new "particles" Fermions? Does the Pauli exclusion principle still apply? What is the value of *n* for the highest occupied level? Is the electrical conductivity going to change because of this pairing?

Note: What you have described in this scenario is superconductivity: The electrons pairs are called "cooper pairs", and their behaviour is completely different from that of unpaired electrons. Cooper pair formation in lithium is possible, but only at high pressure and low temperature P > 48 gigapascals and T < 20 K.

Additional information

Schrødinger equation in 1D:

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+U(x)\right]\Psi(x)=E\Psi(x)$$



Schrødinger equation in 3D (radial coordinates):

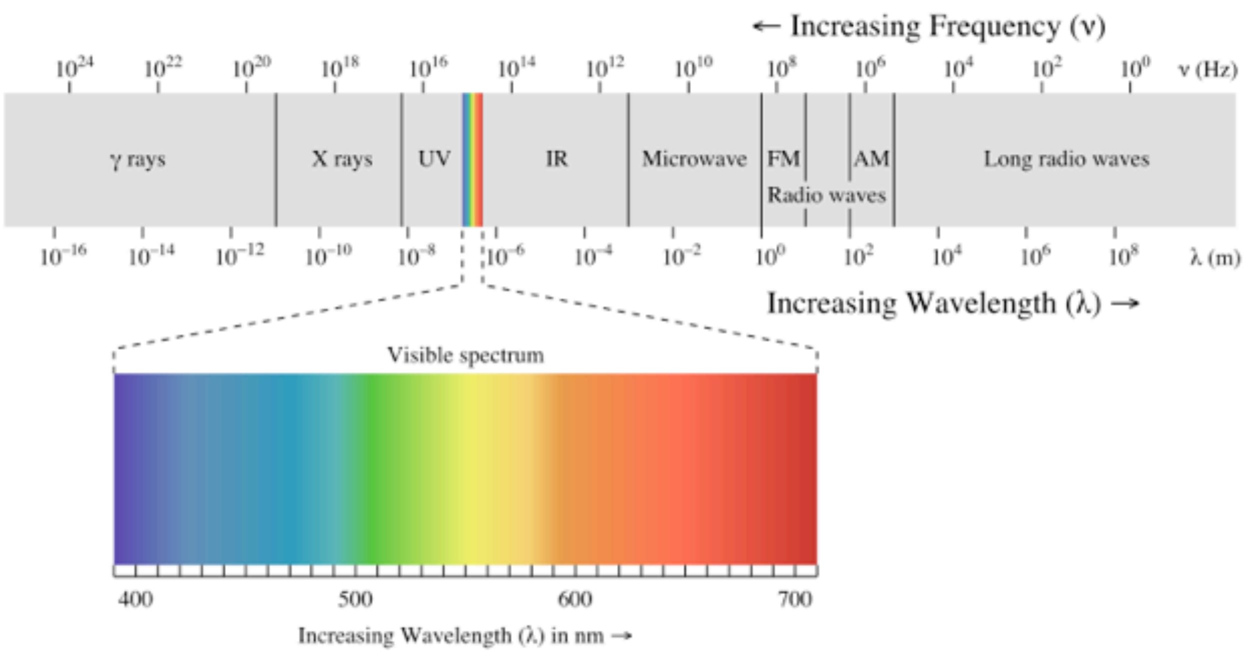
$$\frac{-\hbar^2}{2m_r}\nabla^2\psi(r,\theta,\varphi)+U(r)\psi(r,\theta,\varphi)=E\psi(r,\theta,\varphi)$$

Quantum mechanical probability:

$$\mathbb{P}=\int\psi(x)\psi^*(x).dx$$

The Coulomb force: $$F=k\frac{Q_1Q_2}{r^2}$$

The electromagnetic spectrum:



Fundamental constants

Quantity	Symbol	Approximate value
Acceleration of free fall (Earth's surface)	g	$9.81\,\mathrm{m\,s^{-2}}$
Gravitational constant	G	$6.67\times10^{-11}\,\mathrm{N\,m^{2}\,kg^{-2}}$
Avogadro's constant	N_{A}	$6.02\times10^{23}\,\mathrm{mol^{-1}}$
Gas constant	R	$8.31\,\mathrm{J\,K^{-1}\,mol^{-1}}$
Boltzmann's constant	k	$1.38\times10^{-23}\,\mathrm{J\,K^{-1}}$
Stefan–Boltzmann constant	σ	$5.67\times10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$
Coulomb constant	k	$8.99\times10^{9}\,\mathrm{N\,m^{2}\,C^{-2}}$
Permittivity of free space	ε_0	$8.85\times10^{-12}\,\mathrm{C^{2}\,N^{-1}\,m^{-2}}$
Permeability of free space	μ_0	$4\pi\times10^{-7}\,\mathrm{T\,m\,A^{-1}}$
Speed of light in vacuum	c	$3.00\times10^{8}\,\mathrm{m\,s^{-1}}$
Planck's constant	h	$6.63\times10^{-34}\,\mathrm{J\,s}$
Elementary charge	e	$1.60\times10^{-19}\,\mathrm{C}$
Electron rest mass	m_{e}	$9.110\times10^{-31}\,\mathrm{kg} = 0.000549\,\mathrm{u} = 0.511\,\mathrm{MeV\,c^{-2}}$
Proton rest mass	m_{p}	$1.673\times10^{-27}\,\mathrm{kg} = 1.007276\,\mathrm{u} = 938\,\mathrm{MeV\,c^{-2}}$
Neutron rest mass	m_{n}	$1.675\times10^{-27}\,\mathrm{kg} = 1.008665\,\mathrm{u} = 940\,\mathrm{MeV\,c^{-2}}$
Unified atomic mass unit	u	$1.661\times10^{-27}\,\mathrm{kg} = 931.5\,\mathrm{MeV\,c^{-2}}$