8 Exam Question V2021

1. Quantum mechanical model of electrons in solids: free electron gas

   (a) Describe most important assumptions used to derive the quantum mechanical free electron gas model.

   (b) What properties of materials can this model describe?

   (c) Equation below contains parameter a which has units of length. What is described by this parameter? What is the meaning of n here? What are E and E_1?

$$E = (n_x^2 + n_y^2 + n_z^2)E_1$$

$$E_1 = \frac{\hbar^2 \pi^2}{2ma^2}$$

   (d) What is described by parameter E_F, the Fermi energy? How is it connected to the equations in the previous question (Q1.3)?

   (e) A gold crystal has an fcc lattice (unit cell has a cubic shape) a lattice constant of 4.08 Å and 4 gold atoms in the unit cell. Each gold atom donates one electron to the free electron gas. Calculate the electron number density, Fermi energy, Fermi velocity and electron wavelength in a gold crystal.

   (f) What aspects of electron behaviour in solid materials are not captured by the free electron gas model.

2. Standing waves

   (a) What are standing waves? Give some examples.

   (b) Design and describe an experiment in which you demonstrate the existence of standing waves. You can use couple of small speakers (size of 1cm) that generates sound with a frequency of 50Hz (connected to a single signal generator) and a small microphone (1mm) connected to a PC. Alternatively you can use a water tank with one or two wave-generating devices (Ripple tank) and a sensor that is able to measure water level height. Derive mathematical expression that describes the standing wave in your experiment and plot wave profile you expect to detect.

   Hint: Two waves that are generated by sources that are in phase but are separated by a distance L can be described by:

$$y_1 = A\sin(kx - \omega t + \phi_0)$$

$$y_2 = A\sin(k(x - L) - \omega t + \phi_0)$$

   (c) Understanding of standing waves is important in the design of musical instruments. Give some examples. What are harmonics in this context? Would a guitar sound differently when played on the space station? Would a flute sound differently inside a air-filled diving bell in which the pressure p is 4 atmospheres. Support your answers by calculations.

   (d) Are standing wave phenomena important in quantum mechanics or atomic physics? Explain and if they are, give some examples.

3. Propagating waves.

(a) A propagating sinusoidal wave can be described by the following equation:

$$y(x,t) = A\cos(\omega t - kx + \phi)$$

where $y(x,t)$ describes the amplitude at location and at time.

- The same wave can also be described using a complex notation. Introduce this notation.
- Use complex notation or the formula given above to derive a formula that can be used to calculate the distance between two points along the x-axis that oscillate in phase.

(b) Consider a one-dimensional system with two independent and identical wave sources: S1 located at $x = 0$ and S2 at $x = \Delta x$. Derive the expression for time-dependent amplitude at a observation point O located at the distance x_0 from S1. Consider only situations where $x_0 > \Delta x$. Plot amplitude at as the function of the separation distance between sources Δx

4. You have made a line pattern in a thin metal film deposited on a glass substrate (see figure, line spacing is d and line width is a). You need to verify that this periodic structure has been fabricated correctly. You look around the lab, but the only instrument you find is a laser pointer, that uses green light with a wavelength λ= 550nm. You also have your phone with you and can use the phone camera. Explain how you would investigate this.

   Hint: What physical phenomena can you use to measure d and a? Can you estimate what range of sizes can be measured? Can both a and d be measured and can this be done in the same experiment? Support your answers by calculations.

5. Atomic Physics:

   (a) What steps and assumptions are necessary to use mater waves and time-independent Schrödinger equation to describe 1 electron atom like hydrogen? Why is the situation more complex for atoms that contain more than 1 electron?

   (b) What experimental observations can be used to verify prediction from the quantum mechanical model of the one-electron atom?

   (c) What aspects of the one-electron atom model can be used to describe multi-electron atoms?