8 Exam Question V2022

1. Quantum mechanical model of electrons in solids: free electron gas (30p)

   (a) Describe most important assumptions used to derive the quantum mechanical free electron gas model.

   (b) Consider Lithium metal. For an alkali metal, one normally assumes that one electron per atom is "free" to travel within the sample (i.e. one electron per atom is delocalised and contributes to the metallic bonding and the electrical conductivity).

   What is the reason for this assumption?

   (c) Imagine that we have a chain of Li atoms of length 1 cm. How many atoms (and how many delocalised electrons) does it contain?

   (d) Assuming that T = 0 K, what is the energy of the highest occupied state (i.e. the Fermi energy) for this 1D system?

   (e) What is the Fermi velocity for electrons in this 1D system? What is the energy separation of the states at the Fermi energy? Is this consistent with it being a metal at room temperature?

   (f) One of the most powerful applications of nanoscience is to be able to make structures such that the material properties are exactly what we want for a certain application.

   Imagine that you want to make an 3D object with a shape of a cube, which is metallic at 100°C, but poorly conducting at room temperature. If you should make it from lithium, what size should it be?

   Useful constants: Density of crystalline Lithium : $\rho_{Li} = 0.534\,\mathrm{g\,cm^{-3}}$, Li atomic radius 152 pm. M_W=6.94 g mol^{-1} .

2. WAVES 1 (20p)

   (a) Explain in what way, was the concept of standing waves important in the development of Bohr model of the hydrogen atom. What assumptions were needed. What were the main successes of the Bohr model? What did it fail to describe?

   (b) Why will a guitar string vibrate with only certain frequency/frequencies? Is it one or more? What physical parameters of the string are important to be able to predict the sound waved generated by such string? Explain and illustrate with drawings and equations.

   (c) What is defined by the term "wave packet". Illustrate with drawings, considering both frequency and spatial/time domains.

3. WAVES 2 (30p)

   (a) A model airplane is flying with a velocity of $20\,\mathrm{m\,s^{-1}}$ towards a wall. An observer is located 2000m from that wall and can hear the sound that comes directly from the airplane and the sound that is reflected from the wall. If the airplane motor generates a sound with a frequency of 1000 Hz, what frequencies will be heard by the observer assuming that there is no interference between direct and reflected waves. Speed of sound: $343\,\mathrm{m\,s^{-1}}$.

(b) What will change if we assume that there is interference between the direct wave and the reflected wave? Can you derive a formula that will describe the amplitude of the sound wave that can be heard by the observer? Assume that sound velocity is constant and does not depend on the sound frequency.

4. Answer one of the questions below (20p)

(a) Explain reasons for the diffraction limit in a optical system.

(b) Outline Planck's theory of Black body radiation

(c) What is the photoelectric effect and in what way was it important in the development of quantum mechanics.

(d) What is a diffraction grating? Derive formula that allows you to calculate wave intensity after it has passed through such grating.