

# i Kopi av FrontpageTFY4190

Department of Physics, NTNU

Examination paper for TFY4190 Instrumentation

Examination date: June 10, 2022

Examination time (from-to): 9-13

Permitted examination support material: A / All support material is allowed

Academic contact during examination: Steinar Raaen

Phone: 48 29 67 58

Technical support during examination: Orakel support services

Phone: 73 59 16 00

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can reopen and edit your answer as long as the test is open. If no questions are answered by the time the examination time expires, your answer will not be submitted. This is considered as “did not attend the exam”.

**Withdrawing from the exam:** If you become ill during the exam or wish to submit a blank answer/withdraw from the exam for another reason, go to the menu in the top right-hand corner and click “Submit blank”. This cannot be undone, even if the test is still open.

**Accessing your answer post-submission:** You will find your answer in Archive when the examination time has expired.

## 1 p1v3TFY4190v2022

The 2 complement representation of the decimal number -188 is:

**Select one alternative:**

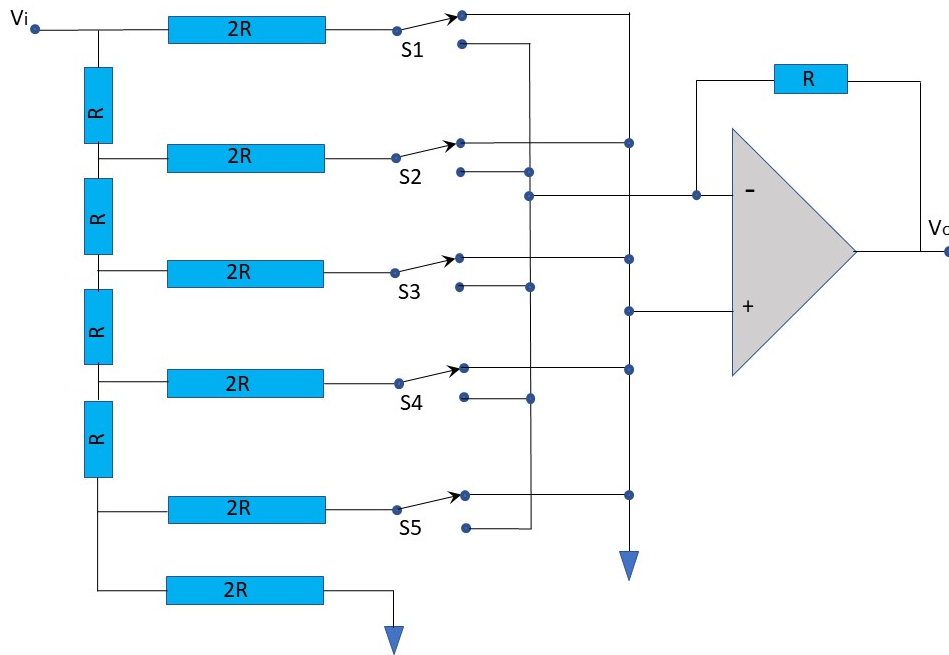
- 1011010101
- 1101100101
- 1010000110
- 101000100
- 1001100010

(2 points for correct answer)

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Maximum marks: 2

## 2 p2v2TFY4190v2022



The DAC circuit shown above consists of resistors with resistances  $R$  and  $2R$  as indicated, switches  $S1$  to  $S5$ , and an ideal operational amplifier. The reference voltage is  $V_i = 5V$  and the output voltage is  $V_o$ .

What is the ratio of currents running through  $S2$  and  $S5$  ( $I_{S2}/I_{S5}$ ) ?

Enter the answer here: .

What will the output voltage  $V_o$  be for a digital input corresponding to the number 24 ?

Enter the answer here:  V.

(2 points for each correct answer)

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Maximum marks: 4

**3 p3v2TFY4190v2022**

A double precision binary number consists of 64 bits. The MSB gives the sign, the next 11 bits the exponent, and the 52 remaining bits give the fraction. A bias of 1023 is used for the exponent. What is the decimal value of the double precision number given in hexadecimal form by BD28 C000 0000 0000 ?

Enter the  $\log_{10}|number|$  here:

Enter the sign of the number here (use 1 for positive sign, and -1 for negative sign):

(1 point for each correct answer)

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Maximum marks: 2

**4 p4v3TFY4190v2022**

The pressure in a vacuum chamber is measured 15 times. The results are: 2.1, 2.2, 2.0, 2.3, 1.9, 2.1, 2.2, 2.0, 1.8, 2.0, 2.3, 2.0, 2.1, 1.8, 2.1 (in units  $10^{-7}$  Pa). What is the precision in % for the sixth measurement (value  $2.1 \cdot 10^{-7}$  Pa) ?

**Select one alternative:**

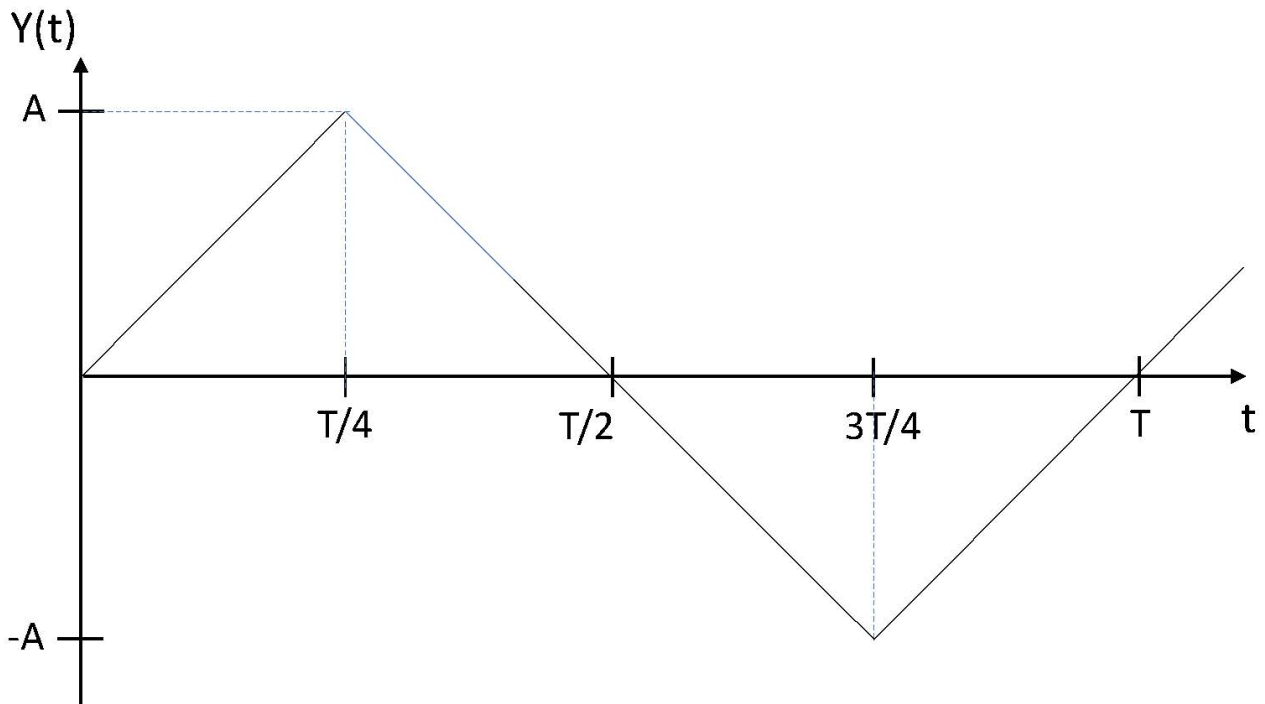
- 90 %
- 98 %
- 95 %
- 92 %
- 96 %

(2 points for correct answer)

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Maximum marks: 2

## 5 p5v1TFY4190v2022



One period of a periodic signal is shown in the figure above. The maximum value of the signal is  $A=4V$ , and the period is  $T$ .

What is the RMS value of the signal?

Enter the answer here:  V.

What is the rectified average of the signal?

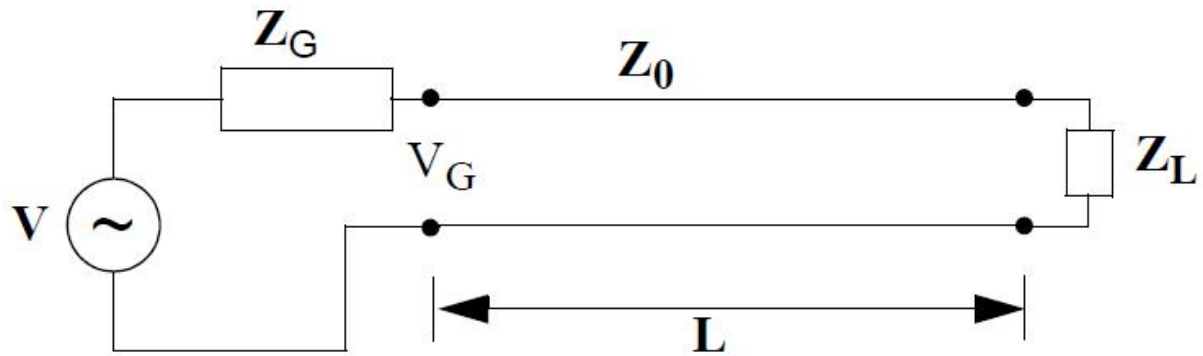
Enter the answer here:  V.

(1 point for each correct answer)

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Maximum marks: 2

## 6 p6v3TFY4190v2022



The figure above shows a generator circuit that transmit a high frequency signal on a transmission line of characteristic impedance  $Z_0 = 75\Omega$ . At the end of the line is a load impedance  $Z_L = 450\Omega + i200\Omega$ .

Which statement is correct?

**Select one alternative:**

- the voltage decreases when moving away from the load
- the voltage signal is a minimum at the load
- the voltage signal is a maximum at the load
- the voltage increases when moving away from the load

What is the magnitude of the voltage standing wave ratio (VSWR) ?

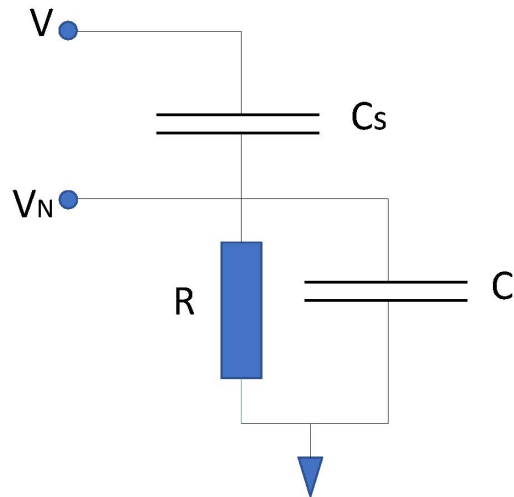
**Select one alternative**

- 7.2
- 5.1
- 8.3
- 6.5
- 4.2

(2 points for each correct answer)

Maximum marks: 4

## 7 p7v2TFY4190v2022



The input circuit of an amplifier consists of a  $R = 200\Omega$  resistor in parallel with a  $C = 100\text{pF}$  capacitor as shown in the figure above. A power line of frequency  $f = 650\text{Hz}$  and peak to peak voltage  $V = 500\text{V}$  is coupled to the input circuit by a stray capacitance  $C_s = 60\text{pF}$ , which results in a noise voltage of  $V_N$  at the input of the amplifier.

What is the amplitude of the noise voltage  $V_N$ ?

Select one alternative:

- 30.8mV
- 18.8mV
- 9.4mV
- 6.2mV
- 24.5mV

(4 points for correct answer)

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Maximum marks: 4

**8 p8v3TFY4190v2022**

A 20 bit ADC has operating range from -5 to +5V. The output signal is given in 2 complement binary format in hexadecimal representation  $V_{out} = 8CDE1$ .

What is the resolution of the ADC?

Enter the answer here:  mV.

What is the input voltage?

Enter the answer here:  V.

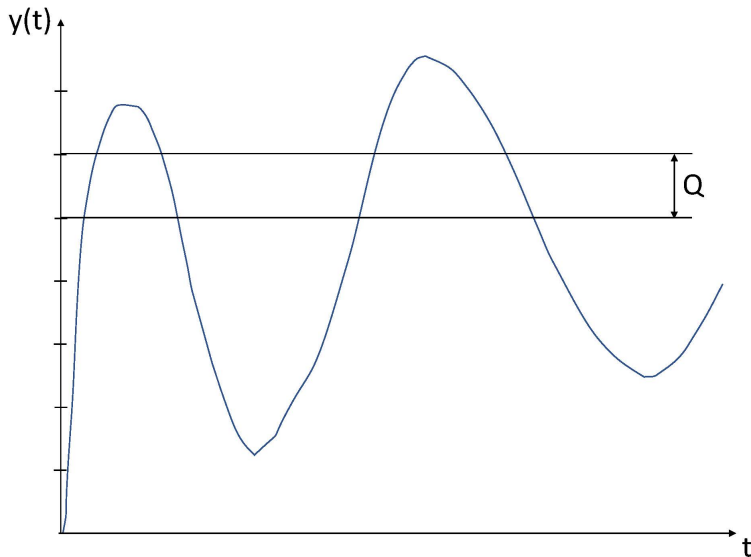
(2 points for each correct answer)

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Maximum marks: 4



## 9 p9v1TFY4190v2022



An analogue signal as shown in the figure above is to be digitized. The resolution of the ADC is  $Q = 2\text{mV}$ . Assuming uniform distribution of deviation from the analogue signal, what is the value of the quantization error of the digitized signal?

Select one alternative:

- 0.58 mV
- 1.45 mV
- 0.87 mV
- 0.95 mV
- 1.23 mV

The maximum rate of change of the signal is  $10\text{ V/s}$ . Estimate the maximum aperture time that is required for a proper sampling of the signal?

Select one alternative

- 0.08 ms
- 0.13 ms
- 0.03 ms
- 0.11 ms
- 0.05 ms

(1 point for each correct answer)

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Maximum marks: 2

**10 p10v3TFY4190v2022**

A sound signal has bandwidth from 15 to 25000Hz. Only frequencies up to 19000Hz are of interest for a particular application. Assume that a digital sampling is to be done of the signal. What should the minimum sampling frequency be to avoid aliasing?

**Select one alternative**

- 19000 Hz
- 25000 Hz
- 38000 Hz
- 50000 Hz

What should the cut-off frequency be for the low-pass filter?

**Select one alternative**

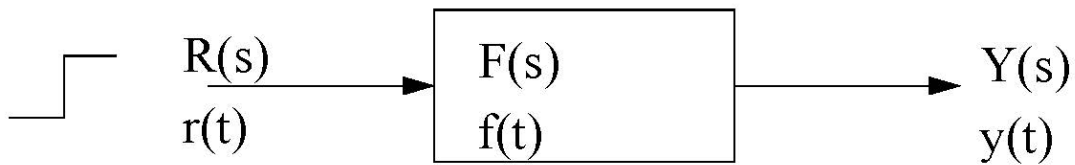
- 25000 Hz
- 19000 Hz
- 50000 Hz
- 38000 Hz

(1 point for each correct answer)

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Maximum marks: 2

## 11 p11v2TFY4190v2022



A step disturbance is impinging on a system of transfer function  $F(s)$  as shown above. The transfer function of the measurement system is given by:  $F(s) = \frac{s}{(s+1)(s+3)}$ . The output  $y(t)$  of the system is measured in volts.

What is the value of the maximum disturbance in  $y(t)$  ?

Enter the answer here:  V.

At what time does the maximum disturbance occur?

Enter the answer here:  s.

How large is the disturbance after  $t=5s$ ?

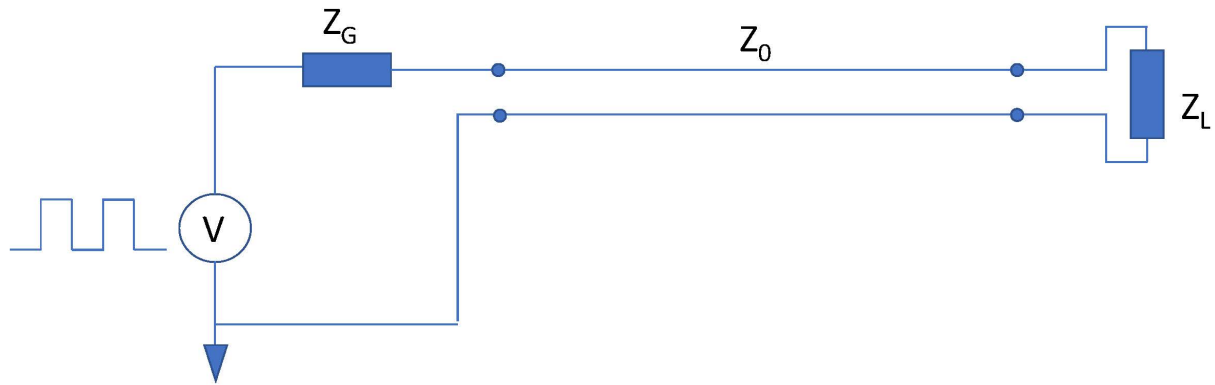
Enter the answer here:  mV.

(1 points for each correct answer)

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Maximum marks: 3

## 12 p12v1TFY4190v2022



A high frequency pulse train enters a transmission line as shown in the figure. At the end of the transmission line is a load. The line impedance  $Z_0 = 75\Omega$ , and the load impedance consists of a parallel coupling of resistor  $R = 200\Omega$  and a capacitor  $C = 0.4\mu F$ . The frequency of the pulse train is  $f = 1000Hz$ .

Find the reflection coefficients at the load. Give the answer in amplitude and phase angle.

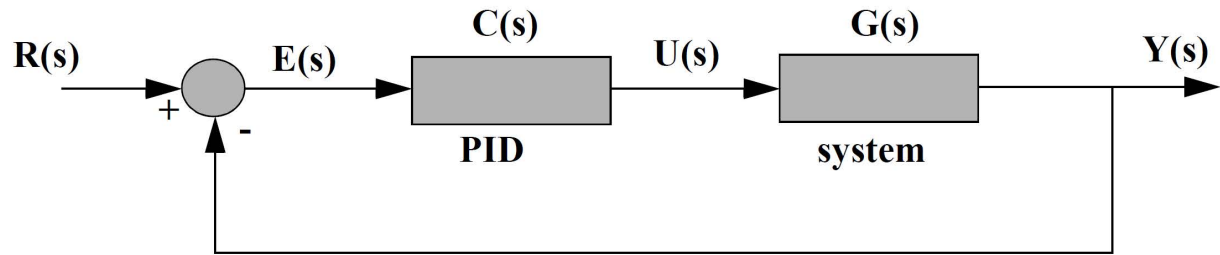
The amplitude of the reflection coefficient is: .

The phase of the reflection coefficient is: ° (degrees).

(2 points for each correct answer)

Maximum marks: 4

## 13 p13v2TFY4190v2022



A PID regulated system is shown in the figure above.

The transfer function of the system is given by  $G(s) = \frac{1}{s^3 + 2s^2 + 65s + 70}$

Assume proportional control and harmonic signal and find the angular frequency where the regulated system starts to oscillate.

Enter the answer here:  rad/s.

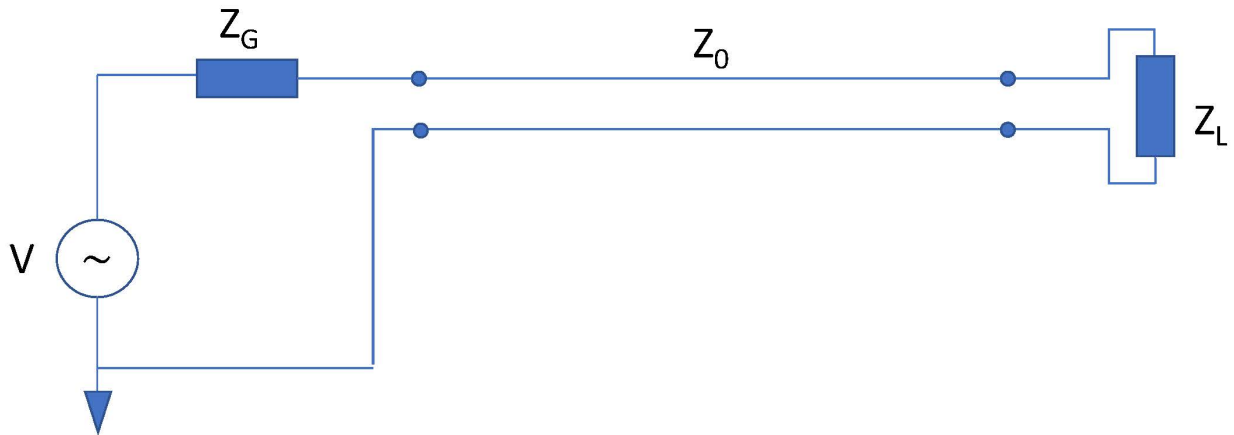
Find the period of the oscillations at this point.

Enter the oscillation period here:  s.

(2 points for each correct answer)

Maximum marks: 4

## 14 p14v2TFY4190v2022



A voltage signal of angular frequency  $\omega = 1 \cdot 10^8 \text{ s}^{-1}$  is incident on a transmission line of characteristic line impedance  $Z_0$ , as shown in the figure above. The inductance and capacitance per unit length of the transmission line are  $L = 250 \text{ nH/m}$  and  $C = 100 \text{ pF/m}$ , respectively. Assume loss-less transmission line. The load impedance is  $Z_L = 200 \Omega$ .

Find the magnitude of the line impedance  $Z(d)$  at distance  $d = 35 \text{ cm}$  from the load.

Select one alternative:

- 126  $\Omega$
- 148  $\Omega$
- 171  $\Omega$
- 179  $\Omega$
- 163  $\Omega$

(2 points for correct answer)

Maximum marks: 2

15 **p15v1TFY4190v2022**

A current of  $I = 0.2\text{nA}$  is to be measured using an electrometer that is kept at temperature  $T = 280\text{K}$ . The internal bandwidth of the electrometer is  $4\text{kHz}$  and the internal resistance is  $120\text{M}\Omega$ .

Estimate the thermal noise current:

**Select one alternative:**

- 0.72pA
- 0.83pA
- 0.76pA
- 0.66pA
- 0.91pA

Estimate the shot noise current:

**Select one alternative**

- 0.69pA
- 0.71pA
- 0.38pA
- 0.49pA
- 0.51pA

(1 point for each correct answer)

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Maximum marks: 2

**16 p16v2TFY4190v2022**

A signal of frequency  $f = 7000\text{Hz}$  is being sampled by sampling frequency  $f_s = 15000\text{Hz}$ . Which of the following frequencies may be present in the digitized signal in the absence of a low-pass filter?

**Select one or more alternatives:**

- 15000 Hz
- 14000 Hz
- 16000 Hz
- 22000 Hz
- 30000 Hz
- 38000 Hz

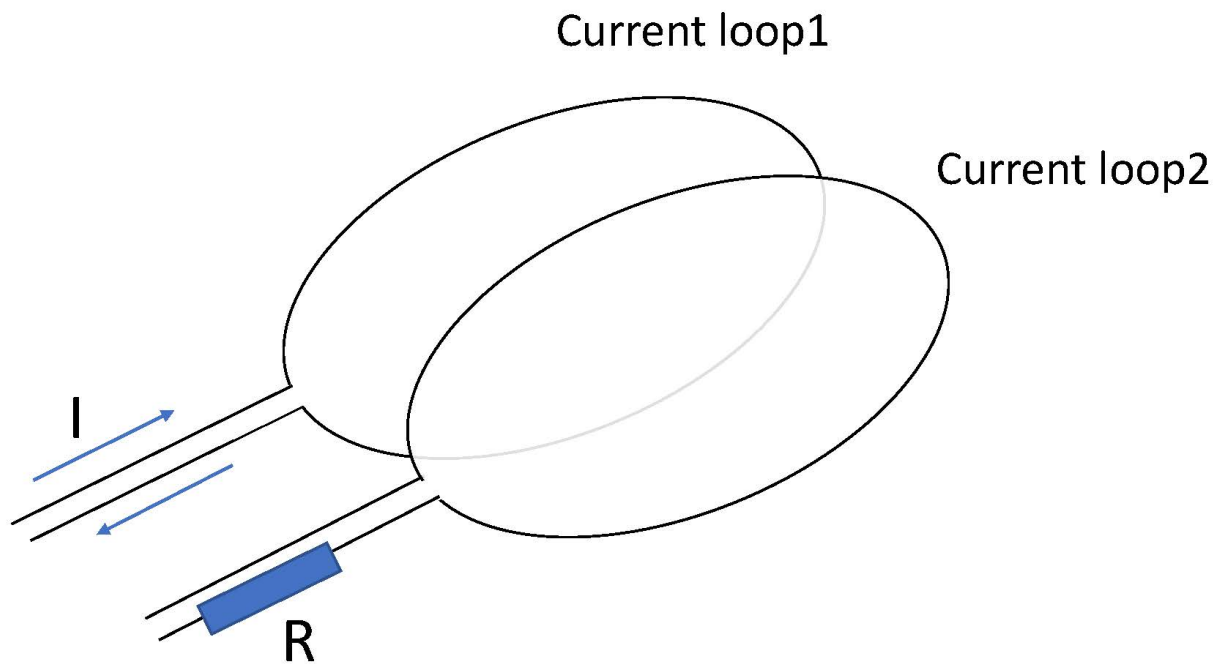
(1 points for each correct answer, -1 points for each wrong answer, minimum score is 0 points)

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Maximum marks: 3



## 17 p17v1TFY4190v2022



An **AC** current  $I=2\text{A}$  of frequency  $f=500\text{Hz}$  in **loop1** induces a voltage and current in nearby **loop2** as shown in the figure. The mutual inductance of the current loops is  $M=0.5\mu\text{H}$ .

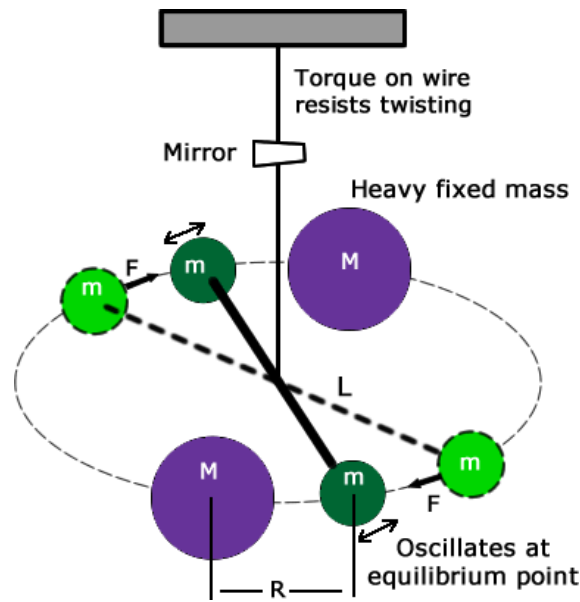
What is the magnitude of the voltage induced in loop1:  mV.

(2 points for correct answer)

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Maximum marks: 2

## 18 p18v3TFY4190v2022



The gravitational constant  $G = 2\pi^2 \frac{x \cdot R^2 \cdot L}{T^2 \cdot Y \cdot M}$  may be estimated from the Cavendish experiment as illustrated above. Here  $Y$  is the distance from mirror to ruler,  $x$  is the distance on the ruler which measures the angle of rotation,  $R$  is the distance between masses  $M$  and  $m$ ,  $L$  is the length of the torsion arm, and  $T$  is the period of oscillation.

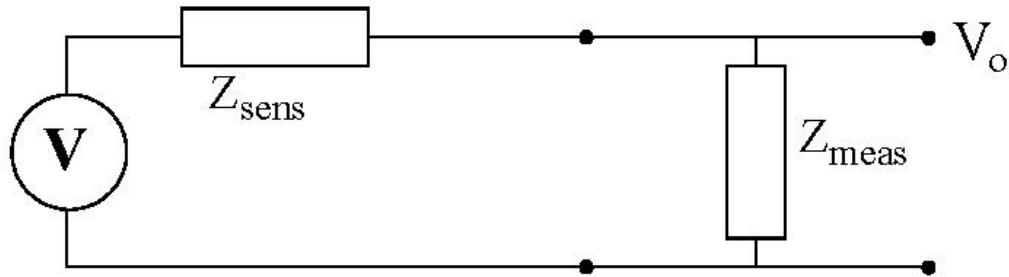
Use Gauss' law of error propagation to estimate the relative uncertainty in  $G$  when the relative uncertainties in the parameters are given by  $\frac{\Delta Y}{Y} = 0.002$ ,  $\frac{\Delta x}{x} = 0.004$ ,  $\frac{\Delta R}{R} = 0.001$ ,  $\frac{\Delta T}{T} = 0.002$ ,  $\frac{\Delta L}{L} = 0.002$  and  $\frac{\Delta M}{M} = 0.001$ .

Enter the answer in % here:  %.

(2 points for correct answer)

Maximum marks: 2

## 19 p19v1TFY4190v2022



An AC voltage source has amplitude  $V = 100V$  and output impedance  $Z_{sens} = (100 + j50)\Omega$ . The input of an amplifier circuit is described by the input impedance  $Z_{meas} = (500 - j100)\Omega$ . See figure above.

Find the power  $P$  which is transmitted into the amplifier circuit:  W.

(2 points for correct answer)

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Maximum marks: 2

### Problem1

v1:  $-414 = -(256+128+0+0+16+8+4+2+0) = -110011110 \rightarrow$  (add MSB) 0110011110  $\rightarrow$  (2komp) 1001100010  
v2:  $-378 = -(256+0+64+32+16+8+0+2+0) = -101111010 \rightarrow$  (add MSB) 0101111010  $\rightarrow$  (2komp) 1010000110  
v3:  $-188 = -(128+0+32+16+8+4+0+0) = -10111100 \rightarrow$  (add MSB) 010111100  $\rightarrow$  (2komp) 101000100

### Problem2

DAC R-2R ladder

v1:  $I_5/I_1 = 16; I_1 = V_i/2R. I = I_1 + I_3 + I_4 = I_1(1 + 1/4 + 1/8) = -V_o/R \Rightarrow V_o = -(V_i/2)11/8 = -3.44V$   
v2:  $I_5/I_1 = 8; I_1 = V_i/2R. I = I_1 + I_2 = I_1(1 + 1/2) = -V_o/R \Rightarrow V_o = -(V_i/2)3/2 = -3.75V$   
v3:  $I_5/I_1 = 4; I_1 = V_i/2R. I = I_1 + I_3 + I_4 = I_1(1 + 1/2 + 1/8) = -V_o/R \Rightarrow V_o = -(V_i/2)13/8 = -4.06V$

### Problem3

v1: BC28 C000 0000 0000 = 1|011 1100 0010|1000 1100 00.....  
sign -, exponent 0111 1000 010 = 962 -bias(1023)=-61, fraction= $1/2 + 1/32 + 1/64 = 0.546875$   
number =  $-1.546875 \cdot 2^{-61} = -6.7085 \cdot 10^{-19}$ ,  $-\log_{10}(|\text{number}|) = -18.17$   
v2: BD28 C000 0000 0000  $\Rightarrow -\log_{10}(|\text{number}|) = -13.36$   
v3: BE28 C000 0000 0000  $\Rightarrow -\log_{10}(|\text{number}|) = -8.54$

### Problem4

v1:  $\langle x \rangle = 2.067[\text{units}]$ ,  $p_n = 1 - |x_5 - \langle x \rangle| / |\langle x \rangle| = 0.919[\text{units}] = 91.9\%$   
v2: 94.94%  
v3: 98.06%

### Problem5

$$Y(t) = 4At/T, Y_{RMS} = \sqrt{\frac{4}{T} \int_0^T \left(\frac{4A}{T}t\right)^2 dt} = \frac{A}{\sqrt{3}} = 0.577A, Y_{rectified} = \frac{4}{T} \int_0^T \frac{4A}{T}t dt = \frac{A}{2} = 0.50A$$

v1:  $A=4, Y_{RMS}=2.31, Y_{rectified}= 2.00$   
v2:  $A=6, Y_{RMS}=3.46, Y_{rectified}= 3.00$   
v3:  $A=7, Y_{RMS}=4.04, Y_{rectified}= 3.50$

### Problem6

$$\Gamma_R = \frac{Z_L - Z_0}{Z_L + Z_0}, \quad |\Gamma_R| = \sqrt{\frac{(\text{Re}(Z_L) - Z_0)^2 + \text{Im}(Z_L)^2}{(\text{Re}(Z_L) + Z_0)^2 + \text{Im}(Z_L)^2}}, \quad VSWR = \frac{1 + |\Gamma_R|}{1 - |\Gamma_R|}$$

v1:  $Z_L = 375 + i300 \Omega$ , voltage increases away from load,  $VSWR = 8.28$   
v2:  $Z_L = 375 - i200 \Omega$ , voltage decreases away from load,  $VSWR = 6.47$   
v3:  $Z_L = 450 + i200 \Omega$ , voltage increases away from load,  $VSWR = 7.21$

### Problem7

$$V_N = V \frac{j\omega R C_s}{1 + j\omega R(C + C_s)}, \quad \left| \frac{V_N}{V} \right| = \frac{R}{\sqrt{\frac{1}{\omega^2 C_s^2} + R^2 \left(1 + \frac{C}{C_s}\right)^2}} \approx R\omega C_s$$

v1: R=200, f=600, C=100e-12; Cs=50e-12; V=500; V<sub>N</sub>=18.84mV

v2: R=200, f=650, C=100e-12; Cs=60e-12; V=500; V=500, V<sub>N</sub>=24.50mV

v3: R=200, f=700, C=100e-12; Cs=70e-12; V=500, V=500, V<sub>N</sub>=30.79mV

### Problem8

$$\text{resolution} = \text{LSB} = \frac{5 - (-5)}{2^{20} - 1} = 9.54e-6 \text{ V}$$

v1: ABC23 = 1010 1011 1100 0010 0011 =>

sign -, 2komp = 0101 0100 0011 1101 1101 =  $2^{18} + 2^{16} + 2^{14} + 2^9 + 2^8 + 2^7 + 2^6 + 2^4 + 2^3 + 2^2 + 2^0 = 345053$

input voltage = -345053 x 9.54e-6 = -3.29068 V

v2: 7AD3B = 0111 1010 1101 0011 1011 = 503099,

input voltage = +503099 x 9.54e-6 = +4.807V

v3: 8CDE1 = 1000 1100 1101 1110 0001 => 2comp = 0111 0011 0010 0001 1111 = 471583

input voltage = -471583 x 9.54e-6 = -4.499V

### Problem9

Uniform distribution  $f(y) = 1/Q = \text{constant}$ .

$$\text{Quantization error: } E_Q = \sqrt{\langle y^2 \rangle} = \sqrt{\int_{-\frac{Q}{2}}^{\frac{Q}{2}} y^2 f(y) dy} = \frac{Q}{\sqrt{12}} = 0.289Q$$

$$\text{Aperture (sampling) time: } \left(\frac{dy}{dt}\right)_{\max} = \frac{Q}{4\tau} \Rightarrow \tau = \frac{Q}{4\left(\frac{dy}{dt}\right)_{\max}}$$

v1: Q=2mV, E<sub>Q</sub>=0.578 mV, t = 0.050 ms

v2: Q=3mV, E<sub>Q</sub>=0.867 mV, t = 0.075 ms

v3: Q=5mV, E<sub>Q</sub>=1.445 mV, t = 0.125 ms

### Problem10

Nyquist sampling theorem

v1: f<sub>s</sub>=50000 Hz, f<sub>cut-off</sub>=19000 Hz

v2: same

v3: same

### Problem11

v1: Output is  $Y(s) = \frac{1}{s(s+1)(s+2)}$ , from Laplace table  $y(t) = \frac{1}{2-1}(e^{-t} - e^{-2t})$ ,  $y(5s) = 6.7\text{mV}$

$$\frac{dy}{dt} = 2e^{-2t} - e^{-t} = 0 \Rightarrow t = \ln(2) = 0.693, \text{ and } y_{\max} = e^{-\ln 2} - e^{-2\ln 2} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

v2:  $Y(s) = \frac{1}{s(s+1)(s+3)}$ , Laplace =>  $y(t) = \frac{1}{3-1}(e^{-t} - e^{-3t})$ ,  $y(5s) = 3.4\text{mV}$

$$\frac{dy}{dt} = \frac{1}{2}(3e^{-3t} - e^{-t}) = 0 \Rightarrow t = \frac{1}{2}\ln(3) = 0.549, \text{ and } y_{\max} = \frac{1}{2}(e^{-\frac{1}{2}\ln 3} - e^{-\frac{3}{2}\ln 3}) = \frac{1}{2}(3^{-\frac{1}{2}} - 3^{-\frac{3}{2}}) = 0.192$$

v3:  $Y(s) = \frac{1}{s(s+1)(s+4)}$ , Laplace =>  $y(t) = \frac{1}{4-1}(e^{-t} - e^{-4t})$ ,  $y(5s) = 2.3\text{mV}$

$$\frac{dy}{dt} = \frac{1}{3}(4e^{-4t} - e^{-t}) = 0 \Rightarrow t = \frac{1}{3}\ln(4) = 0.462, \text{ and } y_{\max} = \frac{1}{3}(e^{-\frac{1}{3}\ln 4} - e^{-\frac{4}{3}\ln 4}) = 0.157$$

**Problem12**

$$\frac{1}{Z_L} = \frac{1}{R} + j\omega C \Rightarrow Z_L = \frac{R}{1+j\omega RC} = \frac{R-j\omega R^2 C}{1+\omega^2 R^2 C^2} = A-jB$$

v1:

$$f=1000\text{Hz} \Rightarrow \omega=6280 \text{ s}^{-1}, C=0.4\mu\text{F}, R=200\Omega,$$

$$Z_L = 159.66 - j80.03, \Gamma_R = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{84.66 - j80.25}{234.66 - j80.25} = \frac{(1.055 - j)(2.924 + j)}{(2.924 - j)(2.924 + j)} = \frac{4.085 - j1.869}{9.55} = 0.4277 - j0.1957$$

$$|Z_L| = 0.470, \angle Z_L = \tan^{-1}(-0.1957/0.4277) = -24.59$$

$$v2: f=1500\text{Hz}, |Z_L| = 0.489, \angle Z_L = -35.96$$

$$v3: f=2000\text{Hz}, |Z_L| = 0.512, \angle Z_L = -46.43$$

**Problem13**

$$G(s) = \frac{1}{s^3 + 2s^2 + As + 70} \xrightarrow{s=j\omega} G(\omega) = \frac{1}{70 - 2\omega^2 + j(A\omega - \omega^3)} = \frac{70 - 2\omega^2 - j(A\omega - \omega^3)}{(70 - 2\omega^2)^2 + (A\omega - \omega^3)^2}$$

$$\angle G(\omega) = \tan^{-1}\left(\frac{\text{Im}}{\text{Re}}\right) = \tan^{-1}\left(\frac{\omega^3 - A\omega}{70 - 2\omega^2}\right), \text{ since } \tan(n180^\circ) = 0 \text{ we get } \omega(n180^\circ) = \sqrt{A} \text{ and } T = 2\pi/\omega$$

$$v1: A=50, \omega=7.07\text{s}^{-1}, T=0.89\text{s}$$

$$v2: A=65, \omega=8.06\text{s}^{-1}, T=0.78\text{s}$$

$$v3: A=80, \omega=8.94\text{s}^{-1}, T=0.70\text{s}$$

**Problem14**

$$\gamma = \alpha + j\beta \xrightarrow{\alpha=0} j\beta = j\omega\sqrt{LC}, Z(d) = Z_0 \frac{1 + \Gamma_R e^{-2\gamma d}}{1 - \Gamma_R e^{-2\gamma d}} = Z_0 \frac{1 + \Gamma_R \cos(2\beta d) - j\Gamma_R \sin(2\beta d)}{1 - \Gamma_R \cos(2\beta d) + j\Gamma_R \sin(2\beta d)}, Z_0 = \sqrt{\frac{L}{C}} = 50\Omega$$

$$|Z(d)| = Z_0 \sqrt{\frac{(1 + \Gamma_R \cos(2\beta d))^2 + (\Gamma_R \sin(2\beta d))^2}{(1 - \Gamma_R \cos(2\beta d))^2 + (\Gamma_R \sin(2\beta d))^2}}, \Gamma_R = \frac{Z_L - Z_0}{Z_L + Z_0} = 0.6$$

$$v1: d=25\text{cm}, Z(d)=179\Omega$$

$$v2: d=35\text{cm}, Z(d)=163\Omega$$

$$v3: d=45\text{cm}, Z(d)=148\Omega$$

**Problem15**

$$I_{\text{thermal}} = \sqrt{\frac{4k_B T \Delta f}{R}}, I_{\text{shot}} = \sqrt{2eI\Delta f}$$

$$v1: I=0.2\text{nA}, T=280\text{K}, \Delta f=4\text{kHz}, R=120\text{M}\Omega, I_{\text{thermal}}=0.72\text{pA}, I_{\text{shot}}=0.51\text{pA}$$

$$v2: I=0.15\text{nA}, T=290\text{K}, \Delta f=5\text{kHz}, R=140\text{M}\Omega, I_{\text{thermal}}=0.76\text{pA}, I_{\text{shot}}=0.49\text{pA}$$

$$v3: I=0.25\text{nA}, T=310\text{K}, \Delta f=6\text{kHz}, R=150\text{M}\Omega, I_{\text{thermal}}=0.83\text{pA}, I_{\text{shot}}=0.69\text{pA}$$

**Problem16**

$$v1: 16, 19, 38\text{kHz}$$

$$v2: 14, 15, 30\text{kHz}$$

$$v3: 22, 24, 48\text{kHz}$$

**Problem17**

$$\text{Faraday } V = M \frac{d(Ie^{j\omega t})}{dt} = j\omega MI \Rightarrow |V| = \omega MI$$

$$v1: I=2\text{A}, f=500\text{Hz}, M=0.5\mu\text{H}, V=3.14\text{mV}$$

$$v2: I=5\text{A}, f=1500\text{Hz}, M=0.2\mu\text{H}, V=9.42\text{mV}$$

$$v3: I=3\text{A}, f=1200\text{Hz}, M=0.3\mu\text{H}, V=6.78\text{mV}$$

**Problem18**

$$\frac{\Delta G}{G} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(2\frac{\Delta R}{R}\right)^2 + \left(\frac{\Delta L}{L}\right)^2 + \left(2\frac{\Delta T}{T}\right)^2 + \left(\frac{\Delta Y}{Y}\right)^2 + \left(\frac{\Delta M}{M}\right)^2}$$

v1: 0.57%

v2: 0.62%

v3: 0.67%

**Problem19**

$$P = R_{meas} I_{RMS}^2 = \frac{1}{2} R_{meas} |I|^2 = \frac{R_{meas}}{2} \left( \frac{V}{Z_{sens} + Z_{meas}} \right)^2 = \frac{1}{2} \frac{R_{meas} V^2}{(R_{sens} + R_{meas})^2 + (X_{sens} + X_{meas})^2}$$

v1: V=100V, Z<sub>sens</sub>=100+j50Ω, Z<sub>meas</sub>=500-j100Ω, => P=6.90W

v2: V=100V, Z<sub>sens</sub>=100+j50Ω, Z<sub>meas</sub>=500-j250Ω, => P=6.25W

v3: V=100V, Z<sub>sens</sub>=100+j50Ω, Z<sub>meas</sub>=400-j100Ω, => P=7.92W