NTNU
Norwegian University of Science and Technology
Department of Physics

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TFY4195 Optikk (Optics – basic course)

Examination May 31th, 2006, Time: 09.00 – 13.00

Allowed aid
Level C: “Specified printed and handwritten references allowed. Simple electronic calculator (scientific).”
Mathematical reference books are allowed, such as “BETA Mathematics Handbook” (Råde; Westergren) or “Matematisk Formelsamling” (Rottmann) or “Fysikaliske Størrelser” (Øgrim).
No lap-top computer, electronic notebook, or similar, is allowed. There is no particular restriction on electronic calculator capacity.
Correct solutions (suggestions) will be posted outside D5-107 during the examination.
Grades to be announced before June 21th, 2006.

The examination problems have been reviewed by an external “sensor” prior to the examination. The same sensor will review and pass all the examinations.

Sensor: Dr Ingve Simonsen, Institute for Transport and Economics, Dresden

Evaluation/grades
Total number of points of the written examination is 100. These will constitute the basis for evaluation. The following table recommended by NTNU will be used for converting to A, B, C, …-scale.
A: better than 85 points
B: better than 75 points
C: better than 65 points
D: better than 55 points
E: better than 35 points
F: less than 35 points

Section A: Problems to which ONLY answers shall be given (each max 7 pts). Hand in ONLY answers on separate sheets.

1: Youngs experiment:

a) Sketch the interference pattern in the observation plane (I(y)) formed in the Youngs two slit experiment (assume observation in the far zone), with a point source. [2p]

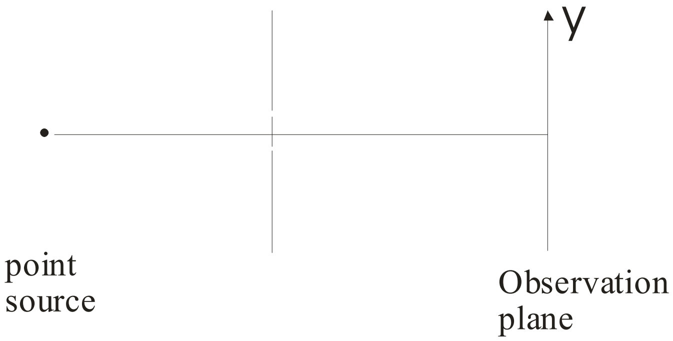


Figure 1.

b) Describe with words what happens to the interference pattern upon increasing the source width. [2p]

c) Sketch the *visibility* (or degree of coherence) as a function of the source width. [3p]

2: An object is placed 7 cm in front of a negative thin lens (f = - 4 cm) which in turn is placed 6 cm in front of a positive thin lens (f = 5 cm).

A) Determine the position and magnification of the image. [3p]

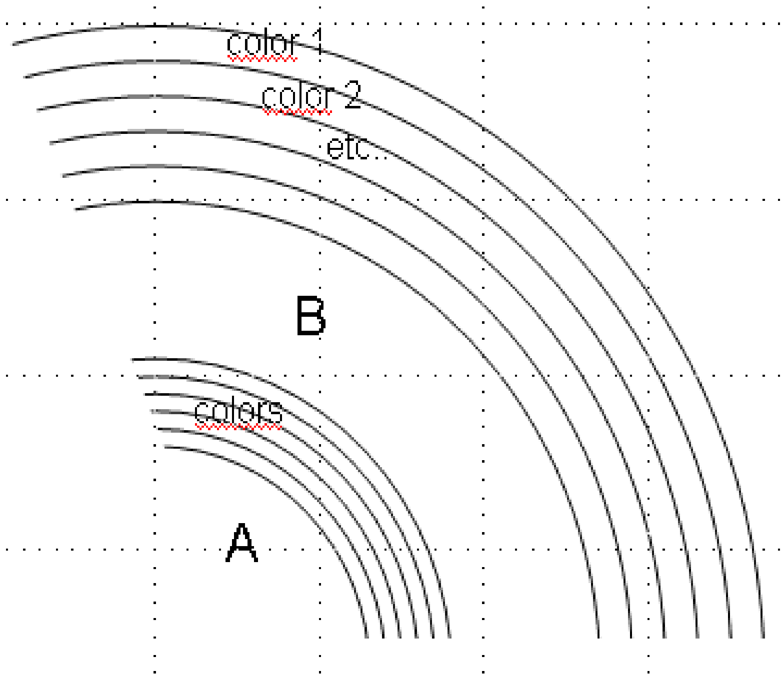
B) Provide a ray diagram via the intermediate image/object. [4p]

3: A long horizontal slit of height 0.5 mm is used to examine diffraction phenomena using a He-Ne laser (λ = 632.8 nm). The diffraction pattern is displayed on a screen 20 m away.

A) Sketch the shape of the observed diffraction pattern in relation to the horizontal slit. [3p]

B) Determine the distance between the first minima located on each side of the main spot. You may assume that the extension of the slit is "unlimited" in the horizontal direction. [4p]

4: A strong rainbow is formed and it is possible to also see the "secondary bow".
(not relevant in 2007 curriculm)
A) Draw the rainbows and fill in colors as indicated below. Use at least the colors green, red, yellow, blue and orange. [5p] (if you do not have color-pen, simply write the colors name)



B) Comparing the light backscattered from the sun from the regions A and B; which region appears being the brightest? [2p]

5: Optical discoveries
The following 4 scientists made great discoveries within optics. Ole Römer, Johannes Kepler, Thomas Young, James Clerk Maxwell.

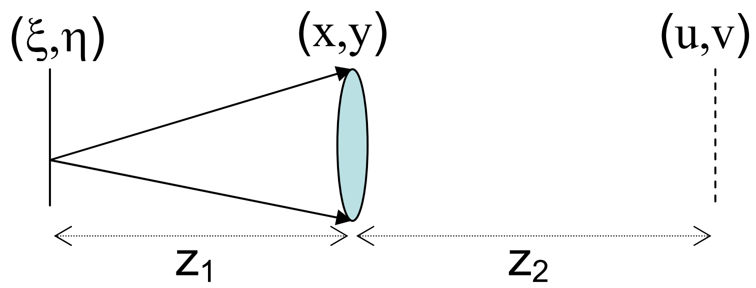
A) Approximate at what time did they live and were active (give a 50 years span for each). [3p]

B) Who made what discovery? Here are some hints: Considered the father of interference and first suggested that a light wave is "transversal" in nature. Created the theoretical foundation for light as an electromagnetic phenomenon and unified "dynamic" and "static" electromagnetics. He discovered the phenomenon "total internal reflection". He was first to prove that the speed of light is finite and also made a good estimate. [4p]

6: A) Write an expression for a linearly polarized light-wave of wavelength λ and amplitude E_0 propagating along the x-axis. The resulting amplitude is zero at t = 0 and x = 0. The plane of electric vibration is making 25^o with the xy-plane. [4p]

B) Write down the associated B-field component. [3p]

7: Generally, the diffraction phenomenon using a lens can be formulated by analyzing an array of individual point sources as shown below,



The point sources are located in the (ξ,η)-plane. Consider one point source emitting a spherical wave of the form: $\frac{e^{i(\bar{k}\cdot\bar{r}-\omega t)}}{r}$. Applying the small angle approximation (also called the paraxial or Fresnel approximation) and treating the influence of the thin lens using spherical surfaces resulting in the focal length *f,* the application of rigorous diffraction theory leads to the following formidable result:

$$U(u,v)=\frac{Ae^{i\gamma}}{\lambda^2 z_1 z_2}\iint_{lens\ aperture} e^{ik\left(\frac{1}{2}\left(\frac{1}{z_1}+\frac{1}{z_2}-\frac{1}{f}\right)(x^2+y^2)-\left[\left(\frac{\xi}{z_1}+\frac{u}{z_2}\right)x+\left(\frac{\eta}{z_1}+\frac{v}{z_2}\right)y\right]\right)}\,dx\,dy$$

The function U describes the field distribution in the (u,v) plane. ***A*** is a constant, λ the wavelength and the parameter γ contains phase-factors that are unimportant in case we consider only the intensity distribution.

A) What is the condition for obtaining the “clean” Fourier transform of the point source at z_2? [3p]

B) Identify and write down the spatial frequencies. [4p]

Section B: Problems to which also solutions must be handed in (each max 17 points). Provide derivations and answers on separate sheets.

8: Ray optics and system matrix applications

A Huygens eye-piece consists of two positive, thin lenses with focal lengths f_1 and f_2 (lens 1 left of lens 2) separated by a distance L equal to the average of their focal lengths, i.e., $L = \dfrac{f_1 + f_2}{2}$.

A) Assume en eye-piece with $f_1 = 3.125$ cm and $f_2 = 2.083$ cm. Determine the positions of the 6 cardinal points $N_1, N_2, F_1, F_2, H_1, H_2$. Draw an optical system with the cardinal points indicated roughly to scale. [6p]

B) Determine the position and magnification of an object placed 10 cm in front of the eye-piece. [6p]

C) Draw in the figure (A) the rays for image formation using the relevant cardinal points. [5p]

The following system matrices and relations might be useful:

Translation matrix: $$M = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}$$

Thin-lens matrix: $$M = \begin{bmatrix} 1 & 0 \\ -\dfrac{1}{f} & 1 \end{bmatrix}$$

Refraction matrix, spherical interface: $$M = \begin{bmatrix} 1 & 0 \\ \dfrac{n_1 - n_2}{R n_2} & \dfrac{n_1}{n_2} \end{bmatrix}$$

Refraction matrix, plane interface: $$M = \begin{bmatrix} 1 & 0 \\ 0 & \dfrac{n_1}{n_2} \end{bmatrix}$$

Lens-maker's formula: $$\frac{1}{f} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

There are more useful expressions on the last page.

9: Polarization components and their applications

A laser is giving out coherent vertical linearly polarized light. The appropriate normalized Jones vector is then given as: $J = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. A student heard that in order to control the laser output power as well as polarization in an experiment, it is necessary to use a polarizer and a so-called "wave-plate". The student is well aware of the quarter-wave plate (QWP). Its corresponding Jones matrix can be written as:

$$J_{QWP} = \begin{bmatrix} \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\cos 2\theta & \frac{i}{\sqrt{2}}\sin 2\theta \\ \frac{i}{\sqrt{2}}\sin 2\theta & \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\cos 2\theta \end{bmatrix}$$

The angle θ is here the angle between the fast axis of the wave-plate and the vertical axis, so it is possible to calculate its influence on polarized light for any angular position it is set to. However, the student has also heard of another kind of wave-plate, the half-wave plate (HWP). In an optics book he finds its Jones matrix defined in the same reference frame as,

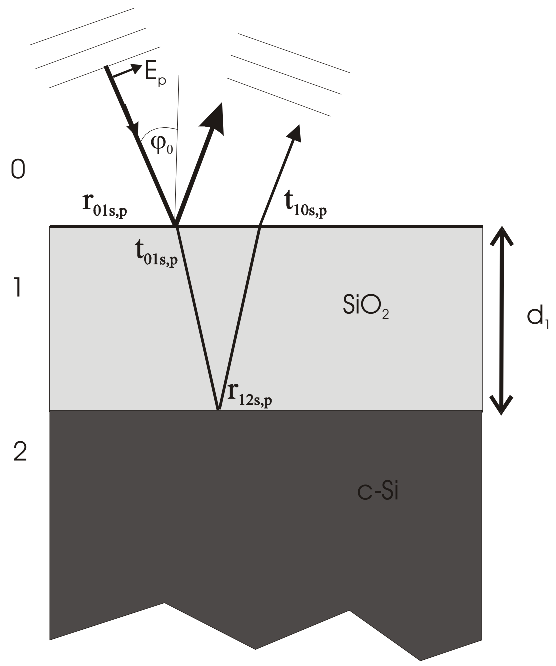
$$J_{HWP} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}.$$

A) Determine the Jones vector resulting from the linearly polarized laser beam described above going through the QWP for the three angles 0, 45^{o} and 90^{o}. Then do the same for the HWP. [5p]

B) How to arrange a polarizer and one wave-plate (which wave-plate?) to get "controlled" output power, always vertically polarized. It should be possible to control between 0 power (no light) and to the maximum output (we neglect influence from reflections in the optical components). Draw a scheme including the laser and the laser-beam and describe the basic function in words. [6p]

C) Determine the angular settings of the wave-plate and the polarizer in order to obtain 0, 50%, and 100% output power (always vertical). [6p]

10: Thin film interference patterns and reflectivity measurements



Consider the two reflected beams from a plane incident wave as indicated in Figure 1 (assume multiple reflections to be masked away). Assume that the phase shift $\delta = \frac{4\pi d_1}{\lambda}\sqrt{n_1^2 - n_0^2 \sin^2 \varphi_o}$ has already been derived from geometrical considerations. n_0 and n_1 is the refractive index of incident medium 0 (air, n_o=1), and the film respectively. φ_o is the angle of incidence, and λ is the free space wavelength.

A) Write an expression for the reflection coefficient for the film system for both s and p polarized light in terms of the single interface Fresnel reflection coefficients $\left(r_{012}^{s,p}\right)$. [6p]

B) For certain phases (δ), the reflection coefficient is that of the substrate with no film $\left(r_{02}^{s,p}\right)$. (Hint: what happens when the thickness d_1 goes to zero). State these phases, and give an expression for $\left(r_{02}^{s,p}\right)$ as a function of the Fresnel reflection and transmission coefficients in part (A). [6p]

C) Derive an expression for the reflectivity as a function of the Fresnel reflection and transmission coefficients, to obtain the form

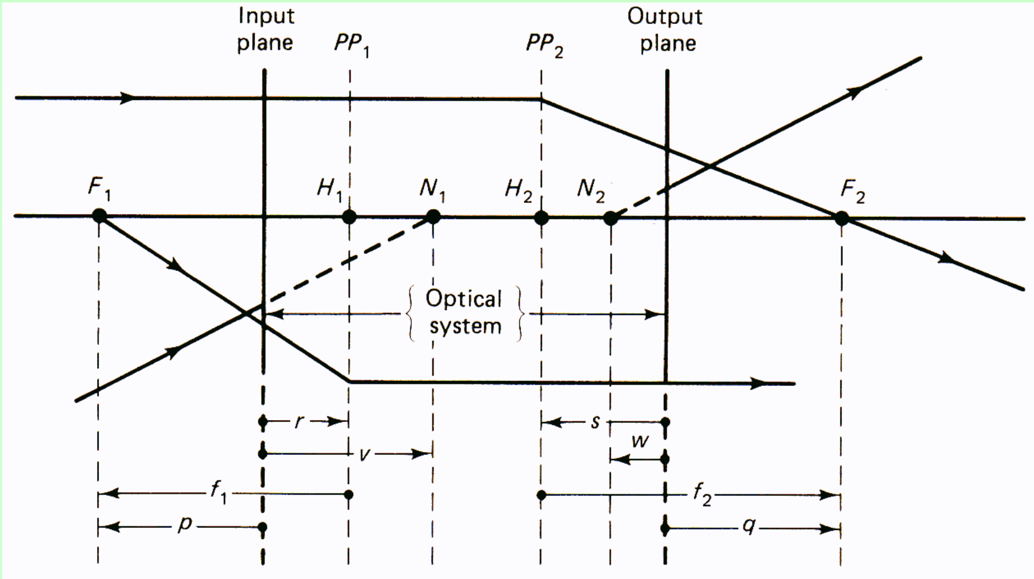
$$R_{s,p} = \left|r_{012}^{s,p}\right|^2 = a + b\cos(\delta)$$

[5p].

System matrix relations

$$\begin{bmatrix} y_f \\ \alpha_f \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_0 \\ \alpha_0 \end{bmatrix}$$

y: distance from optical axis (height)
α: angle (within small angle approxaimation)



Location designations for the six cardinal points of an optical system. The rays associated with the nodal points and principal planes are also shown.

$$\left.\begin{array}{lr} p = \dfrac{D}{C} & F_1 \\[2ex] q = -\dfrac{A}{C} & F_2 \\[2ex] r = \dfrac{D - n_0/n_f}{C} & H_1 \\[2ex] s = \dfrac{1 - A}{C} & H_2 \\[2ex] v = \dfrac{D - 1}{C} & N_1 \\[2ex] w = \dfrac{n_0/n_f - A}{C} & N_2 \end{array}\right\} \text{Located relative to input (1) and output (2) reference planes}$$

$$\left.\begin{array}{lr} f_1 = p - r = \dfrac{n_0/n_f}{C} & F_1 \\[2ex] f_2 = q - s = -\dfrac{1}{C} & F_2 \end{array}\right\} \text{Located relative to principal planes}$$

Cardinal point locations in terms of system matrix elements.