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NTNU
Norwegian University of Science and Technology
Department of Physics

Associate Professor Morten Kildemo, Phone: 73593211; Mobile: 93287744

TFY4195 Optikk (Optics – basic course)

Examination, May 26 th, 2009, Time: 09.00 – 13.00

Allowed aid
Level C: "Specified printed and handwritten references allowed. Simple electronic calculator (scientific)." Mathematical reference books are allowed, such as "BETA Mathematics Handbook" (Råde; Westergren) or "Matematisk Formelsamling" (Rottmann) or "Fysikaliske Størrelser" (Øgrim). No lap-top computer, electronic notebook, or similar, is allowed.

Evaluation/grades
Total number of points of the written examination is 100. These will constitute the basis for evaluation. The following table recommended by NTNU will be used for converting to A, B, C, …-scale.

A: 100-90 points
B: 89-80 points
C: 79-60 points
D: 59-50 points
E: 49-40 points
F: 39-0 points

Section A

1. Fundamental concepts [22p]

a) State Fermat's principle for a geometric ray. Describe the more accurate form of Fermat's principle. Give examples of both cases. [4p]

b) The focal length on the image side of a spherical refractive surface is given by:

$$\frac{P'}{n'} = \frac{1}{f'} = \frac{(n'-n)}{n'R}, \qquad \text{(A.1)}$$

where the radius of curvature R is positive for a convex surface (see the appendix).

State the main approximations involved in deriving equation (A.1). State the most important approximations involved in Gaussian first order optics. [5p]

c) Figure 2 shows the principal and focal points of a general optical system. State the definition of the principal planes (H and H'), the focal points (F and F'), and the Nodal points (N and N'). Use these definitions, and perform Graphical imaging (draw at least 3 rays, maximum 4) from an off axis object point (P_O) to an image point on the image side (P_I), see Figure A.1 and attached answer sheet. [7p]

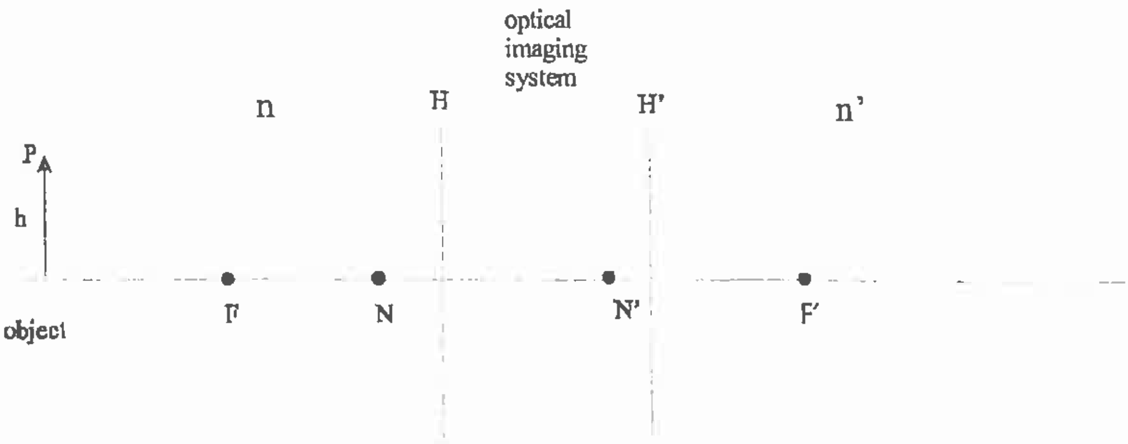


Figure A.1. General optical system, with principal and focal points. The object-space is on the left side, and the image space is on the right hand side.

d) Derive the Newtonian and Gaussian image equation from your figure in question (c). (You will need that $\frac{n'}{f'} = \frac{n}{f}$). Give also the transversal magnification. [6p]

A2. Analysis of optical systems – achromatic systems [25p]

a) Show (e.g. by using the paraxial system transfer matrices) that the lensmakers equation for a thin lens is given by

$$P = \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right), \qquad \text{(A2.1)}$$

where P is the refractive power and f is the focal length of the thin lens. [6p]

b) Explain the main effect of dispersion on an imaging system. [4p]

c) For a two lens system, separated by a distance L, see Figure A2a, find the paraxial system transfer matrix from the first to the last refracting surface. Give the expression for the refractive power for the combined system. [4p]

d) Let the lenses in (c) be made of the **same material**, but have **different refractive power** (different focal lengths). Find the distance L that makes the system achromatic in the neighbourhood of a design wavelength, λ_D. If we choose the calculated L in our design, what kind of optical system do you get? [5p]

e) The general compound lens in Figure A2.3 (unknown curvatures of surfaces) is achromatic under certain conditions. We require an overall positive lens (i.e. P>0) .Calculate the general conditions required for such a compound lens to be achromatic, with respect to $P_1(=1/f_1)$, $P_2(=1/f_2)$ and the dispersion. [6p]

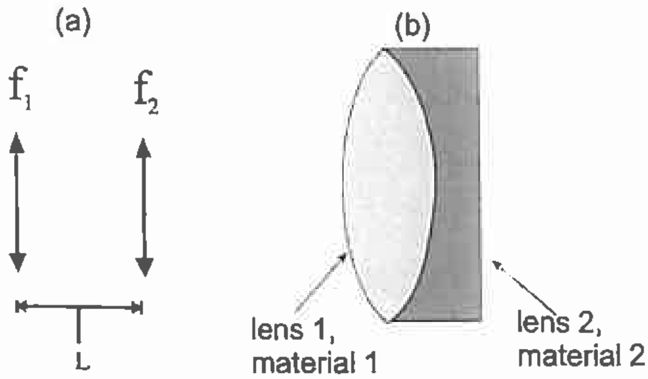


Figure A2. Figure (a) shows the two-lens system separated by a distance L. Figure (b) shows a sketch of a compound lens to be made achromatic.

Section B:

B1. General properties of waves –polarisation and Jones analysis. [20 p]

a) A plane s-polarized wave ($\perp$ to the incidence plane) is incident on a surface with an angle of incidence θ_0, with respect to the surface normal. The incident electric field is given by

$$\vec{E}(\vec{r},t) = \hat{e}_s E_{0s} \exp\left[i\left(\vec{k}\cdot\vec{r} - \omega t\right)\right]$$

Let the incidence plane be the x-z plane, see Figure B1. Derive the p-component of the **incident magnetic field** (|| to the incidence plane). Write the magnetic field along its Cartesian coordinates (x,y,z). [5p]

b) Describe at least three methods of producing linearly polarised light from an un-polarised light source. [5p]

c) A (uniaxial) quartz plate has been cut such that its optic axis is in the surface plane (i.e. a standard waveplate). The birefringence for quartz is given by $\Delta n = (n_e - n_o)$, with $n_e = 1.5462$ and $n_o = 1.5553$. Find the thickness necessary to produce a quarter wave plate and a half wave plate for λ=546 nm. [4p]

d) Incoherent (unpolarised) monochromatic light propagating along the z-axis, is incident on a polarizer, oriented with the transmission axis along the laboratory x-axis. The polarizer is followed by a waveplate, oriented with the fast axis at 45 degrees with respect to the transmission axis of the polarizer. Make a sketch of the system (including the laboratory coordinates). Let the retardance of the waveplate be $\delta = \frac{3\pi}{2}$. Find the outgoing polarisation state (Jones vector) defined by the laboratory coordinates (x,y). Sketch the polarisation ellipse of this state. (See the appendix for the definition of the polarisation ellipse). [6p]

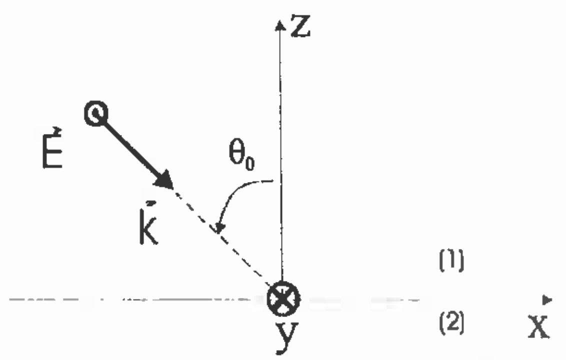


Figure B1. Standard geometry for question B1a. $\hat{e}_s$ is out of the paper, while the y axis is into the paper. The incidence plane is the x-z plane.

B3. X-ray reflectivity of Si3N4/c-Si [14p].

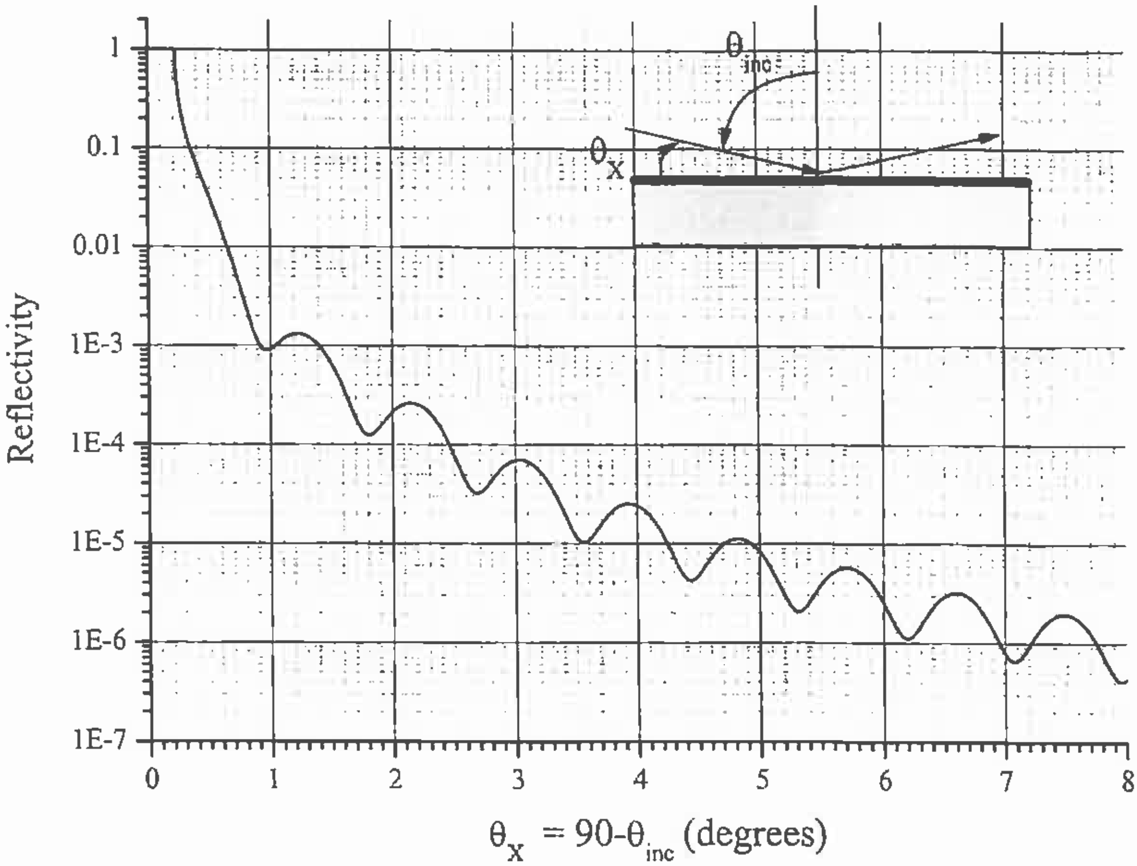


Figure B3. The reflectivity of a thin film of Si_3N_4 on c-Si. as measured with monochromatic light with wavelength λ=1.5406 Å.

Figure B3 shows the measurement* of a thin Si_3N_4 film on c-Si, using x-ray reflectivity, i.e. recording the specular beam as a function of grazing incidence angle (θ_X), see Figure B3. The surfaces are assumed perfectly smooth. It is assumed that we may model the reflectivity with the thin film Fresnel reflection model.

Let the wavelength of the monochromatic x-ray be λ=1.5406 Å. The refractive index of air is unity (n_{air}=1.0). The refractive index of Si_3N_4 is $n_{Si3N4} = 1.0 - 1.13\times10^{-5}$ and the refractive index of c-Si is $n_{c-Si} = 1.0 - 7.67\times10^{-6}$.

a) Write the reflection coefficient for the thin film on a substrate as a sum of reflected beams. You may assume known that the phase shift between reflected beams is $\frac{4\ d_{film}}{\quad} n_{film} \cos {}_{1}$. [5p]

b) Make an estimate of the thickness of the film from the interference fringes, by reading off Figure B3. [5p]

c) Why is the reflectivity unity for small θ_X angles. [4p]

* (Here simulated using the standard Fresnel formulas for a thin film on a substrate)

B4. Diffraction Grating and Fourier Optics [19p]

a) Using the Fraunhofer approximation, find the electric field and the intensity in the observation plane (X,Y,z), using the set-up in Figure B4.a, and the single rectangular aperture in Figure B4.b. [5p]

b) If we let Lx=20 μm, Ly= 40 μm, λ=633 nm, and z=2 m, verify the validity of the Fraunhofer approximation in the set-up in Figure B4.a. [4p]

c) We are about to use the set-up in Figure B4.a, but now to study N+1 rectangular apertures equally spaced by the distance a, as shown in Figure B4.c. Using the Fraunhofer approximation, find the electric field and the intensity, in terms of a, Lx, Ly, z and λ, in the observation plane (X,Y,z). [5p]

e) Let $a=\frac{3}{2}Ly$. Sketch the intensity, from part (c), in the observation plane along the Y-axis (i.e. for X=0). [5p]

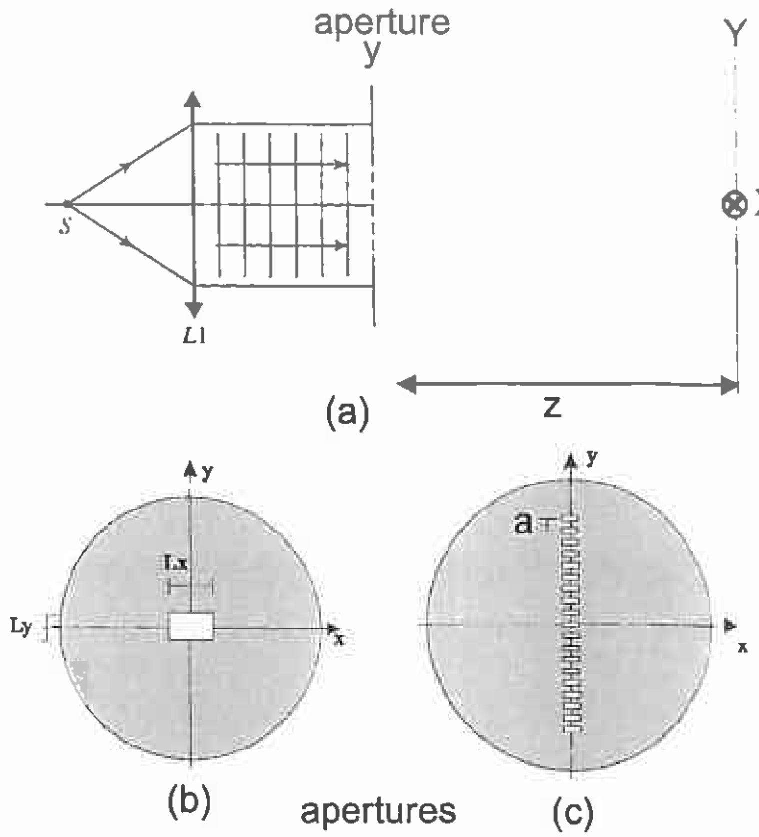


Figure B4. Figure (a) shows the set-up for Fraunhofer diffraction. A spatially coherent monochromatic source is collimated by lens 1. A plane wave falls onto the aperture. Figure (b) shows the rectangular aperture, dimensions Lx and Ly. Figure (c) shows the N+1 identical rectangular apertures, spaced by a distance, a, along the y axis.

Appendix

The First Rayleigh-Sommerfeld solution:

$$U(X,Y,z)=\frac{1}{i\lambda}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}U(x,y,0)\frac{\exp(ikr)}{r}\cos(\theta)dxdy$$

The Fresnel Diffraction integral is given by :

$$U(X,Y,z)=\frac{1}{i\lambda z}e^{ikz}e^{ik\frac{(X^2+Y^2)}{2z}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}U(x,y,0)\exp\left[\frac{ik}{2z}\left(x^2+y^2\right)-i\left(k_Xx+k_Yy\right)\right]dxdy$$

The 2D Fourier Transform of a general aperture transmission function is given by :

$$T(k_X,k_Y)=\mathfrak{F}\left\{t(x,y)\right\}=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}t(x,y)\exp\left[-i\left(k_Xx+k_Yy\right)\right]dxdy$$

The Fraunhofer diffraction integral is given by :

$$U(X,Y,z)=\frac{1}{i\lambda z}e^{ikz}e^{ik\frac{(X^2+Y^2)}{2z}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}U(x,y,0)\exp\left[-i\left(k_Xx+k_Yy\right)\right]dxdy$$

where

$$k_X=\frac{kX}{z},\ k_Y=\frac{kY}{z}\ ,\ k=\frac{2\pi}{\lambda}$$

or alternatively (depending on how the approximations are performed)

$$U(X,Y,z)=\frac{1}{i\lambda R}e^{ikR}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}U(x,y,0)\exp\left[-i\left(k_Xx+k_Yy\right)\right]dxdy$$

where

$$k_X=k\sin\theta_X=\frac{kX}{R}\approx\frac{kX}{z},\ k_Y=k\sin\theta_Y=\frac{kY}{R}\approx\frac{kY}{z}\quad ,\ R=\sqrt{X^2+Y^2+z^2}$$

Some useful functions in optics, Fourier Transforms and its properties

The circ function is defined by:

$$circ(\frac{\rho}{b})\ =\begin{cases}1 & for\ |\rho|<b\\ 0 & elsewise\end{cases}$$

The rect function is defined by :

$$rect(\frac{x}{w})\ =\begin{cases}1 & for\ |x|<\frac{w}{2}\\ 0 & elsewise\end{cases}$$

The dirac delta function

$$\delta(x) = \begin{cases} \infty & for\ x=0 \\ 0 & elsewise \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(x)dx = 1$$

The Fourier Transform of the circ function with radius d:

$$T(\kappa) = \mathfrak{F}\left\{circ(\frac{\rho}{d})\right\} = \pi d^2\, 2\frac{J_1(\kappa d)}{\kappa d},$$

where $\rho = \sqrt{x^2+y^2}$, and $\kappa = \sqrt{k_X{}^2 + k_Y{}^2}$,
and J_1 *is the Bessel function of first Kind.*

Table Appendix.1. *Properties of the "Besinc" function* $\left(2\frac{J_1(\pi B)}{\pi B}\right)$.

B	0	1.22	1.63	2.33	2.68	3.33
$2\frac{J_1(\pi B)}{\pi B}$	1	0	0.017	0	0.004	0

The Fourier Transform of the rect function

$$\mathfrak{F}\left\{rect(\frac{x}{w})\right\} = w\frac{\sin(uw/2)}{uw/2},$$

Properties of the sinc function :
Define

$$\operatorname{sinc}(x) = \frac{\sin(x)}{x},$$

Then $\operatorname{sinc}(x) = 1$ *for x=0,*

$|\operatorname{sinc}(x)|$ - *maximum for* $x=(2m+1)\pi/2$, $m=0,1,2,...$

$\operatorname{sinc}(x) \to 0$ *for* $x=m\pi$, $m=1,2,3,...$

The Fourier Transform of the Dirac delta function

$$\mathfrak{F}\{\delta(x)\} = 1$$

The Fourier transform of a comb function, (sum of 2N+1 delta functions):

$$\mathfrak{F}\left\{\sum_{n=-N}^{n=N} \delta(x-na)\right\} = \frac{\sin(2N+1)k_X a/2)}{\sin(k_X a/2)}$$

The function $\frac{\sin^2((2N+1)k_X a/2)}{\sin^2(k_X a/2)} \to (2N+1)^2$, *for* $k_X a/2 = m\pi$, *where* $m = 0, \pm 1, \pm 2, ...$

The shift property

$$\mathfrak{F}\{f(x-x_0)\} = \exp(ik_x x_0)\,\mathfrak{F}\{f(x)\}$$

The convolution theorem :

The convolution is defined by : $$g(x) = \int_{-\infty}^{\infty} f(\xi)h(x-\xi)d\xi = f(x)\otimes h(x)$$

then by the convolution theorem $$\mathfrak{F}\{g(x)\} = \mathfrak{F}\{f(x)\}\mathfrak{F}\{h(x)\}$$

The sifting property of the Dirac delta function :

$$\int_{-\infty}^{\infty} \delta(x-a)f(x)dx = f(x-a)$$

The Linearity + scaling property

$$\mathfrak{F}\{\alpha g(x)+\beta f(x)\} = \alpha\mathfrak{F}\{g(x)\}+\beta\mathfrak{F}\{f(x)\}$$

Fresnel reflection and transmission coefficients:

$$r_\perp \equiv r_s = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t}; \quad t_\perp \equiv t_s = \frac{2n_i\cos\theta_i}{n_i\cos\theta_i + n_t\cos\theta_t}$$

$$r_\parallel \equiv r_p = \frac{n_t\cos\theta_i - n_i\cos\theta_t}{n_i\cos\theta_t + n_t\cos\theta_i}; \quad t_\parallel \equiv t_p = \frac{2n_i\cos\theta_i}{n_i\cos\theta_t + n_t\cos\theta_i}$$

Boundary conditions for reflection/transmission at interface:

$$B_{2z} - B_{1z} = 0$$

$$D_{2z} - D_{1z} = \varepsilon_2 E_{2z} - \varepsilon_1 E_{1z} = 0$$

$$\mathbf{E}_{2t} - \mathbf{E}_{1t} = 0$$

$$\mathbf{H}_{2t} - \mathbf{H}_{1t} = (\mathbf{B}_{2t} - \mathbf{B}_{1t})/\mu_0 = 0$$

Maxwell's equations:

$$\nabla\times\mathbf{E} + \frac{\partial\mathbf{B}}{\partial t} = 0$$

$$\nabla\times\mathbf{H} - \frac{\partial\mathbf{D}}{\partial t} = 0$$

$$\nabla\cdot\mathbf{B} = 0$$

$$\nabla\cdot\mathbf{D} = 0$$

Material equations

$$\mathbf{D} = \varepsilon\mathbf{E}$$

$$\mathbf{B} = \mu\mathbf{H}$$

The ray equation is given by :

$\nabla n(x) = \frac{d}{d\sigma}(n\bar{s})$, where $\bar{s} = \frac{d\bar{r}}{d\sigma}$, σ is the geometric path length and $\bar{r} = (x, y, z)$.

Polarisation, Jones matrices, definition of polarisation ellipse

Linear Polarizer:

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

General waveplate:

$$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\varepsilon} \end{bmatrix}$$

Rotation matrix :

$$R(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

Jones Matrix T rotated by α:

$$R(-\alpha)TR(\alpha)$$

Jones vector for a polarisation ellipse along its principal axis (x',y') in the Figure Pol. ellipse :

$$\overline{J}_{x',y'} = \begin{bmatrix} \cos\varepsilon \\ i\sin\varepsilon \end{bmatrix} = \cos\varepsilon \begin{bmatrix} 1 \\ i\tan\varepsilon \end{bmatrix}, \; where \; \tan\varepsilon = \frac{E_{0y'}}{E_{0x'}}$$

The tilted polarisation ellipse Jones vector along the laboratory coordinates is given by :

$$\overline{J}_{x,y} = R(-\alpha)\overline{J}_{x',y'}$$

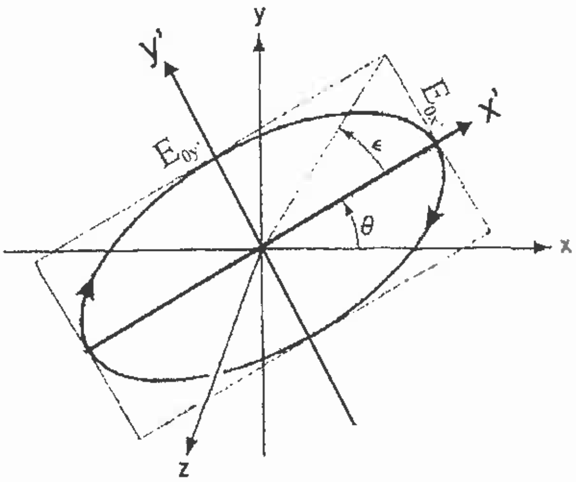
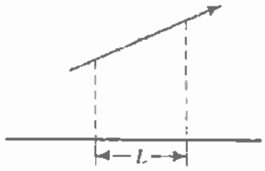


Figure Polarisation ellipse.

TABLE 18-1 SUMMARY OF SOME SIMPLE RAY-TRANSFER MATRICES

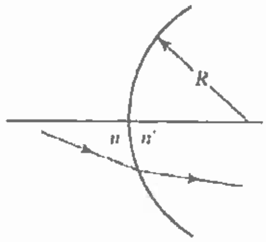
Translation matrix:

$$M = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}$$



Refraction matrix,
spherical interface:

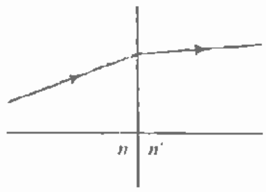
$$M = \begin{bmatrix} 1 & 0 \\ \dfrac{n-n'}{Rn'} & \dfrac{n}{n'} \end{bmatrix}$$



(+R): convex
(−R): concave

Refraction matrix,
plane interface

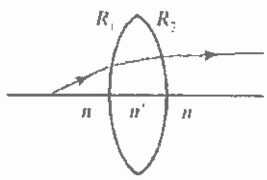
$$M = \begin{bmatrix} 1 & 0 \\ 0 & \dfrac{n}{n'} \end{bmatrix}$$



Thin-lens matrix:

$$M = \begin{bmatrix} 1 & 0 \\ -\dfrac{1}{f} & 1 \end{bmatrix}$$

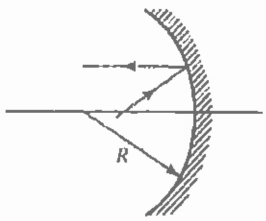
$$\frac{1}{f} = \frac{n'-n}{n}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$



(+f): convex
(−f): concave

Spherical mirror
matrix:

$$M = \begin{bmatrix} 1 & 0 \\ \dfrac{2}{R} & 1 \end{bmatrix}$$



(+R): convex
(−R): concave

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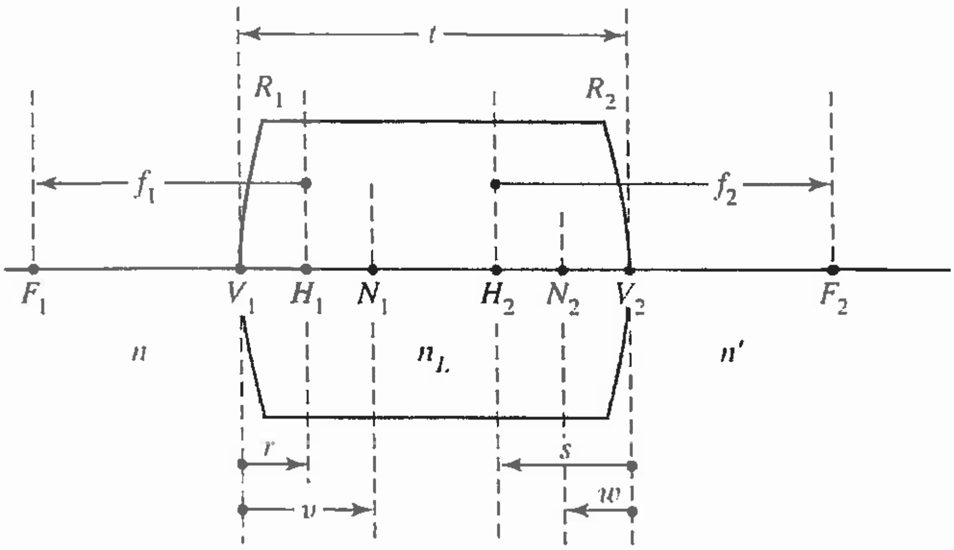

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TABLE 18-2 CARDINAL POINT LOCATIONS IN TERMS OF SYSTEM MATRIX ELEMENTS

$p = \frac{D}{C}$	F_1	Located relative to input (1) and output (2) reference planes
$q = -\frac{A}{C}$	F_2	
$r = \frac{D - n_0/n_f}{C}$	H_1	
$s = \frac{1 - A}{C}$	H_2	
$v = \frac{D - 1}{C}$	N_1	
$w = \frac{n_0/n_f - A}{C}$	N_2	
$f_1 = p - r = \frac{n_0/n_f}{C}$	F_1	Located relative to principal planes
$f_2 = q - s = -\frac{1}{C}$	F_2	

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Recall that in table 18-2, all distances are taken as positive to the right of the plane it refers to, and negative to the left of the plane it refers to.

TABLE 2-1 SUMMARY OF GAUSSIAN MIRROR AND LENS FORMULAS

	Spherical surface	Plane surface
Reflection	$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f},\ f=-\frac{R}{2}$ $m=-\frac{s'}{s}$ Concave: $f>0,\ R<0$ Convex : $f<0,\ R>0$	$s'=-s$ $m=+1$
Refraction Single surface	$\frac{n_1}{s}+\frac{n_2}{s'}=\frac{n_2-n_1}{R}$ $m=-\frac{n_1 s'}{n_2 s}$ Concave: $R<0$ Convex : $R>0$	$s'=-\frac{n_2}{n_1}s$ $m=+1$
Refraction Thin lens	$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ $\frac{1}{f}=\frac{n_2-n_1}{n_1}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ $m=-\frac{s'}{s}$ Concave: $f<0$ Convex : $f>0$	

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Newtonian imaging equation :

$$ff'=zz'$$

where z,z' refers to focal points.

Answer sheet, question A1c.