NTNU
Det skapende universitet

Department of Physics

Exam TFY4195 Optics

Contact during exam:	**Dag Werner Breiby**
Tel.:	**9845 4213**
Exam date:	**23 May 2016**
Exam time (from-to):	**09:00 – 13:00**

Allowed aids:

Code C:
Approved calculator, with empty memory.
K. Rottmann: Matematisk Formelsamling
S. Barnett & T.M. Cronin: Mathematical Formulae
O. Øgrim & B.E. Lian: Størrelser og enheter i fysikk og teknikk

Other information:

The exam set is made by prof. Dag W. Breiby and proofread by Eirik T. Skjønsfjell and prof. Emil J. Samuelsen. Every subtask a), b), etc. of exercise 1-3 counts equally, summing up to 100 % for the 10 subtasks.

Language:	**English**
Number of pages:	**Total of 6 pages**
Number of pages attachment:	**2 pages (formulae)**

Controlled by:

Date Sign

Task 1

a) Outline the traditional statement of Fermat's principle. How can Fermat's principle be understood in terms of modern physics concepts? By using mirrors, the same object can be observed simultaneously in different directions – does this violate Fermat's principle?

b) A camera has a single thin lens of focal length f = 50.0 mm. A 1.70 m tall woman stands 10.0 m in front of the camera. Calculate the length of the image of the woman (expression and numerical answer).

c) A ray of natural light comes in towards an air-glass interface with an incidence angle of 40°. Calculate the reflectance. Assume n = 1.55 for the glass.

Task 2 Geometrical optics

A thin lens L of focal length $f > 0$ stands in air (with index of refraction $n = 1$). The lens is mounted in a frame with inner diameter D_L .

a) Show that the matrix **M** given by

$$\mathbf{M} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} A & \dfrac{-1}{f} \\ s_1 + s_2 - \dfrac{s_1 s_2}{f} & 1 - \dfrac{s_2}{f} \end{pmatrix} \qquad (1)$$

describes the path of rays from the object plane in a distance $s_1 > f$ in front of the lens to a plane at a distance s_2 after the lens. Determine also the matrix element A.

What conditions must generally be fulfilled for this kind of matrix formalism to be valid?

b) We now put an aperture with adjustable circular opening of radius r_B in the plane X a distance x after the lens, with $0 < x \le f$.
Assume that a ray starts on the optical axis in the object plane and is marginal at L (that is, it just barely makes within the frame of the lens).
- Show by using the matrix from a) that the y-position of this ray in the plane X is given by $y_X = \frac{1}{2} D_L x \left(\frac{1}{s_1} + \frac{1}{x} - \frac{1}{f} \right)$.
- Use this to express a condition for r_B which makes the aperture the *aperture stop* of the system.
- Find expressions for the location of the entrance and exit pupils of the system (given that the aperture is the aperture stop).

c) Explain shortly the connection between the stop and a *telecentric* system for the special case that the matrix element $D = 0$ in equation (1). What is the practical understanding of a telecentric system?

Task 3 Diffraction by gratings and a hole

a) Show that the diffraction angles for a planar grating consisting of parallel slits are given by $m\lambda = d(\sin\theta_r - \sin\theta_i)$, where *m* is the diffraction order, λ the wavelength of the incoming light, θ_i is the angle of the incoming ray, and θ_r is the angle of the outgoing (diffracted) ray. A 30 mm long diffraction grating yields 33° for 2. order diffraction with the incoming light perpendicular to the grating, using light with λ = 600 nm. How many slits does the diffraction grating have in total?

b) Assume that we have a diffraction grating with a large number of slits with repetition distance *d*, illuminated by coherent incoming light with a planar wavefront.
- Sketch qualitatively what the diffraction pattern will look like in the Fraunhofer regime if the slits are assumed to be very narrow compared with the distance between them.
- Next, use the convolution theorem to explain how the pattern is modified if each individual slit is *not* narrow, but rather has a width equal to 1/4 of the distance between the slits.

c) The diffracted field in task b) is imaged by setting a lens of focal length *f* immediately after the grating. Explain shortly where any diffraction peaks can be observed in this case.

d) When light goes through a circular aperture, one gets a Fraunhofer diffraction pattern given by

$$I \propto \left[\frac{2J_1(ka\rho / R)}{ka\rho / R}\right]^2, \qquad (1)$$

where J_1 is a Bessel function of the first order, *k* is the wave number, *a* is the radius of the aperture, ρ is the radius in the detector plane and *R* is the distance from the aperture to the detector plane. It is given that the first zero of $J_1(u) = 0$ is with u = 3,83.

- Show with the necessary calculations how this leads to the Rayleigh criterion $(\Delta\varphi)_{\min} = 1{,}22\lambda / 2a$, where $(\Delta\varphi)_{\min}$ is the smallest resolved angle. Make a sketch!
- The primary mirror of the Hubble telescope has a diameter of 2.4 m. What is the shortest separation between two object that can be resolved on Saturn? (The shortest distance to Saturn from Earth is 1.2 billion km. Assume λ = 550 nm.)

Formelliste for emnet TFY4195 Optikk (VEDLEGG)

Vektorstørrelser er i **uthevet** skrift.

Fysiske konstanter:

Ett mol: $M(^{12}C) = 12$ g	$1u = 1{,}6605 \cdot 10^{-27}$ kg	$N_A = 6{,}0221 \cdot 10^{23}$ mol^{-1}
$k_B = 1{,}3807 \cdot 10^{-23}$ J/K	$R = N_A k_B = 8{,}3145$ J mol^{-1} K^{-1}	$0°C = 273{,}15$ K
$\varepsilon_0 = 8{,}8542 \cdot 10^{-12}$ C^2/Nm^2	$\mu_0 = 4\pi \cdot 10^{-7}$ N/A^2	
$e = 1{,}6022\cdot 10^{-19}$ C	$m_e = 9{,}1094 \cdot 10^{-31}$ kg	
$c = 2{,}998 \cdot 10^8$ m/s	$h = 6{,}6261 \cdot 10^{-34}$ Js	$g = 9{,}81$ m/s^2

Elektrisitet og magnetisme:

$$\oint_S \mathbf{E}\cdot d\mathbf{A} = \oint_S E_n dA = \frac{Q_{inni}}{\varepsilon_0} \qquad \nabla\cdot\mathbf{D} = \rho$$

$$\oint_S \mathbf{B}\cdot d\mathbf{A} = \oint_S B_n dA = 0 \qquad \nabla\cdot\mathbf{B} = 0$$

$$\oint_C \mathbf{E}\cdot d\mathbf{s} = \mathcal{E} = -\frac{d\Phi_m}{dt} = -\frac{d}{dt}\int_S B_n dA = -\int_S \frac{\partial B_n}{\partial t} dA \qquad \nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\oint_C \mathbf{B}\cdot d\mathbf{s} = \mu_0(I_{inni} + I_d),\ I_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \int_S \frac{\partial E_n}{\partial t} dA \qquad \nabla\times\mathbf{B} = \mu_0 J + \mu_0\varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\mathbf{S} = c^2\varepsilon_0 \mathbf{E}\times\mathbf{B} \qquad I \equiv \langle S\rangle_T = \frac{\varepsilon_0 c}{2} E_0^2 = \varepsilon_0 c \langle E^2\rangle_T$$

$$\mathbf{D} = \varepsilon\mathbf{E} \qquad \mathbf{B} = \mu\mathbf{H}$$

Bølger, refraksjon og refleksjon:

$$\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \qquad (\nabla^2 + k^2)U = 0 \qquad I(r) = |U(r)|^2 \qquad I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

$$U(\mathbf{r}) = \exp(\mathrm{i}\,\mathbf{k}\cdot\mathbf{r}) \qquad U(r) = \frac{A}{r}\exp(ikr) \approx \frac{A}{z}\exp(ikz)\exp\left[ik\frac{x^2+y^2}{2z}\right]$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$r_s = r_\perp = \left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t} \qquad r_p = r_\parallel = \left(\frac{E_{0r}}{E_{0i}}\right)_\parallel = \frac{n_t\cos\theta_i - n_i\cos\theta_t}{n_i\cos\theta_t + n_t\cos\theta_i}$$

$$t_s = t_\perp = \left(\frac{E_{0t}}{E_{0i}}\right)_\perp = \frac{2n_i\cos\theta_i}{n_i\cos\theta_i + n_t\cos\theta_t} \qquad t_p = t_\parallel = \left(\frac{E_{0t}}{E_{0i}}\right)_\parallel = \frac{2n_i\cos\theta_i}{n_i\cos\theta_t + n_t\cos\theta_i}$$

_____ Jones og Stokes vektorer:_____

$$\mathbf{E} = \begin{bmatrix} E_x(t) \\ E_y(t) \end{bmatrix} \qquad \mathbf{S} = \begin{bmatrix} 2I_0 \\ 2I_1 - 2I_0 \\ 2I_2 - 2I_0 \\ 2I_3 - 2I_0 \end{bmatrix}$$

_____ Geometrisk optikk:_____

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad \frac{1}{f} = \frac{n_{lens} - n_{medium}}{n_{medium}} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\begin{bmatrix} n\alpha \\ y \end{bmatrix} \qquad R_1 = \begin{pmatrix} 1 & -P_1 \\ 0 & 1 \end{pmatrix}, \; P_1 = \frac{n_{t1} - n_{i1}}{R} \;\; T_{21} = \begin{pmatrix} 1 & 0 \\ d_{21} / n_{t1} & 1 \end{pmatrix}$$

_____ Diffraksjon:_____

$$U(X,Y,z) = \frac{1}{i\lambda z} e^{ikz} e^{ik\frac{(X^2+Y^2)}{2z}} \iint U(x,y,0) \exp\left[\frac{ik}{2z}(x^2 + y^2) - i(k_x x + k_y y) \right] dxdy$$

$$k_x = \frac{kX}{z}, k_y = \frac{kY}{z}, \; k = \frac{2\pi}{\lambda}$$

_____ Fouriertransformasjon:_____

$$F(k_x, k_y) = \iint f(x,y) \exp(i(k_x x + k_y y)) dxdy$$

$$F\{f(x - x_0)\} = \exp(ik_x x_0) F\{f(x)\}$$

$$h(x) = f(x) \otimes g(x) \Rightarrow F\{h(x)\} = F\{f(x)\} F\{g(x)\}$$

$$F\left\{ \mathrm{circ}\left(\frac{\rho}{a} \right) \right\} = \pi a^2 \, 2 \frac{J_1(\kappa a)}{\kappa a}, \quad \kappa = \sqrt{k_x^2 + k_y^2}, \quad \rho = \sqrt{x^2 + y^2}, \; J_1(3{,}83) = 0. \;\; \rho_1 = 1{,}22 \frac{R\lambda}{2a}$$

$$F\left\{ \mathrm{rect}\left(\frac{x}{w} \right) \right\} = w \frac{\sin(k_x w / 2)}{k_x w / 2}$$

$$F\left\{ \sum_{n=1}^{N} \delta(x - na) \right\} = e^{ik_x a(N+1)/2} \frac{\sin(k_x a N / 2)}{\sin(k_x a / 2)}$$