NTNU

Department of Physics

Examination paper for TFY4195 Optics

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Examination date: **20. desember 2017**

Examination time (from-to): **09:00 – 13:00**

Permitted examination support material:

Code C:
Approved calculator with empty memory.
K. Rottmann: Matematisk Formelsamling
S. Barnett & T.M. Cronin: Mathematical Formulae
O. Øgrim & B.E. Lian: Størrelser og enheter i fysikk og teknikk

Other information:

The examination set is developed by Prof. Dag W. Breiby and read through by Prof. Morten Kildemo. Each subtask a) b) etc. i task 1-3 will be given equal weight, with in total 100 % for the 10 subtasks.

Number of pages (front page excluded): **5**

Number of pages enclosed: **2**

Informasjon om trykking av eksamensoppgave

Originalen er:

1-sidig ☒ **2-sidig** ☐

sort/hvit ☒ **farger** ☐

skal ha flervalgskjema ☐

Checked by:

14.12.17 B Syvertsen IFY

Date Signature

Students will find the examination results in Studentweb. Please contact the department if you have questions about your results. The Examinations Office will not be able to answer this.

Task 1. Mixed challenges.

a) Give approximately the wavelength range of the visible part of the electromagnetic spectrum. What are the corresponding frequencies? What are the wavelength, linear momentum and frequency of an X-ray photon having energy of 8040 eV (Cu K_{α} radiation)?

b) Write down an equation for the electric field component of a plane wave travelling in a transparent homogeneous and uniform medium of refractive index *n*, for a general direction **k** and general polarization state. Explain briefly the connection to the Jones formalism.

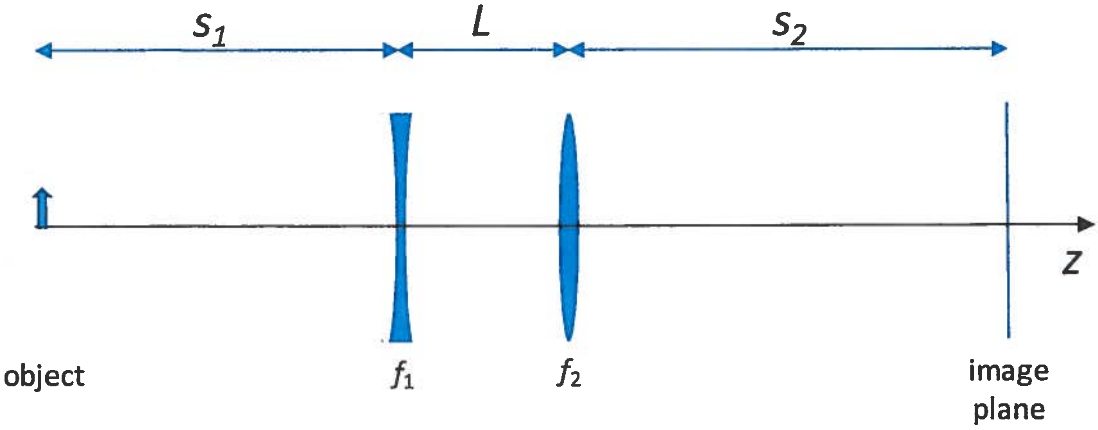
c) Using a negative (concave) thin lens with focal length $f = -0.50$ m, an object is imaged with a magnification of $M_T = 1/3$. Where is the object, and where is the image with respect to the lens? Make a sketch.

d) Describe briefly how *i*) refractive (prism) and *ii*) diffractive (grating) optics can be used for dispersion of white light. In particular, explain in both cases whether the dispersive properties increase or decrease with increasing frequency.

Task 2. Analysis of optical system (using the geometrical optics approximation)

The system sketched in the figure below consists of a negative lens (f_1 = -10.0 cm, concave) followed by a positive lens (f_2 =15.0 cm, convex), separated by L = 14.0 cm. Both lenses have a physical aperture (diameter) of D. A detector screen (image plane) is placed a distance s_2 = 50.0 cm after the last lens. The object is a distance s_1 before the first lens.

Note: We use the Hecht convention with rays described by vectors $\begin{bmatrix} n\alpha \\ y \end{bmatrix}$, where n is the index of refraction, α the angle with respect to the optical axis, and y the distance from the optical axis.



a) Write down the expression for the multiplication of matrices needed to calculate the transfer matrix **M** from the object to the image plane. The multiplication into a single matrix is given for your convenience (you are *not* requested to prove this expression):

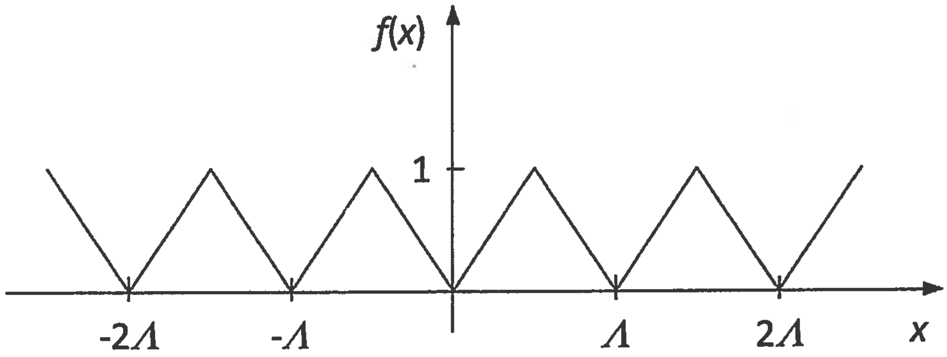
$$\mathbf{M} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \frac{1}{15} - \frac{3s_1}{50} & \frac{-3}{50} \\ \frac{52}{3} - \frac{3s_1}{5} & \frac{-3}{5} \end{pmatrix}$$

A ray is parallel to the optical axis and has y = 2.00 cm at the object position. What is the position y' and angle α' of this ray in the image plane?

b) In terms of the matrix elements, what is the condition for imaging? Calculate the distance s_1 which fulfills the imaging condition. What is the transversal magnification M_T of the image? Is the image upright?

c) Determine what serves as aperture stop for the system. Find the z-position of the entrance pupil.

Task 3. Diffraction and Fourier optics



The periodic triangle function f(x) plotted in the figure above can be expressed by the Fourier series

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{\Lambda}, \text{ with}$$

$a_n = \frac{4}{n^2\pi^2}(2\cos\frac{n\pi}{2} - \cos n\pi - 1)$,	if $n = 2, 6, 10, 14, \ldots$
$a_n = 1/2$,	if $n = 0$,
$a_n = 0$,	*otherwise.*

a) Rewrite $f(x)$ using complex exponentials. Show that the corresponding Fourier transform, when including terms up to $n \leq 6$, is given by

$$F(k_x) \propto a_0\delta(k_x) + \frac{a_2}{2}\delta(k_x \pm \frac{2\pi}{\Lambda}) + \frac{a_6}{2}\delta(k_x \pm \frac{6\pi}{\Lambda})$$

Hint: $\cos(u) = \frac{1}{2}[\exp(iu) + \exp(-iu)]$, and $\sin(u) = \frac{1}{2i}[\exp(iu) - \exp(-iu)]$

b) A transparency with transmission function $t(x,y) = f(x)\cdot\text{rect}(y/w)$ is used as a diffraction grating. w is the width of the grating. Assume that the grating has spatial period $\Lambda = 2.00$ μm and is illuminated with coherent laser light with wavelength $\lambda = 633$ nm.

Sketch the intensity distribution $I(X)$ on a screen located a distance $L = 4.00$ m downstream from the grating (Fraunhofer diffraction regime). Calculate the coordinates and the relative intensities (with respect to the central maximum at $X = 0$) of the diffraction peaks for $n \leq 6$. Comment on the Y-dependence of the diffracted intensities when it is given that $w >> \lambda$.

c) The grating from c) is placed in the "input" (object) plane of a 4-f correlator. By using a single slit as low-pass filter, only the $n = \{0, +2\}$ diffraction orders should be allowed to pass the Fourier plane. Make a qualitative sketch of the system with the slit in the correct position. If $f = 0.20$ m, what is the minimum and maximum possible width of the slit opening?

Formelliste for emnet TFY4195 Optikk (VEDLEGG)

Vektorstørrelser er i **uthevet** skrift.

Fysiske konstanter:

Ett mol: $M(^{12}C) = 12{,}000$ g
$k_B = 1{,}3807 \cdot 10^{-23}$ J/K
$\varepsilon_0 = 8{,}8542 \cdot 10^{-12}$ C²/Nm²
$e = 1{,}6022 \cdot 10^{-19}$ C
$c = 2{,}998 \cdot 10^{8}$ m/s

1u = $1{,}6605 \cdot 10^{-27}$ kg
$R = N_A k_B = 8{,}3145$ J mol^{-1} K^{-1}
$\mu_0 = 4\pi \cdot 10^{-7}$ N/A²
$m_e = 9{,}1094 \cdot 10^{-31}$ kg
$h = 6{,}6261 \cdot 10^{-34}$ Js

$N_A = 6{,}0221 \cdot 10^{23}$ mol^{-1}
0°C = 273,15 K
$g = 9{,}81$ m/s²

Elektrisitet og magnetisme:

$$\oint_S \mathbf{E}\cdot d\mathbf{A} = \oint_S E_n dA = \frac{Q_{inni}}{\varepsilon_0} \qquad \nabla\cdot\mathbf{D} = \rho$$

$$\oint_S \mathbf{B}\cdot d\mathbf{A} = \oint_S B_n dA = 0 \qquad \nabla\cdot\mathbf{B} = 0$$

$$\oint_C \mathbf{E}\cdot d\mathbf{s} = \varepsilon = -\frac{d\Phi_m}{dt} = -\frac{d}{dt}\int_S B_n dA = -\int_S \frac{\partial B_n}{\partial t} dA \qquad \nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\oint_C \mathbf{B}\cdot d\mathbf{s} = \mu_0(I_{inni} + I_d),\ I_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \int_S \frac{\partial E_n}{\partial t} dA \qquad \nabla\times\mathbf{B} = \mu_0 J + \mu_0\varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\mathbf{S} = c^2\varepsilon_0 \mathbf{E}\times\mathbf{B} \qquad I \equiv \langle S\rangle_T = \frac{\varepsilon_0 c}{2} E_0^2 = \varepsilon_0 c \langle E^2\rangle_T$$

$$\mathbf{D} = \varepsilon\mathbf{E} \qquad \mathbf{B} = \mu\mathbf{H}$$

Bølger, refraksjon og refleksjon:

$$\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \qquad (\nabla^2 + k^2)U = 0 \qquad I(r) = |U(r)|^2 \qquad I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

$$U(\mathbf{r}) = \exp(i\,\mathbf{k}\cdot\mathbf{r}) \qquad U(r) = \frac{A}{r}\exp(ikr) \approx \frac{A}{z}\exp(ikz)\exp\left[ik\frac{x^2+y^2}{2z}\right]$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$r_s = r_\perp = \left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t} \qquad r_p = r_\parallel = \left(\frac{E_{0r}}{E_{0i}}\right)_\parallel = \frac{n_t\cos\theta_i - n_i\cos\theta_t}{n_i\cos\theta_t + n_t\cos\theta_i}$$

$$t_s = t_\perp = \left(\frac{E_{0t}}{E_{0i}}\right)_\perp = \frac{2n_i\cos\theta_i}{n_i\cos\theta_i + n_t\cos\theta_t} \qquad t_p = t_\parallel = \left(\frac{E_{0t}}{E_{0i}}\right)_\parallel = \frac{2n_i\cos\theta_i}{n_i\cos\theta_t + n_t\cos\theta_i}$$

Jones og Stokes vektorer:

$$\mathbf{E} = \begin{bmatrix} E_x(t) \\ E_y(t) \end{bmatrix} \qquad \mathbf{S} = \begin{bmatrix} 2I_0 \\ 2I_1 - 2I_0 \\ 2I_2 - 2I_0 \\ 2I_3 - 2I_0 \end{bmatrix}$$

Geometrisk optikk:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad \frac{1}{f} = \frac{n_{lens} - n_{medium}}{n_{medium}} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\begin{bmatrix} n\alpha \\ y \end{bmatrix} \qquad R_1 = \begin{pmatrix} 1 & -P_1 \\ 0 & 1 \end{pmatrix}, \; P_1 = \frac{n_{t1} - n_{i1}}{R} \; T_{21} = \begin{pmatrix} 1 & 0 \\ d_{21} / n_{t1} & 1 \end{pmatrix}$$

Diffraksjon:

$$U(X,Y,z) = \frac{1}{i\lambda z} e^{ikz} e^{ik\frac{(X^2+Y^2)}{2z}} \iint U(x,y,0) \exp\left[\frac{ik}{2z}(x^2 + y^2) - i(k_x x + k_y y) \right] dxdy$$

$$k_x = \frac{kX}{z}, k_y = \frac{kY}{z}, \; k = \frac{2\pi}{\lambda}$$

Fouriertransformasjon:

$$F(k_x, k_y) = \iint f(x,y) \exp(i(k_x x + k_y y)) dxdy$$

$$\mathcal{F}\{f(x - x_0)\} = \exp(ik_x x_0) \mathcal{F}\{f(x)\}$$

$$h(x) = f(x) \otimes g(x) \Rightarrow \mathcal{F}\{h(x)\} = \mathcal{F}\{f(x)\}\mathcal{F}\{g(x)\}$$

$$\mathcal{F}\left\{ \mathrm{circ}\left(\frac{\rho}{a} \right) \right\} = \pi a^2 2 \frac{J_1(\kappa a)}{\kappa a}, \quad \kappa = \sqrt{k_x^2 + k_y^2}, \quad \rho = \sqrt{x^2 + y^2}, \; J_1(3{,}83) = 0. \qquad \rho_1 = 1{,}22 \frac{R\lambda}{2a}$$

$$\mathcal{F}\left\{ \mathrm{rect}\left(\frac{x}{w} \right) \right\} = w \frac{\sin(k_x w / 2)}{k_x w / 2}$$

$$\mathcal{F}\left[\sum_{n=1}^{N} \delta(x - na) \right] = e^{ik_x a(N+1)/2} \frac{\sin(k_x a N / 2)}{\sin(k_x a / 2)}$$