NTNU

Department of Physics

# Examination paper for TFY4195 Optics


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| **Examination date:** | **___ August 2018** |
| **Examination time (from-to):** | **09:00 – 13:00** |

**Permitted examination support material:**

Code C:
Approved calculator with empty memory.
K. Rottmann: Matematisk Formelsamling
S. Barnett & T.M. Cronin: Mathematical Formulae
O. Øgrim & B.E. Lian: Størrelser og enheter i fysikk og teknikk

**Other information:**

Each subtask a) b) etc. in task 1-4 will be given equal weight, with in total 100 % for the 11 subtasks.

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| **Number of pages (front page excluded):** | **4** |
| **Number of pages enclosed:** | **2** |

**Informasjon om trykking av eksamensoppgave**

**Originalen er:**

**1-sidig** □ **2-sidig** □

**sort/hvit** □ **farger** □

**skal ha flervalgskjema** □

**Checked by:**

Date Signature

**Task 1 Mixed challenges**

a) Give shortly Fermat's principle. Derive Snell's law from Fermat's principle. You may choose the coordinate system and variables yourself, but you should write down the assumptions you make.

b) A ray of light in air falls onto a planar glass surface with index of refraction $n_2$. We assume no absorption. Sketch the reflectance $R_s$ and $R_p$ as function of incidence angle $\theta_1$. What is the interpretation of the Brewster angle $\theta_B$?

We now assume that the light is unpolarized and that the incidence angle equals the Brewster angle, $\theta_i = \theta_B$. The net transmittance is known to be $T = 0.86$. What is the reflectance (reflected intensity ratio) of the *s*-polarized light?

c) A thin plano-convex lens made of glass with index of refraction $n$ has diameter $D = 40$ mm and radius of curvature $R = 100$ mm. When the focal length $f$ is 180 mm in air for $\lambda = 632.8$ nm (He-Ne laser), what is the value of $n$ for this wave length? Would you normally expect that the focal length is shorter or longer for $\lambda = 454.6$ nm (Argon laser)? Explain.

**Task 2 Geometrical optics**

A thin lens $L$ of focal length $f > 0$ stands in air (with index of refraction $n = 1$). The lens is mounted in a frame with inner diameter $D_L$.

a) Show that the matrix **M** given by

$$\mathbf{M} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} A & \dfrac{-1}{f} \\ s_1 + s_2 - \dfrac{s_1 s_2}{f} & 1 - \dfrac{s_2}{f} \end{pmatrix} \qquad (1)$$

describes the path of rays from the object plane in a distance $s_1 > f$ in front of the lens to a plane at a distance $s_2$ after the lens. Determine also the matrix element $A$.
What conditions must generally be fulfilled for this kind of matrix formalism to be valid?

b) We now put an aperture with adjustable circular opening of radius $r_B$ in the plane X a distance $x$ after the lens, with $0 < x \le f$.
Assume that a ray starts on the optical axis in the object plane and is marginal at $L$ (that is, the ray just barely makes it within the frame of the lens).
- Show by using the matrix from a) that the $y$-position of this ray in the plane X is given by

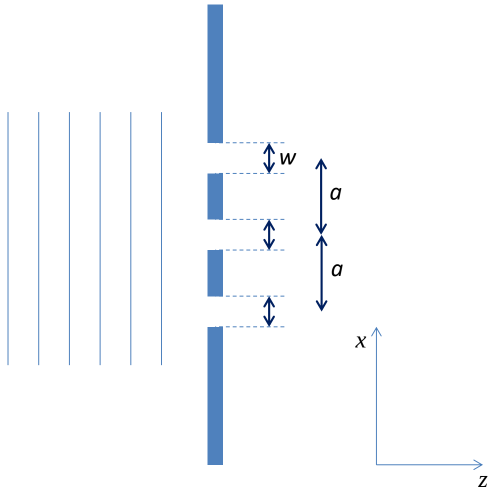
$$y_X = \frac{1}{2} D_L x \left( \frac{1}{s_1} + \frac{1}{x} - \frac{1}{f} \right).$$

- Use this to express a condition for $r_B$ which makes the aperture the *aperture stop* of the system.
- Find expressions for the location of the entrance and exit pupils of the system (given that the aperture is the aperture stop).

c) Explain shortly the connection between the stop and a *telecentric* system for the special case that the matrix element $D = 0$ in equation (1). What is the practical understanding of a telecentric system?

*Note:* We use the Hecht-convention where rays are represented with vectors $\begin{bmatrix} n\alpha \\ y \end{bmatrix}$, where $n$ is the index of refraction, $\alpha$ is the angle between the ray and the optical axis, and $y$ is the distance from the optical axis.

**Task 3 Interference from grating**




We have a diffraction grating consisting of three slits of width $w$ = 10 μm, separated by a distance $a$ = 100 μm, as shown in the sketch. A monochromatic light source ($\lambda$ = 633 nm) sends light (approximately plane wave) through the lattice. The three slits all have the same rectangular cross section that can be described by $f_n(x) = f_0 \, \mathrm{rect}(x/w)$, where $w$ is the slit width and the function rect($u$) is defined by

$$\mathrm{rect}(u) = \begin{cases} 1, & |u| \le \frac{1}{2} \\ 0, & |u| > \frac{1}{2} \end{cases}$$

a) Find an expression for the intensity distribution in the Fraunhofer-regime (far field) for *one single* of the three slits. At what angle $\theta_1$ is the first minimum seen? What happens to $\theta_1$ if $w$ increases?

b) Explain how the intensity from the three slits together at a long distance from the slits can be described by

$$I \propto \left| F \quad f_{tot}(x) \right|^2 = \left| F\left\{ \sum_{n=-1}^{1} f_n(x) \otimes \delta(x - na) \right\} \right|^2, \qquad (1)$$

where $F$ denotes Fourier transformation and $\otimes$ is convolution. Have we made any assumptions about the coherence of the light source to arrive at this expression?
Find a more specific expression for the intensity, starting out from eq. (1). Comment on use of the convolution theorem.

c) A positive lens is used to image the lattice. The lens has a focal length $f$ = 15.0 cm and diameter $D$ = 2.00 cm. If the lens is placed at a distance $s$ = 18.0 cm behind the lattice, how many diffraction orders will pass through the lens, and what consequences will this have for the imaging?

**Task 4 You as a consultant**

a) Explain shortly why laser light is
   - (approximately) monochromatic
   - (approximately) parallel

b) Show that the relative phase change $\Delta\varphi$ between the ordinary and extraordinary ray when going through a uniaxial crystal with its optical axis parallel to the entrance plane is given by

$$\Delta\varphi = \frac{2\pi}{\lambda}\left|n_o - n_e\right| d\ .$$

Use this to calculate the thickness $d$ for a piece of quartz to act as a quarter wave plate, given the wavelength $\lambda$ = 632 nm.

Finally, argue why the Jones matrix

$$\mathrm{M}_{1/4} = \exp(i\pi/4)\begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}$$

mathematically can serve as model for a quarter wave plate.

*Given:* For quartz, $n_o$ = 1.5442 and $n_e$ = 1.5533.

**Formelliste for emnet TFY4195 Optikk** **(VEDLEGG)**

Vektorstørrelser er i **uthevet** skrift.

## _____ Fysiske konstanter:_____

Ett mol: $M(^{12}C) = 12{,}000$ g | 1u = $1{,}6605 \cdot 10^{-27}$ kg | $N_A = 6{,}0221 \cdot 10^{23}$ mol$^{-1}$

$k_B = 1{,}3807 \cdot 10^{-23}$ J/K | $R = N_A k_B = 8{,}3145$ J mol$^{-1}$ K$^{-1}$ | 0°C = 273,15 K

$\varepsilon_0 = 8{,}8542 \cdot 10^{-12}$ C$^2$/Nm$^2$ | $\mu_0 = 4\pi \cdot 10^{-7}$ N/A$^2$

$e = 1{,}6022 \cdot 10^{-19}$ C | $m_e = 9{,}1094 \cdot 10^{-31}$ kg

$c = 2{,}998 \cdot 10^8$ m/s | $h = 6{,}6261 \cdot 10^{-34}$ Js | $g = 9{,}81$ m/s$^2$

## _____ Elektrisitet og magnetisme:_____

$$\oint_S \mathbf{E}\cdot d\mathbf{A} = \oint_S E_n dA = \frac{Q_{inni}}{\varepsilon_0} \qquad \nabla\cdot\mathbf{D} = \rho$$

$$\oint_S \mathbf{B}\cdot d\mathbf{A} = \oint_S B_n dA = 0 \qquad \nabla\cdot\mathbf{B} = 0$$

$$\oint_C \mathbf{E}\cdot d\mathbf{s} = \varepsilon = -\frac{d\Phi_m}{dt} = -\frac{d}{dt}\int_S B_n dA = -\int_S \frac{\partial B_n}{\partial t} dA \qquad \nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\oint_C \mathbf{B}\cdot d\mathbf{s} = \mu_0(I_{inni} + I_d),\ I_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \int_S \frac{\partial E_n}{\partial t} dA \qquad \nabla\times\mathbf{B} = \mu_0 J + \mu_0\varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\mathbf{S} = c^2\varepsilon_0 \mathbf{E}\times\mathbf{B} \qquad I \equiv \langle S\rangle_T = \frac{\varepsilon_0 c}{2} E_0^2 = \varepsilon_0 c \langle E^2\rangle_T$$

$$\mathbf{D} = \varepsilon\mathbf{E} \qquad \mathbf{B} = \mu\mathbf{H}$$

## _____ Bølger, refraksjon og refleksjon:_____

$$\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \qquad (\nabla^2 + k^2)U = 0 \qquad I(r) = |U(r)|^2 \qquad I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

$$U(\mathbf{r}) = \exp(i\,\mathbf{k}\cdot\mathbf{r}) \qquad U(r) = \frac{A}{r}\exp(ikr) \approx \frac{A}{z}\exp(ikz)\exp\left[ik\frac{x^2+y^2}{2z}\right]$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$r_s = r_\perp = \left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t} \qquad r_p = r_\parallel = \left(\frac{E_{0r}}{E_{0i}}\right)_\parallel = \frac{n_t\cos\theta_i - n_i\cos\theta_t}{n_i\cos\theta_t + n_t\cos\theta_i}$$

$$t_s = t_\perp = \left(\frac{E_{0t}}{E_{0i}}\right)_\perp = \frac{2n_i\cos\theta_i}{n_i\cos\theta_i + n_t\cos\theta_t} \qquad t_p = t_\parallel = \left(\frac{E_{0t}}{E_{0i}}\right)_\parallel = \frac{2n_i\cos\theta_i}{n_i\cos\theta_t + n_t\cos\theta_i}$$

## _____ Jones og Stokes vektorer:

$$\mathbf{E} = \begin{bmatrix} E_x(t) \\ E_y(t) \end{bmatrix} \qquad \mathbf{S} = \begin{bmatrix} 2I_0 \\ 2I_1 - 2I_0 \\ 2I_2 - 2I_0 \\ 2I_3 - 2I_0 \end{bmatrix}$$

## _____ Geometrisk optikk:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad \frac{1}{f} = \frac{n_{lens} - n_{medium}}{n_{medium}} \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\begin{bmatrix} n\alpha \\ y \end{bmatrix} \qquad R_1 = \begin{pmatrix} 1 & -P_1 \\ 0 & 1 \end{pmatrix},\ P_1 = \frac{n_{t1} - n_{i1}}{R}\ T_{21} = \begin{pmatrix} 1 & 0 \\ d_{21}/n_{t1} & 1 \end{pmatrix}$$

## _____ Diffraksjon:

$$U(X,Y,z) = \frac{1}{i\lambda z} \mathrm{e}^{ikz} \mathrm{e}^{ik\frac{(X^2+Y^2)}{2z}} \iint U(x,y,0) \exp\left[ \frac{ik}{2z}(x^2+y^2) - i(k_x x + k_y y) \right] dxdy$$

$$k_x = \frac{kX}{z},\ k_y = \frac{kY}{z},\ k = \frac{2\pi}{\lambda}$$

## _____ Fouriertransformasjon:

$$F(k_x,k_y) = \iint f(x,y) \exp(i(k_x x + k_y y)) dxdy$$

$$\mathcal{F}\{ f(x - x_0)\} = \exp(ik_x x_0) \mathcal{F}\{ f(x)\}$$

$$h(x) = f(x) \otimes g(x) \Rightarrow \mathcal{F}\{h(x)\} = \mathcal{F}\{ f(x)\} \mathcal{F}\{g(x)\}$$

$$\mathcal{F}\left\{ \mathrm{circ}\left( \frac{\rho}{a} \right) \right\} = \pi a^2 2 \frac{J_1(\kappa a)}{\kappa a},\quad \kappa = \sqrt{k_x^2 + k_y^2},\quad \rho = \sqrt{x^2+y^2},\ J_1(3{,}83) = 0. \qquad \rho_1 = 1{,}22 \frac{R\lambda}{2a}$$

$$\mathcal{F}\left\{ \mathrm{rect}\left( \frac{x}{w} \right) \right\} = w \frac{\sin(k_x w/2)}{k_x w/2}$$

$$\mathcal{F}\left\{ \sum_{n=1}^{N} \delta(x - na) \right\} = \mathrm{e}^{ik_x a(N+1)/2} \frac{\sin(k_x aN/2)}{\sin(k_x a/2)}$$