NTNU

Department of Physics

Examination paper for TFY4195 Optics

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Examination date:	**13 December 2018**
Examination time (from-to):	**09:00 – 13:00**

Permitted examination support material:

Code C:
Approved calculator with empty memory.
K. Rottmann: Matematisk Formelsamling
S. Barnett & T.M. Cronin: Mathematical Formulae
O. Øgrim & B.E. Lian: Størrelser og enheter i fysikk og teknikk
C. Angell & B.E. Lian: Fysiske størrelser og enheter – navn og symboler

Other information:

The examination set is developed by Prof. Dag W. Breiby. Each subtask a) b) etc. in task 1-3 will be given equal weight, with in total 100 % for the 10 subtasks.

Number of pages (front page excluded):	**4**
Number of pages enclosed:	**1 (front) + 4 (tasks) + 2 (formula) = 7.**

Informasjon om trykking av eksamensoppgave

Originalen er:

1-sidig ☒ **2-sidig** ☐

sort/hvit ☒ **farger** ☐

skal ha flervalgskjema ☐

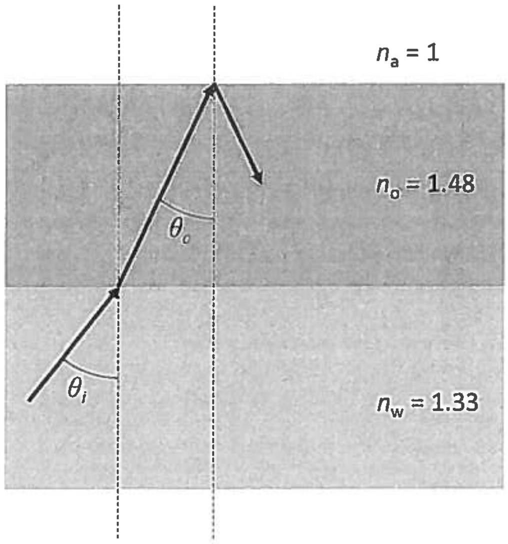
Checked by:

10.12.18 BSY, IFY
Date Signature

Students will find the examination results in Studentweb. Please contact the department if you have questions about your results. The Examinations Office will not be able to answer this.

Task 1. Mixed challenges

a) What must the focal length of a positive thin lens be if the object and the image distances are 90 cm and 45 cm, respectively? What will the transverse magnification be?



b) A tank of water is covered with a 1 cm thick layer of linseed oil ($n_o = 1.48$), above which is air (see figure). What angular restrictions must be imposed on a beam of light, originating from the tank, if no light is to escape at the water-oil interface?

c) Write down the equation for a spherical wave on complex exponential form. Derive the paraxial approximation for a spherical (paraboloidal) wave near the propagation axis (z). Show that the same expression arises if calculating the propagation of light from a point source using the Fresnel diffraction formula.

d) The Fresnel reflectance coefficient for p-polarized light, R_p (or $R_{\parallel}$), can be written

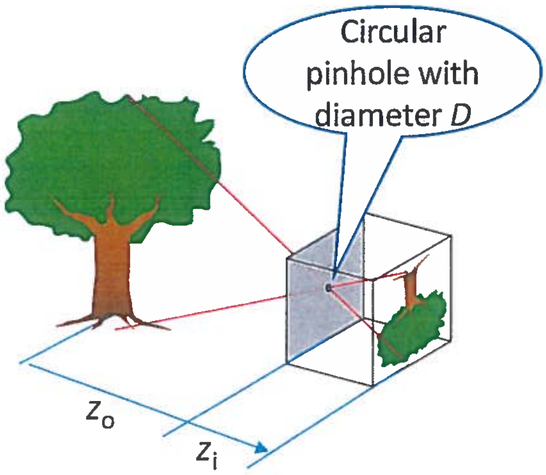
$$R_p = \frac{\tan^2(\theta_i - \theta_t)}{\tan^2(\theta_i + \theta_t)}.$$

Show that the definition of the Brewster angle θ_B leads to the condition $\theta_i + \theta_t = 90°$. Here, θ_i, θ_r and θ_t denote the angles of the incident, reflected and transmitted beams, respectively. Then combine with Snell's law to derive

$$\tan\theta_B = C\frac{n_t}{n_i},$$

where C is a constant to be determined by you. Finally, make a sketch for a p-polarized beam in air incident on a glass surface (with index of refraction $n_t \sim 1.5$) at the Brewster angle. Comment on how the dipole scattering mechanism can account for the Brewster angle.

Task 2. A pinhole camera



A pinhole camera is illustrated in the figure above. Pinhole cameras are historically the first kind of cameras. In this task, the pinhole camera has a circular aperture of diameter D and a fixed distance z_i = 100 mm from the aperture to the image plane. There is a square-shaped area detector located in the image plane having side lengths of 70 mm. We assume $z_o >> z_i$. The wave length λ of the incoming light can be approximated to 550 nm.

a) In the *geometric optics* approximation:
   - Find an expression for the transverse magnification based on the parameters z_o and z_i. What will be the imaged height of a 2.0 m tall man standing z_o = 8.0 m in front of the camera?
   - Where are the aperture stop, entrance pupil, exit pupil, and field stop?
   - What is the angular field of view?

b) Intuitively, decreasing the diameter D of the aperture increases the resolution.
   - Explain how, for a comparably large pinhole (~mm), each point of the object will be imaged to a spot of the same size as the pinhole.
   - Assume that an approximate resolution criterion is $0.75D$ (in the sense that if the centers of the image spots are separated by more than $0.75D$, then the corresponding points in the object plane are resolved). Plot the corresponding resolution as a function of diameter D, for D in the range 0 to 2 mm. Indicate on the sketch roughly the range of validity of the geometric optics approximation.

c) For sufficiently small D, diffraction effects become non-negligible, and at Fresnel numbers $F \sim (D/2)^2/\lambda z_i << 1$, the imaging must be described in the Fraunhofer (far-field) regime.
   - Explain in brief terms the Rayleigh resolution criterion.
   - In the same graph as in task b), plot the Rayleigh resolution as function of D and indicate its range of validity.

(Task 2 continues)

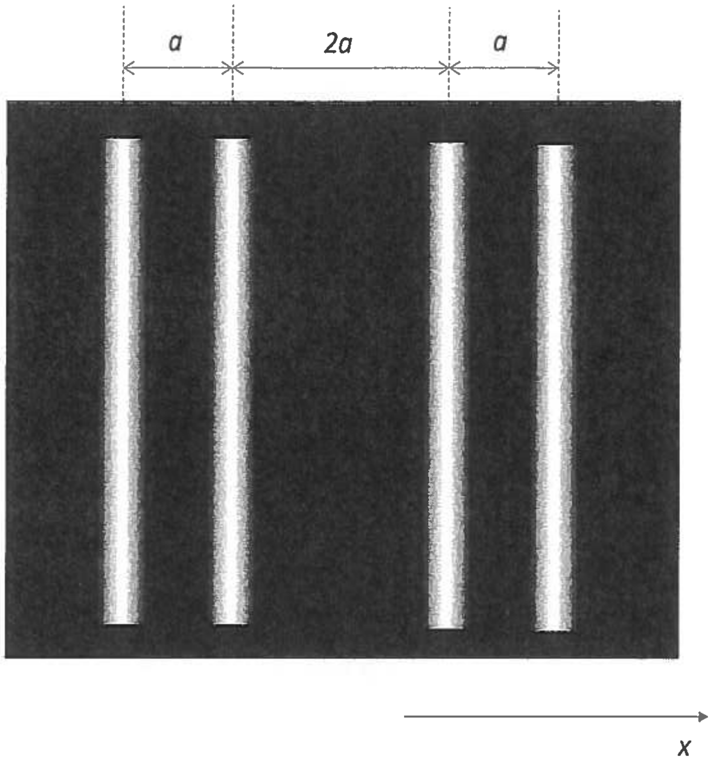
d) It can be shown rigorously that there is in fact an optimal diameter D_{optim} that optimizes the resolution, given implicitly by $z_i = (D_{\text{optim}}/2)^2/\lambda$ (you are not expected to show this).

- Show that with the numbers given in this problem,

  $$D_{\text{optim}} \approx 0.047 \sqrt{z_i}$$

  when both D_{optim} and z_i are expressed in mm. Indicate the location of D_{optim} on the graph from b) and c). In what diffraction regime is D_{optim}?
- How are i) the resolution and ii) the light-gathering properties (the *f*-number) modified if the linear dimensions of the camera (notably z_i and the detector side lengths) are quadrupled while the aperture D_{optim} is rescaled according to the above equation?

Task 3. Fraunhofer diffraction from a 1D structure with four Gaussian lines.



A 1D diffraction grating consists of 4 identical lines with Gaussian transmission function. The spacings between the lines are a, $2a$, and a, as illustrated in the sketch above.

Each of the lines thus has a transmission function ~$\exp(-x^2/w^2)$, where w is a measure of the width of the Gaussians.

a) Derive an expression for the far-field (Fraunhofer) diffraction amplitude $F(k_x)$ in the limit $w \rightarrow 0$. Plot the corresponding (relative) intensity distribution $I(k_x)$.

b) Show that for finite w, the (Fraunhofer) diffracted intensity distribution is given by

$$I(k_x) \sim [\cos(2k_x a) + \cos(k_x a)]^2 \cdot \exp(-w^2 k_x^2 / 2).$$

Make a qualitative sketch explaining how increasing w modifies the diffraction pattern. Comment on the convolution theorem.

Hint: $$\int_{-\infty}^{\infty} e^{-ax^2+bx} dx = e^{\frac{b^2}{4a}} \sqrt{\frac{\pi}{a}}$$

Formula sheet for TFY4195 Optics
Vectors are in **bold**.

Physical constants:

One mole: $M(^{12}C) = 12$ g
$1u = 1.6605 \cdot 10^{-27}$ kg
$N_A = 6.0221 \cdot 10^{23}$ mol^{-1}
$k_B = 1.3807 \cdot 10^{-23}$ J/K
$R = N_A k_B = 8.3145$ J mol^{-1} K^{-1}
0°C = 273.15 K
$\varepsilon_0 = 8.8542 \cdot 10^{-12}$ C^2/Nm^2
$\mu_0 = 4\pi \cdot 10^{-7}$ N/A^2
$e = 1.6022 \cdot 10^{-19}$ C
$m_e = 9.1094 \cdot 10^{-31}$ kg
$c = 2.998 \cdot 10^8$ m/s
$h = 6.6261 \cdot 10^{-34}$ Js
$g = 9.81$ m/s^2

Electricity and magnetism:

$$\oint_S \mathbf{E}\cdot d\mathbf{A} = \oint_S E_n dA = \frac{Q_{inni}}{\varepsilon_0} \qquad \nabla\cdot\mathbf{D} = \rho$$

$$\oint_S \mathbf{B}\cdot d\mathbf{A} = \oint_S B_n dA = 0 \qquad \nabla\cdot\mathbf{B} = 0$$

$$\oint_C \mathbf{E}\cdot d\mathbf{s} = \mathcal{E} = -\frac{d\Phi_m}{dt} = -\frac{d}{dt}\int_S B_n dA = -\int_S \frac{\partial B_n}{\partial t} dA \qquad \nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\oint_C \mathbf{B}\cdot d\mathbf{s} = \mu_0 (I_{inni} + I_d),\ I_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \int_S \frac{\partial E_n}{\partial t} dA \qquad \nabla\times\mathbf{B} = \mu_0 J + \mu_0\varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\mathbf{S} = c^2\varepsilon_0 \mathbf{E}\times\mathbf{B} \qquad I \equiv \langle S\rangle_T = \frac{\varepsilon_0 c}{2} E_0^2 = \varepsilon_0 c \langle E^2\rangle_T$$

$$\mathbf{D} = \varepsilon\mathbf{E} \qquad \mathbf{B} = \mu\mathbf{H}$$

Waves, refraction and reflection:

$$\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \qquad (\nabla^2 + k^2)U = 0 \qquad I(r) = |U(r)|^2 \qquad I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

$$U(\mathbf{r}) = \exp(i\mathbf{k}\cdot\mathbf{r}) \qquad U(r) = \frac{A}{r}\exp(ikr) \approx \frac{A}{z}\exp(ikz)\exp\left[ik\frac{x^2+y^2}{2z}\right]$$

$$n_1\sin\theta_1 = n_2\sin\theta_2$$

$$r_s = r_\perp = \left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t} \qquad r_p = r_\parallel = \left(\frac{E_{0r}}{E_{0i}}\right)_\parallel = \frac{n_t\cos\theta_i - n_i\cos\theta_t}{n_i\cos\theta_t + n_t\cos\theta_i}$$

$$t_s = t_\perp = \left(\frac{E_{0t}}{E_{0i}}\right)_\perp = \frac{2n_i\cos\theta_i}{n_i\cos\theta_i + n_t\cos\theta_t} \qquad t_p = t_\parallel = \left(\frac{E_{0t}}{E_{0i}}\right)_\parallel = \frac{2n_i\cos\theta_i}{n_i\cos\theta_t + n_t\cos\theta_i}$$

Jones and Stokes vectors:

$$\mathbf{E} = \begin{bmatrix} E_x(t) \\ E_y(t) \end{bmatrix} \qquad \mathbf{S} = \begin{bmatrix} 2I_0 \\ 2I_1 - 2I_0 \\ 2I_2 - 2I_0 \\ 2I_3 - 2I_0 \end{bmatrix}$$

Geometrical optics:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \qquad \frac{1}{f}=\frac{n_{lens}-n_{medium}}{n_{medium}}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\begin{bmatrix} n\alpha \\ y \end{bmatrix} \qquad R_1=\begin{pmatrix} 1 & -D_1 \\ 0 & 1 \end{pmatrix},\ D_1=\frac{n_{t1}-n_{i1}}{R} \qquad T_{21}=\begin{pmatrix} 1 & 0 \\ d_{21}/n_{t1} & 1 \end{pmatrix}$$

Diffraction:

$$U(X,Y,z)=\frac{1}{i\lambda z}e^{ikz}e^{ik\frac{(X^2+Y^2)}{2z}}\iint U(x,y,0)\exp\left[\frac{ik}{2z}(x^2+y^2)-i(k_x x+k_y y)\right]dxdy$$

$$k_x=\frac{kX}{z},\ k_y=\frac{kY}{z},\ k=\frac{2\pi}{\lambda}$$

Fourier transform:

$$F(k_x,k_y)=\iint f(x,y)\exp(i(k_x x+k_y y))dxdy$$

$$\mathcal{F}\{f(x-x_0)\}=\exp(ik_x x_0)\mathcal{F}\{f(x)\}$$

$$h(x)=f(x)\otimes g(x)\Rightarrow \mathcal{F}\{h(x)\}=\mathcal{F}\{f(x)\}\mathcal{F}\{g(x)\}$$

$$\mathcal{F}\left\{\mathrm{circ}\left(\frac{\rho}{a}\right)\right\}=\pi a^2 2\frac{J_1(\kappa a)}{\kappa a},\ \kappa=\sqrt{k_x^2+k_y^2},\quad \rho=\sqrt{x^2+y^2},\ J_1(3.83)=0. \qquad \rho_1=1.22\frac{R\lambda}{2a}$$

$$\mathcal{F}\left\{\mathrm{rect}\left(\frac{x}{w}\right)\right\}=w\frac{\sin(k_x w/2)}{k_x w/2}$$

$$\mathcal{F}\left\{\sum_{n=1}^{N}\delta(x-na)\right\}=e^{ik_x a(N+1)/2}\frac{\sin(k_x aN/2)}{\sin(k_x a/2)}$$