SIF 4040 Optics - Suggested solution.

22. may 2001

Problem #1.

a)

$$M_{VV'} = \begin{pmatrix} 1 & 0 \\ -1/f_2 & 1 \end{pmatrix} \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f_1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 - \frac{t}{f_1} & t \\ -\left(\frac{1}{f_1} + \frac{1}{f_2} - \frac{t}{f_1 f_2}\right) & 1 - \frac{t}{f_2} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$f = -\frac{1}{C} = \frac{f_1 f_2}{f_1 + f_2 - t} = \underline{\underline{21.43\ \text{cm}}}$$

b) ~~Use ABCD-law: $d \to \infty$ yields~~

~~$d' \to -\frac{A}{C} = fA = 0.2\ \text{cm}$ Back focal distance.~~

Distance HV: $h = \frac{1-D}{C} = \frac{+t}{f_2 C} = -t\frac{f}{f_2} = \underline{\underline{25.71\ \text{cm}}}$.

Distance V'H': $h' = \frac{1-A}{C} = \frac{+t}{f_1 C} = -t\frac{f}{f_1} = \underline{-17.14\ \text{cm}}$

~~Front focal distance~~

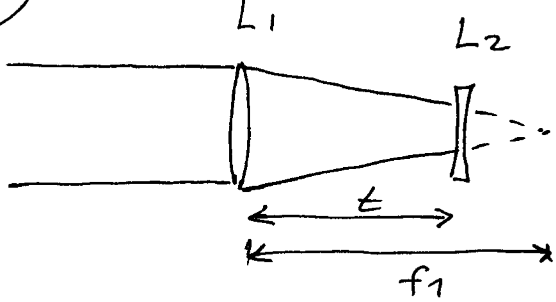
FV: $f + h = \underline{\underline{47.14 \text{ cm}}}$ Front focal distance.

V'F': $f + h' = \underline{\underline{1.29 \text{ cm}}}$. Back focal dist.

Physical length: $f + h' + t = \underline{\underline{16.29 \text{ cm}}}$

(Not a very good telephoto lens).

c)



The ray bundle of the object point on axis is focused by L_1

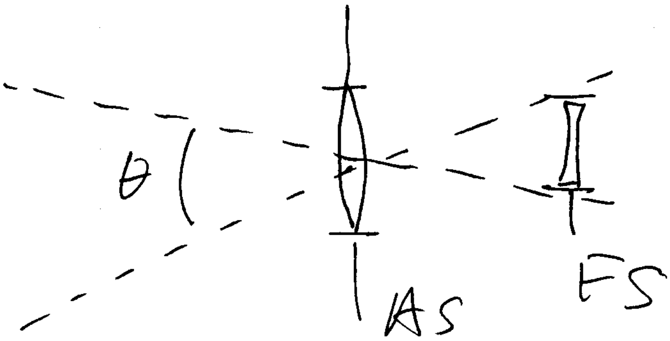
At L_2 it has a diameter D' given by:

$$\frac{D'}{f_1 - t} = \frac{D_1}{f_1} \Rightarrow D' = D_1\left(1 - \frac{t}{f_1}\right) = 1 \text{ cm}$$

which is smaller than $D_2 = 1.5 \text{ cm}$

Therefore: $\underline{L_1 \text{ is the aperture stop}}$

$\underline{L_2 \text{ is the field stop.}}$

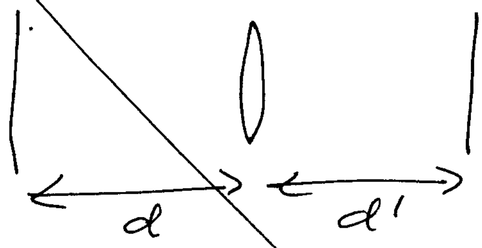


The angular field of view is is ~~the~~.

$$\theta = \frac{D_2}{t}\frac{180}{\pi}$$

$$= \frac{1.5}{12}\frac{180}{\pi} = \underline{\underline{7.17^\circ}}$$

d)



$$m = -\frac{d'}{d} = \text{given.}$$

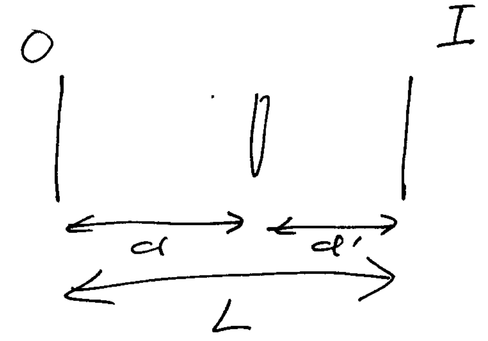
$$\frac{1}{d} + \frac{1}{d'} = \frac{1}{f}$$

$$d' + d = L \Rightarrow d(1-$$

$$\frac{1}{d'}(1 - m) = \frac{1}{f}$$

$$d' - \frac{d'}{m} = L \therefore d' = \frac{L}{1 - \frac{1}{m}}.$$

d)



① $d + d' = L$ ~~given~~ $= 1\,m$ given.

② $m = -\frac{d'}{d}$; ~~given~~ $|m|$ ~~given~~ $= 4$ given

③ $\frac{1}{d} + \frac{1}{d'} = \frac{1}{f}$; We seek d and f.

① & ② $\Rightarrow$ $d = \frac{L}{1-m}$

since $d < L$ we must have $m < 0$,

i.e. $m = -|m|$. $\therefore$ $d = \frac{L}{1+|m|} = \underline{\underline{20\,cm.}}$

$$\Rightarrow \frac{1}{f} = \frac{1}{d} + \frac{1}{d'} = \frac{1}{d}\left(1 - \frac{1}{m}\right) = \frac{1}{d}\left(1 + \frac{1}{|m|}\right)$$

$$= \frac{1+|m|}{L}\,\frac{1+|m|}{|m|} \Rightarrow f = L\frac{|m|}{(1+|m|)^2} = \frac{4m}{25} = \underline{\underline{16\,cm}}$$

Problem #2.

a) $\lambda_m = \frac{2L}{m} \approx 633\text{ nm}$,

$$m \approx \frac{2L}{633\text{nm}} \approx 9.48 \cdot 10^5 \sim 10^6.$$

$$\Delta\lambda = \lambda_m - \lambda_{m+1} = 2L\left(\frac{1}{m} - \frac{1}{m+1}\right) = 2L/m(m+1)$$

$$\approx \frac{2L}{m^2} \quad ; \quad m = \frac{\lambda}{2L} \Longrightarrow$$

$$\Delta\lambda \approx \frac{\lambda^2}{2L} \quad \text{QED}$$

$$\Delta\lambda = 0.6678 \cdot 10^{-12}\text{ m}$$

$$\lambda\nu = c \Longrightarrow \lambda\Delta\nu + \nu\Delta\lambda = 0 \Longrightarrow$$

$$|\Delta\nu| = \nu\frac{\Delta\lambda}{\lambda} = c\frac{\Delta\lambda}{\lambda^2} = \frac{c}{2L} = 50\text{ MHz}$$

b) Resulting intensity

$$I = I_0(1+\cos k_1 s + 1 + \cos k_2 s)$$

$$= I_0\left(2 + 2\cos\left(\frac{k_1+k_2}{2}s\right)\cos\left(\frac{k_1-k_2}{2}s\right)\right)$$

$$= 2I_0\left(1 + \cos\left(\frac{k_1+k_2}{2}s\right)\cos\left(\frac{k_1-k_2}{2}s\right)\right)$$

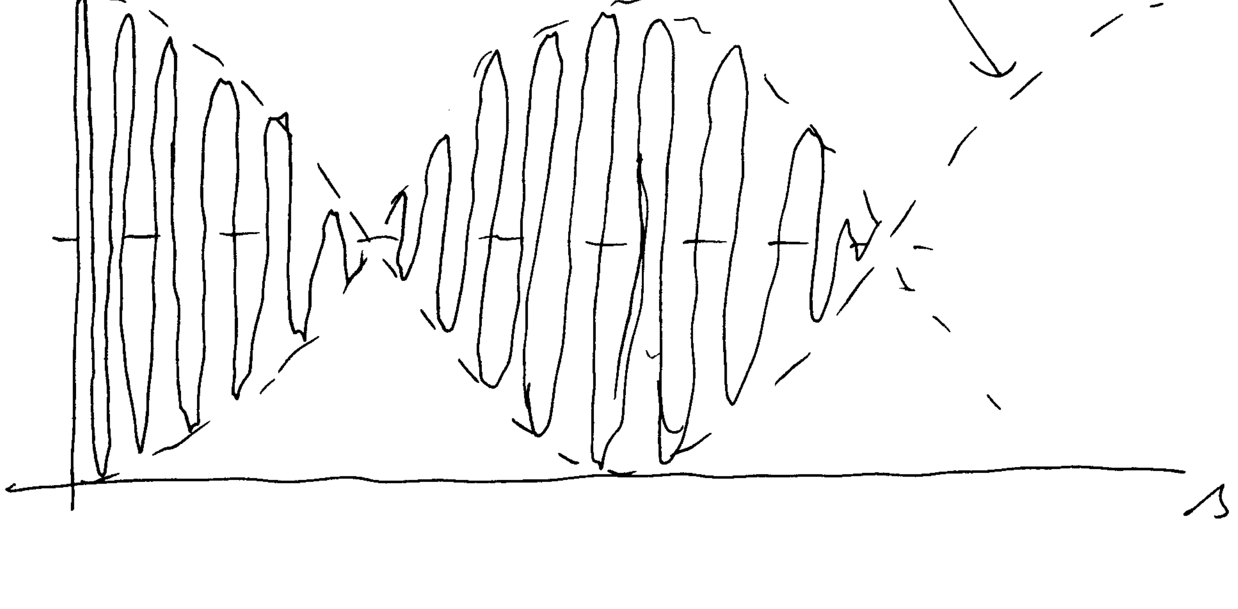
$k_1 = 2\pi/\lambda_1$, $k_2 = 2\pi/\lambda_2$

$$\frac{k_1-k_2}{2} = \pi\left(\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right) = \pi\frac{\lambda_2-\lambda_1}{\lambda_1\lambda_2} \approx \pi\frac{\Delta\lambda}{\lambda^2}$$

$$\frac{k_1+k_2}{2} = \pi\left(\frac{1}{\lambda_1}+\frac{1}{\lambda_2}\right) = \pi\frac{\lambda_1+\lambda_2}{\lambda_1\lambda_2} \approx \pi\frac{2\lambda}{\lambda^2} = 2\pi/\lambda$$

$$I = 2I_0\left(1 + \cos\left(\frac{2\pi s}{\lambda}\right)\cos\left(\pi\frac{\Delta\lambda}{\lambda^2}s\right)\right) \quad \text{QED.}$$

Interference term   envelope



c) Interference term vanishes for

$\cos(\pi \frac{\Delta\lambda}{\lambda^2} s) = 0$, i.e. for $\frac{\Delta\lambda}{\lambda^2} s = m + \frac{1}{2}$

$m = 0, \pm 1$ etc.

$s = (m + \frac{1}{2}) \frac{\lambda^2}{\Delta\lambda} = \underline{(2m+1)L}$ an odd multipleum of L

Maximum interference for

$\cos(\pi \frac{\Delta\lambda}{\lambda^2} s) = \pm 1$, i.e. for $\frac{\Delta\lambda}{\lambda^2} s = m$, $m = 0, \pm 1, ..$

$s = m \frac{\lambda^2}{\Delta\lambda} = \underline{2mL}$ Even multiples of L.

d)
$$\left.\begin{array}{l} I_{max} \sim 1 + |\cos(\pi \frac{\Delta\lambda}{\lambda^2} s)| \\ I_{min} \sim 1 - |\cos(\pi \frac{\Delta\lambda}{\lambda^2} s)| \end{array}\right\} \rightarrow$$

$$V(s) = \left|\cos(\pi \frac{\Delta\lambda}{\lambda^2} s)\right| = \left|\cos\left(\pi \frac{s}{2L}\right)\right|$$

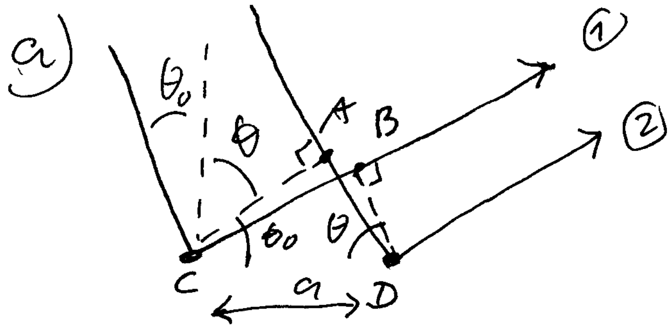
$V \geq 0.5$ for ~~all~~ $|\cos(\pi \frac{s}{2L})| \geq 0.5$

$\frac{\pi s}{2L} = m \pm \frac{\pi}{3}$ ∴ $\underline{S = m2L \pm \frac{2}{3}L}$, $m = 0, \pm 1, \ldots$

$(\cos\frac{\pi}{3} = 0.5)$

Problem #3.

a)



$\Delta = CB - AD = a\sin\theta - a\sin\theta_0 = a(\sin\theta - \sin\theta_0)$ QED

b) $\Delta = m\lambda\,;\ m = 0, \pm 1, \pm 2 \ldots \Longrightarrow$

$$a(\sin\theta_m - \sin\theta_0) = m\lambda$$

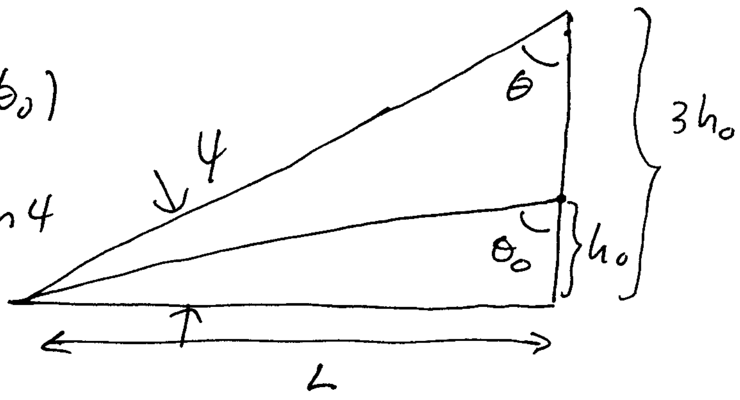
$$a\left(\frac{1}{\sqrt{1+\frac{h_m^2}{L^2}}} - \frac{1}{\sqrt{1+\frac{h_0^2}{L^2}}}\right) = m\lambda.$$

(*) Given formula wrong*: Full credit for any answer here. *) but a fair approximation.

c) $m_1\lambda = -a\sin\theta_0$

$m_2\lambda = a(\sin\theta - \sin\theta_0)$

$m_2 - m_1 = \frac{a}{\lambda}\sin\theta = \frac{a}{\lambda}\cos 4$



$-m_1 = \mathrm{Int}\left(\frac{a}{\lambda}\sin\theta_0\right) =$

$m_2 = \mathrm{Int}\left(\frac{a}{\lambda}(\sin\theta - \sin\theta_0)\right) =$

d)

$$E(x,y,0) = \frac{1}{2}\left[1 + \frac{1}{2}\left(e^{i2\pi\frac{x}{a}} + e^{-2\pi i\frac{x}{a}}\right)\right] A \cancel{e^{ikz}}$$

$$E(x,y,z) = \frac{1}{2} A e^{ikz} + \frac{1}{4} A e^{i\left(2\pi\frac{x}{a} + z\sqrt{k^2 - (2\pi/a)^2}\right)}$$

$$+ \frac{1}{4} A e^{i\left(-2\pi\frac{x}{a} + z\sqrt{k^2 - (2\pi/a)^2}\right)}$$

Sum of 3 plane waves:

$k = 2\pi/\lambda$

$$\sqrt{k^2 - (2\pi/a)^2} = 2\pi\sqrt{\frac{1}{\lambda^2} - \frac{1}{a^2}} = 2\pi i\sqrt{\frac{1}{a^2} - \frac{1}{\lambda^2}}$$

for $a < \lambda$.

$$\Rightarrow E(x,y,z) = \frac{A}{2} e^{ikz} + \frac{A}{2}\cos\left(2\pi\frac{x}{a}\right) e^{-2\pi z\sqrt{\frac{1}{a^2} - \frac{1}{\lambda^2}}}$$

Exponential damping in z-dir.

Only this term will contribute for large z.