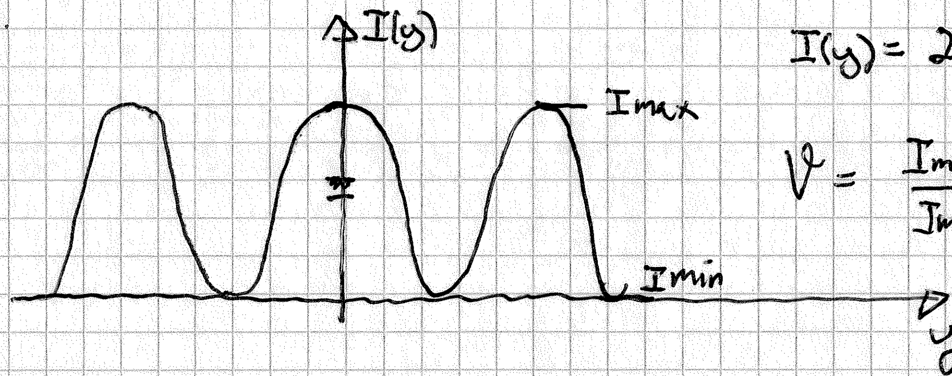
Oppgave 1 – Youngs experiment

a) With a point source, we have high spatial coherence. Assume that diffraction effects can be neglected (i.e. that slit widths are $\gg \lambda$)

Identical slits, $V = 1$

$$I(y) = 2I_0(1 + \cos ks)$$

path length difference

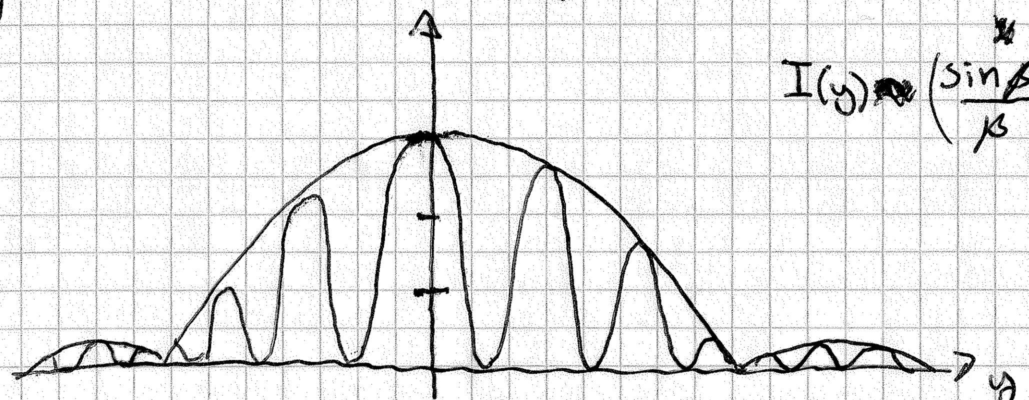


$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

Note, one may include diffraction effects, which become important for $\lambda \to$ slit width. Difficult to realise without the use of a lens.

$$I(y) = \left(\frac{\sin\beta}{\beta}\right)^2 2I_0(1 + \cos ks)$$

$$\beta \propto \frac{\text{slit width}}{\lambda}$$



{note: Both answers are acceptable, since it was not clearly stated that diffraction effects could be neglected}

b) Upon increasing the source width (i.e. upon going from a point source to an extended source), the spatial coherence is reduced $\Rightarrow V \to 0$, effectively ~~[illegible]~~ ~~[illegible]~~ adding intensities in obs. plane from each source point.

The contrast of the interference pattern is reduced and the pattern gets blurred ($V < 1$, and $V \to 0$ with increasing source width)

Note: A number of students may have misread the question, and thought that either the slit width increased or the ~~[illegible]~~ distance between the slits increased.

b) note contd.

increasing slit width ⇒ ~~more~~ less diffraction

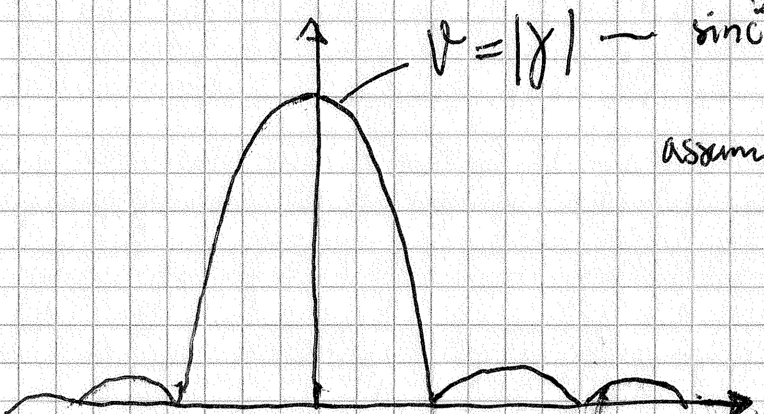
Increasing slit spacing (more likely misunderstanding)
– changes period of oscillations
see also note below.

c) The visibility $\mathcal{V} = \dfrac{I_{max} - I_{min}}{I_{max} + I_{min}} = |\gamma(0,a)|$

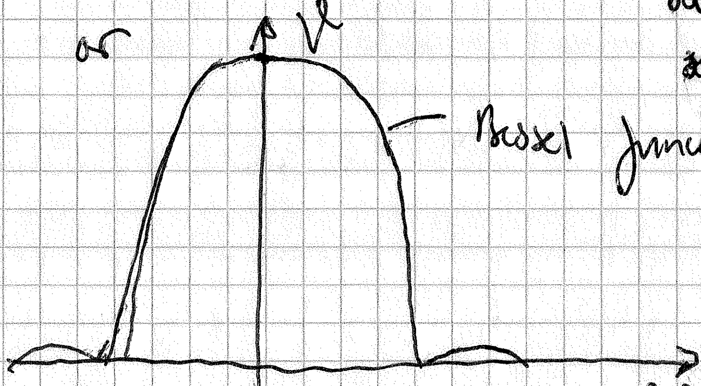
distance between sources

degree of coherence

$|\gamma(0,a)|$ is given by the Fourier Transform of the source. For a rectangular source it becomes a sinc function and for a ~~Bessel~~ circular source it becomes a Bessel function.



or



2. Use $\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f}$ in two steps

1st $\quad \frac{1}{7}+\frac{1}{s_i}=-\frac{1}{4} \Rightarrow \frac{1}{s_i}=-\frac{1}{4}-\frac{1}{7}=\frac{-4-7}{7\cdot 4} \Rightarrow s_i=-\frac{28}{11}$

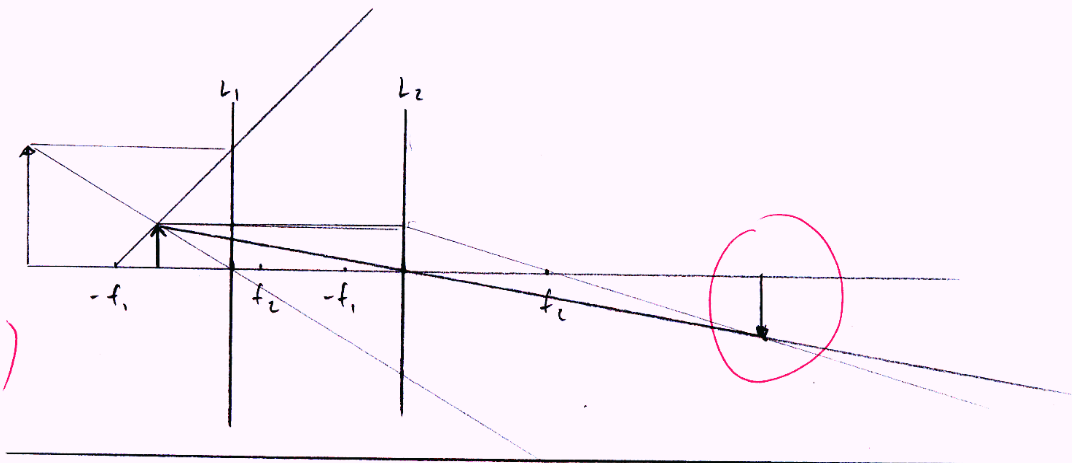
2nd $\quad s_o'=6+\frac{28}{11}=\frac{94}{11}$

$$\frac{11}{94}+\frac{1}{s_i'}=\frac{1}{5} \Rightarrow \frac{1}{s_i'}=\frac{1\cdot 94}{5\cdot 94}-\frac{11\cdot 5}{94\cdot 5}=\frac{94-55}{470} \Rightarrow s_i'=\frac{470}{39}\approx 12.5 \text{ cm}$$ after lens 2.

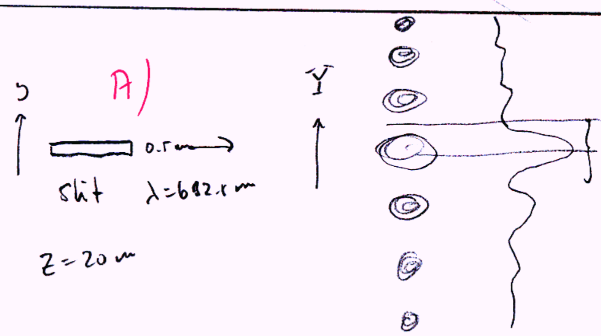
A)

$$M=\left(-\frac{s_i}{s_o}\right)\left(-\frac{s_i'}{s_o'}\right)=\left(-\frac{-\frac{28}{11}}{\frac{7}{1}}\right)\left(-\frac{\frac{470}{39}}{\frac{94}{11}}\right)=\left(\frac{4}{11}\right)\left(-\frac{11\cdot 5}{39}\right)=-\frac{20}{39}$$

B)



3.) A)



$\left(\frac{\sin \pi x}{\pi x}\right)^2$ function

$\updownarrow Y_{min}$

distance between 1st minima on each side

B): $2Y_{min}=\frac{2z\cdot\lambda}{b}=5.06$ cm

$k=\frac{2\pi}{\lambda}$

B) See eq. problem 7: $\iint e^{\frac{ik\cdot Y\cdot y}{z_2}}\,dy$ gives FT

$$\int_{-\frac{b}{2}}^{\frac{b}{2}} e^{i\frac{2\pi}{z\lambda}\cdot Y y}\,dy=\left[\frac{e^{i\alpha y}}{i\alpha}\right]_{-\frac{b}{2}}^{\frac{b}{2}}=\left(\frac{e^{\frac{ib\alpha}{2}}-e^{-\frac{ib\alpha}{2}}}{2i\alpha\frac{1}{2}b}\right)=\frac{\sin\left(\frac{b\alpha}{2}\right)}{\left(\frac{b\alpha}{2}\right)}=\frac{\sin\left(\frac{b\pi Y}{z\lambda}\right)}{\left(\frac{b\pi Y}{z\lambda}\right)}$$

($\alpha = \frac{2\pi}{z\lambda}$)

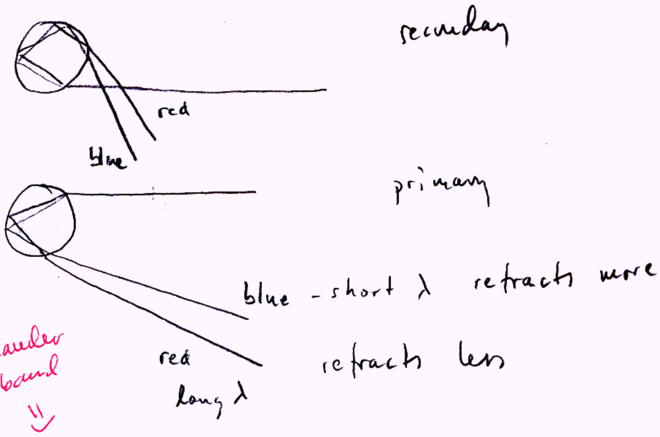
intensity $\Rightarrow$ square $\Rightarrow I_Y \propto \frac{\sin^2\left(\frac{b\pi Y}{z\lambda}\right)}{\left(\frac{b\pi Y}{z\lambda}\right)^2}$

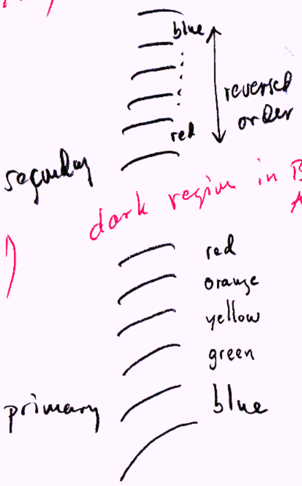
min at

$Y\frac{b\pi}{z\cdot\lambda}=\pi \Rightarrow Y_{min}=\frac{z\cdot\lambda}{b}$

4.

A:)





dark region in B - Alexander band

⇓

region A brightest

B:)

5.

Ole Christian Römer (1644-1710): Proved speed of light finite. Good estimation.

Johannes Kepler (1571-1630): Total internal reflection

Thomas Young (1773-1829): Interference, transversal waves.

James Maxwell (1831-1879): EM-theory: Unifying static & dynamic EM.

↑ A ↑ B

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7. A) Quadratic phase-exponential in integral should vanish i.e. $\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$. The FT is to be found where the point source is imaged!!

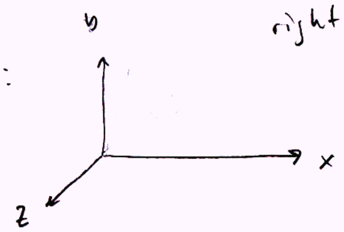
B) $F(f_x) = \int_{-\infty}^{\infty} f(x)\, e^{i2\pi x \cdot f_x}\, dx$ identify: $f_x = \frac{1}{\lambda}\left(\frac{\xi}{z_1} + \frac{u}{z_2}\right)$

$f_y = \frac{1}{\lambda}\left(\frac{\eta}{z_1} + \frac{v}{z_2}\right)$

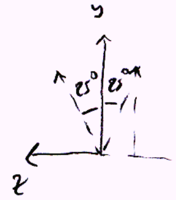
spatial frequency

other conventions are possible

6:



right ON system



both ok

$\Rightarrow E_{0y} = E_0 \cdot \cos 25^\circ$

$E_{0z} = \pm E_0 \sin 25^\circ$

general Amplitude

Generally: $E(x,t) = A \cdot \cos(kx - \omega t + \phi)$

$k = \frac{2\pi}{\lambda} \qquad \frac{\omega}{k} = c \Rightarrow \omega = \frac{2\pi}{\lambda} \cdot c$

$$\Rightarrow E(x,t) = A \cdot \cos\left(\frac{2\pi}{\lambda}(x - ct) + \phi\right)$$

Amplitude zero at $x = 0,\ t = 0 \Rightarrow$

$0 = A \cos(0 + \phi) \Rightarrow \phi = 90^\circ$ (or 270°)

or 270°

A: $\Rightarrow \bar{E}(x,t) = \left(E_0 \cos 25^\circ\, \hat{y} \underset{(-)}{+} E_0 \sin 25^\circ\, \hat{z}\right) \cos\left[\frac{2\pi}{\lambda}(x - ct) + 90^\circ\right]$

B: $\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$

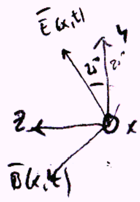
$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y(x,t) & E_z(x,t) \end{vmatrix}$$

$$\hat{x}\left(\underbrace{\frac{\partial E_z}{\partial y}}_{=0} - \underbrace{\frac{\partial E_y}{\partial z}}_{=0}\right) + \hat{y} \cdot \underbrace{\left(0 - \frac{\partial}{\partial x} E_z(x,t)\right)}_{-\frac{\partial}{\partial t} B_y} + \hat{z} \underbrace{\left(\frac{\partial}{\partial x} E_y(x,t) + 0\right)}_{-\frac{\partial}{\partial t} B_z}$$

$\bar{E}$ & $\bar{B}$ must have the same phasor (time & x-dependence)

$\Rightarrow \hat{y}: \left(-\frac{2\pi}{\lambda} \cdot E_0 \sin 25^\circ\right) = -\left(-\frac{2\pi}{\lambda} c\right) \cdot B_y \Rightarrow B_y = -\frac{E_0}{c} \cdot \sin 25^\circ$

$\hat{z}: \frac{2\pi}{\lambda} \cdot E_0 \cos 25^\circ = \left(-\frac{2\pi}{\lambda} c\right) \cdot B_z \Rightarrow B_z = \frac{E_0}{c} \cdot \cos 25^\circ$



$\therefore \bar{B}(x,t) = \frac{1}{c} \cdot \left(\underset{(+)}{-} E_0 \sin 25^\circ\, \hat{y} + E_0 \cos 25^\circ\, \hat{z}\right) \cdot \cos\left[\frac{2\pi}{\lambda}(x - ct) + 90^\circ\right]$

8 Lösning $L = \dfrac{3.125 + 2.083}{2} = 2.604$

Taken together the system matrix will be:

$$\begin{bmatrix} y_{out} \\ \alpha_{out} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{bmatrix}}_{\text{lens 2}} \underbrace{\begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}}_{\text{free prop.}} \underbrace{\begin{bmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{bmatrix}}_{\text{lens 1}} \begin{bmatrix} y_{in} \\ \alpha_{in} \end{bmatrix}$$

include numbers

$$M_{tot} = \begin{bmatrix} 1 - \frac{L}{f_1} & L \\ \frac{1}{f_2}\left(\frac{L}{f_1} - 1\right) - \frac{1}{f_1} & 1 - \frac{L}{f_2} \end{bmatrix} = \begin{bmatrix} 0.16672 & 2.604 \\ -0.4000 & -0.25 \end{bmatrix}$$

A) from formula page

$p = \dfrac{D}{C} = 0.685 \rightarrow F_1$

$q = -\dfrac{A}{C} = 0.4168 \rightarrow F_2$

$r = \dfrac{D-1}{C} = 3.125 \rightarrow H_1$

$s = \dfrac{1-A}{C} = -2.0832 \rightarrow H_2$

$n_0 = n_f$

$v = \dfrac{D-1}{C} = 3.125 \rightarrow N_1$

$w = \dfrac{1-A}{C} = -2.0832 \rightarrow N_2$

B) use thin lens formula

1) $\dfrac{1}{10} + \dfrac{1}{S_i} = \dfrac{1}{3.125} \Rightarrow S_i = 4.5454$

2) new $S_o' \equiv L - 4.5454 = -1.9414$

$\dfrac{-1}{1.9414} + \dfrac{1}{S_i'} = \dfrac{1}{2.083} \Rightarrow S_i' = 1\text{ cm}$

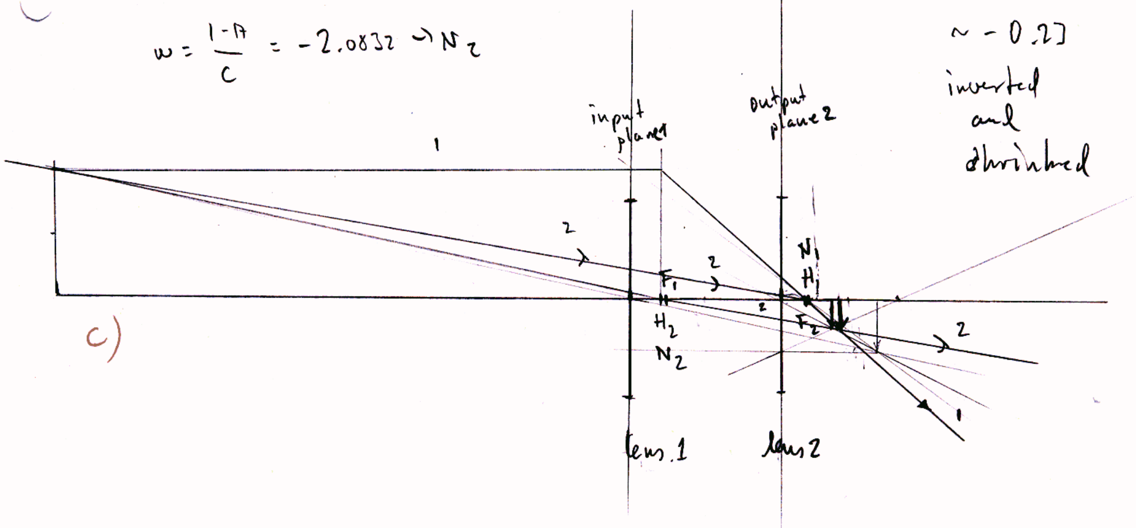
$M_{tot} = M_1 \cdot M_2 = \left(-\dfrac{S_i}{S_o}\right)\left(-\dfrac{S_i'}{S_o'}\right) = \left(\dfrac{4.54}{10}\right)\left(-\dfrac{1}{-1.99}\right)$

$= -0.227$

~ -0.23

inverted and shrinked

c)

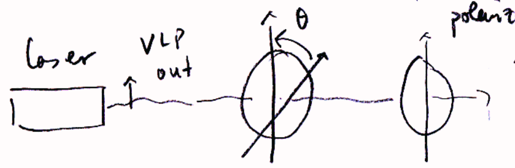


9)

A:)

	0° out	0°	45° out	45°	90° out	90°	in
QWP	$\begin{pmatrix} 0 \\ \frac{1}{\sqrt2}+\frac{i}{\sqrt2} \end{pmatrix}$ VLP	$\begin{pmatrix} \frac{1}{\sqrt2}+\frac{i}{\sqrt2} & 0 \\ 0 & \frac{1}{\sqrt2}-\frac{i}{\sqrt2} \end{pmatrix}$	$\begin{pmatrix} \frac{i}{\sqrt2} \\ \frac{1}{\sqrt2} \end{pmatrix}$ RCP	$\begin{pmatrix} \frac{1}{\sqrt2} & \frac{i}{\sqrt2} \\ \frac{i}{\sqrt2} & \frac{1}{\sqrt2} \end{pmatrix}$	$\begin{pmatrix} 0 \\ \frac{1}{\sqrt2}+\frac{i}{\sqrt2} \end{pmatrix}$ VLP	$\begin{pmatrix} \frac{1}{\sqrt2}-\frac{i}{\sqrt2} & 0 \\ 0 & \frac{1}{\sqrt2}+\frac{i}{\sqrt2} \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
HWP	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ VLP	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ HLP	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ VLP	$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

B)



HWP rotates polarization depending on θ

$$\Rightarrow \text{out} = \begin{pmatrix} \sin 2\theta \\ -\cos 2\theta \end{pmatrix} = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Then vertical polarizer takes out vertical component

$$\text{final} = \begin{pmatrix} 0 \\ -\cos 2\theta \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}}_{\text{vertical polarizer}} \underbrace{\begin{pmatrix} \sin 2\theta \\ -\cos 2\theta \end{pmatrix}}_{\text{from HWP}}$$

intensity $\propto (-\cos 2\theta)^2$

C) 0 out $\Rightarrow$ set $\theta = 45°$ $(\cos 90°)^2 = 0$

50% out $\Rightarrow$ set $\cos^2 2\theta = \frac{1}{2} \Rightarrow \theta = 22.5°$

100% out $\Rightarrow$ $\underbrace{\text{set } \theta = 0°}_{\text{HWP}}$ $(\cos 0°)^2 = 1$

LP always vertical

Problem 10:

(A) $\hat{r}_{012}^{s,p} = r_{01}^{s,p} + t_{01}^{s,p} r_{12}^{s,p} t_{10}^{s,p} e^{i\delta}$

(optional, according to text in ~~[illegible]~~ problem, use that
$t_{01}^{s,p} t_{10}^{s,p} = 1 - (r_{01}^{s,p})^2$, bonus?)

(B) $d_1 \to 0 \Rightarrow \delta \to 0$

$\hat{r}_{02}^{s,p} = \hat{r}_{01}^{s,p} + t_{01}^{s,p} r_{12}^{s,p} t_{10}^{s,p}$ (also accept this answer)

This happens for $\delta = 0, 2\pi, 4\pi, 6\pi, \ldots$

(C) $R^{s,p} = |\hat{r}_{012}^{s,p}|^2 = |r_{01}^{s,p}|^2 + |t_{01}^{s,p}|^2 |t_{10}^{s,p}|^2 |r_{12}^{s,p}|^2 + 2\,\mathrm{Re}\{ r_{01}^{s,p} (t_{01}^{s,p})^* (t_{10}^{s,p})^* r_{12}^{s,p\,*} \times e^{-i\delta} \}$

$= \overbrace{|r_{01}^{s,p}|^2 + |t_{01}^{s,p}|^2 |t_{10}^{s,p}|^2 |r_{12}^{s,p}|^2}^{a} + \overbrace{2 |r_{01}^{s,p}| |t_{01}^{s,p}| |t_{10}^{s,p}| |r_{12}^{s,p}|}^{b} \cos(\delta')$

optional, bonus

May analyze ~~[illegible]~~ the details of δ' to include phase shifts upon reflection:

$r_{01}^{s,p} = |r_{01}^{s,p}| e^{i\delta_{r_{01}}^{s,p}}$, $r_{12}^{s,p} = |r_{12}^{s,p}| e^{i\delta_{r_{12}}^{s,p}}$, $t_{01}^{s,p} = |t_{01}^{s,p}| e^{i\delta_{t_{01}}^{s,p}}$

$\delta' = -\delta + \delta_{r_{01}}^{s,p} - \delta_{t_{01}}^{s,p} - \delta_{t_{10}}^{s,p} - \delta_{r_{12}}^{s,p}$

$\delta_{t_{01}}^{s,p} = \delta_{t_{10}}^{s,p} = 0$ from considerations of transmission coefficient (recall that SiO_2 is transparent) $\{(1 - r_{01}^2) > 0$ & real$\}$

$\delta_{r_{01}}^{s} = \pi$ and $\delta_{r_{12}}^{s} = \pi \Rightarrow \delta' = -\delta$ for s-pol. light, transparent materials

$\delta_{r_{01}}^{p} = 0$ and $\delta_{r_{12}}^{p} = 0$, for $\theta_i < \theta_{Brewster} \Rightarrow \delta' = -\delta$

Hence $a = |r_{01}^{s,p}|^2 + |t_{01}^{s,p}|^2 |t_{10}^{s,p}|^2 |r_{12}^{s,p}|^2$ $\Big\{$ for $\theta_i < \theta_{Brewster}$, and transparent materials

$b = 2 |r_{01}^{s,p}| |t_{01}^{s,p}| |t_{10}^{s,p}| |r_{12}^{s,p}|$ ~~[illegible]~~

$|\delta'| = |\delta|$

(c)

Also accept answers of form:

$$R^{s,p} = (r_{01}^{s,p})^2 + \underbrace{\left(1 - r_{01}^{s,p\,2}\right)^2}_{\text{or } t_{01}^{s,p} t_{10}^{s,p}} r_{12}^{s,p\,2} + 2 r_{01}^{s,p} \underbrace{\left(1 - r_{01}^{s,p\,2}\right)}_{\text{or } t_{01}^{s,p} t_{10}^{s,p}} r_{12}^{s,p} \cos(\delta)$$

where it is clearly stated that all reflection/transmission coefficient are assumed real, and that phase shifts upon reflection have been neglected.