A1

a) principle of least time, or equivalently minimal optical path

• B

$\mathcal{L}[AB] = \int n \, ds$

• A

$\mathcal{L}[AB] = \min$

$\frac{c}{n} = v$ $\qquad$ $s = vt$

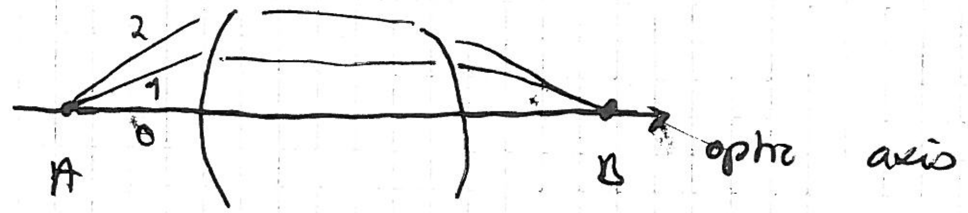
$ds = v \, dt$ $\qquad \rightarrow \qquad \frac{1}{v} = \frac{dt}{ds}$

$n = \frac{c}{v} = c \, d$ $\qquad$ $v = \frac{ds}{dt}$

$\mathcal{L}[AB] = \int n v \, dt = c \int dt$

$\Rightarrow$ equivalent to minimal time.

b)



Consider image from A to B.

perfect image in gen. optics approx
(ignoring astigmatism)
$\Rightarrow$ all rays should be isochronous

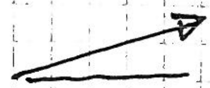
consider rays $\overset{0}{L}[AB] = \overset{1}{L}[AB] = \overset{2}{L}[AB]$
should have identical optical path length or
the time of transit.

The main approximations in Gaussian 1^{st} order optics are:

Assume → 1) Spherical surfaces give perfect imaging 1p

2) Paraxial rays 2p

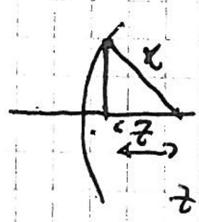
- rays ~~make~~ ~~the~~ making small angle with optic axis



consider $\vec{s} = (\sin\alpha, \sin\beta, \cos\gamma)$

paraxial rays $(\alpha, \beta, 1)$

The ~~from~~ exact refract formula



$$\vec{n}\,(n_2\vec{s}_2 - n_1\vec{s}_1) = 0$$

$z \approx R$

→ simplifies to
$$\begin{bmatrix} y \\ \alpha \end{bmatrix}' = \begin{bmatrix} 1 & 0 \\ \frac{n'}{n} & 1 \end{bmatrix} \begin{bmatrix} y \\ \alpha \end{bmatrix}$$

3) No absorption or reflection losses 1p

4) Rotationally symmetric surfaces. 1p

$\boxed{5p}$

Answer ~~A1~~ A1.d)

$$\nabla n(x) = \frac{d}{d\sigma}\left(n\vec{s}\right)$$

$$\vec{s} = \frac{d\vec{r}}{d\sigma} = \frac{dx}{d\sigma}\hat{x} + \frac{dy}{d\sigma}\hat{y} + \frac{dz}{d\sigma}\hat{z}$$

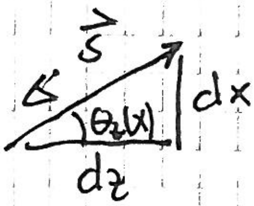
The RAY EQN IN COMPONENT FORM:

1.5p

$$\cancel{\frac{dx}{d\sigma}}\ \frac{dn}{dx} = \frac{d}{d\sigma}\left(n(x)\frac{dx}{d\sigma}\right)$$

$$\frac{dn}{dy} = \frac{d}{d\sigma}\left(n(x)\frac{dy}{d\sigma}\right) \qquad 0 = 0$$

$$\frac{dn}{dz} = \frac{d}{d\sigma}\left(n\frac{dz}{d\sigma}\right) = 0$$



$$\frac{dx}{d\sigma} = \sin\theta_z(x)$$

$$\frac{dz}{d\sigma} = \cos\theta_z(x)$$

$$\frac{d}{d\sigma}\left(n(x)\cos\theta_z(x)\right) = 0$$

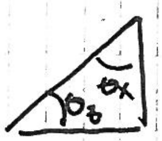
$$n(x)\cos\theta_z(x) = \text{const}$$

$$n(0)\cos\theta_z(0) = C$$

$\boxed{1.5p}$

Here $\underline{n(x)\cos\theta_z(x) = n(0)\cos\theta_z(0)}$, Generalised Snell's law.

May be [illegible]



$$\cos\theta_z = \sin\theta_x$$

May re-express in terms of $\theta_x(x)$

$\Rightarrow$

$$\underline{n(x)\sin\theta_x(x) = n(0)\sin\theta_x(0)}$$

i.e. a generalisation of Snell's law.

$$\sin\theta_x(x) = \frac{n(0)\sin\theta_x(0)}{1.3-|x|}$$

if $\dfrac{n(0)\sin\theta_x(0)}{1.3-|x|} \geq 1$, then ray must turn by total internal reflection

1b RAY TURNS BY TOTAL INTERNAL REFLECTION.

~~[illegible]~~

At turning point $\theta_z(x) = 0$

$$n(x_{tp}) = n(0)\cos\theta_z(0)$$

$$1.3 - |x| = 1.3\cos\theta_z(0)$$

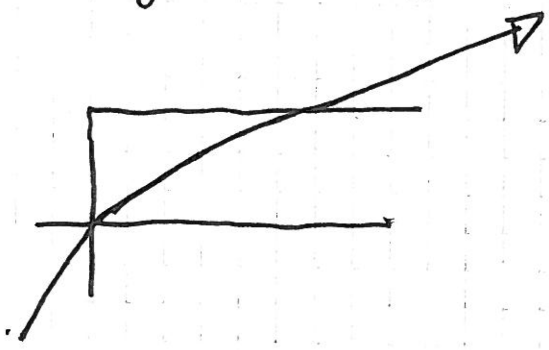
$$|x_{tp}| = 1.3\left(1-\cos\theta_z(0)\right)$$

$$= 1.3\left(1-\sqrt{1-\sin^2\theta_z(0)}\right)$$

$$= 1.3\left(1-\sqrt{1-\left(\tfrac{1}{1.3}\right)^2\sin^2\theta_0}\right)$$

[2p] $\underline{|x_{tp}| = 1.3 - \sqrt{1.3^2 - \sin^2\theta_0}} = 1.3 - \sqrt{1.3^2 - \sin^2 30} = \underline{\underline{0.1}}$

The ray will exit



if $\dfrac{n(0)\cos\theta_z(0)}{1\ \cancel{1.3} = \cancel{1}} < 1$ $\qquad 1.3 - \cancel{B}\,\omega 3 = 1$

$$\cos\theta_z(0) = \frac{1}{n(0)}\sqrt{n(0)^2 - \sin^2\theta_0}$$

$$\sqrt{n^2(0) - \sin^2\theta_0} < 1$$

$$n^2(0) - 1 < \sin^2\theta_0$$

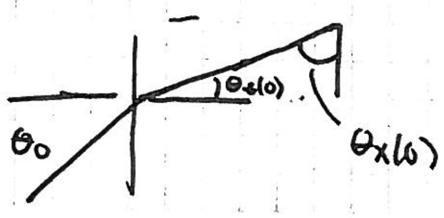
$$\sin\theta_0 > \sqrt{n^2(0) - 1} = \sqrt{1.3^2 - 1} = 0.83$$

[3p] $\underline{\theta_0 > 56.1^\circ}$

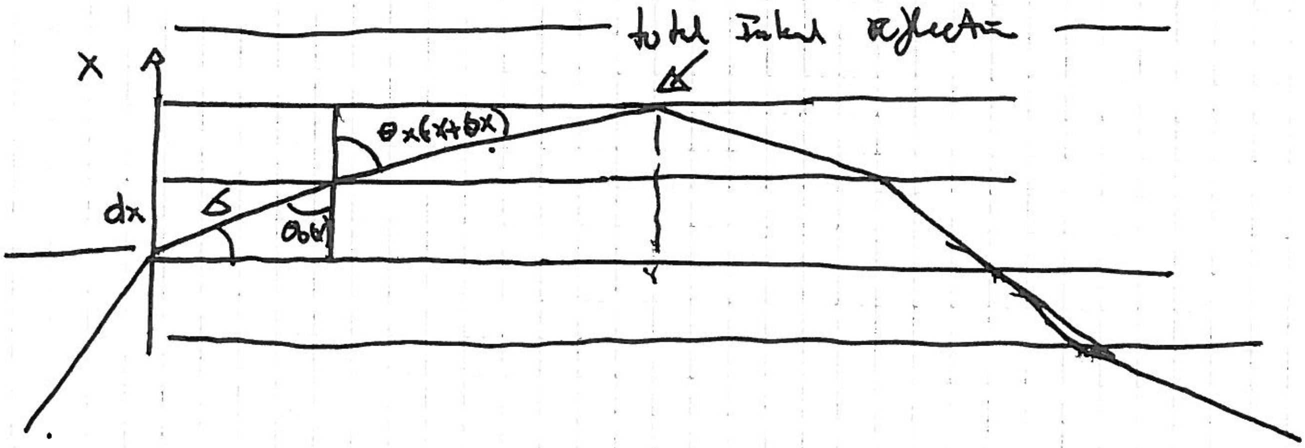
May give some credit for suggested answer 1p

A1 c

The ray will just refract at interface air



Method 1) subdivide into several discrete layers & applying Snell's law for each layer.



$$n(x) \sin\theta_x(x) = n(x+dx) \sin\theta_x(x+dx)$$

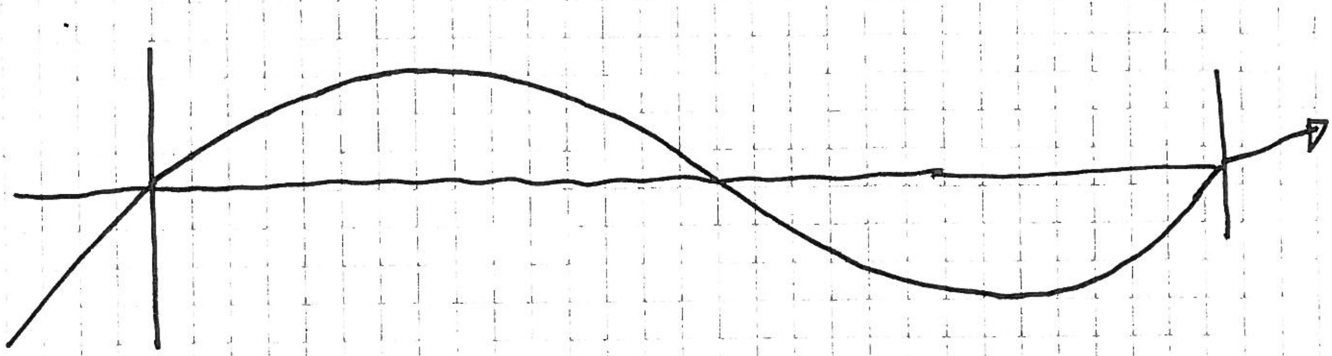
as $n(x+dx) < n(x)$ ray will refract away from the x-axis

~~[illegible]~~

If ~~the~~ $\sin\theta_x(x+dx) = \dfrac{n(x) \sin\theta_x}{n(x+dx)} > 1$

then total internal reflection takes place. The ray will thus turn.

The procedure is identical on each side ⟹

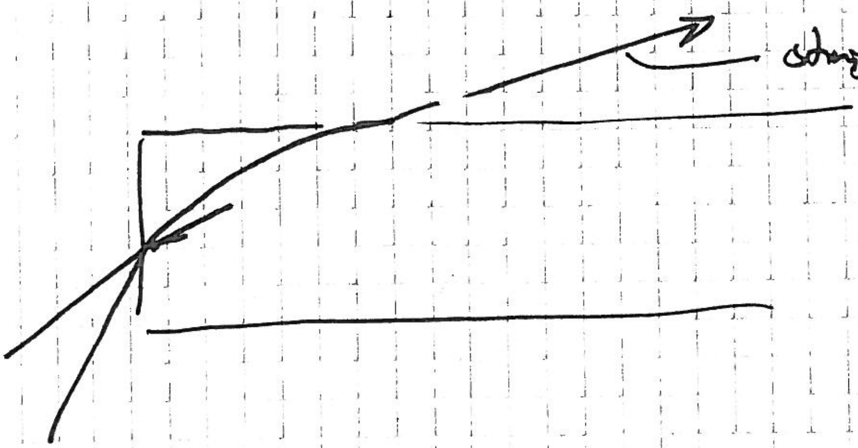


If ray is bound

if the ray to angle θ_0 seems too large,

it is expected that the ray may not turn

between $|x| < 0.3$ ⟹ exits at is lost



straight line prop. in homogeneous medium.

This is identical to not reaching the

total internal reflection condition

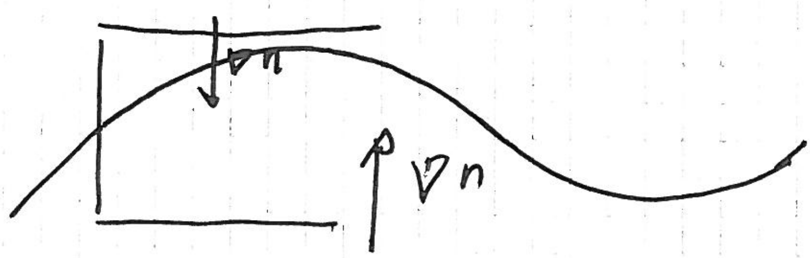
.

Method (2)

The ray eqn. gives

$$\nabla n = \frac{d}{ds}(n\vec{s})$$

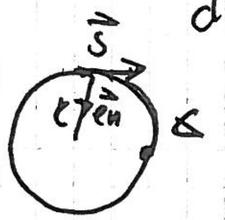
From experience with mirages etc., the ray will turn in the direction of ∇n



"Proof"

$$\nabla n = \frac{dn}{ds}\vec{s} + n\frac{d\vec{s}}{ds}$$

let $\frac{d\vec{s}}{ds} \stackrel{\Delta}{=} \frac{\vec{e_n}}{R}$



R is radius of curvature

$$\vec{e_n}\cdot\nabla n = \frac{n}{R}$$

$R > 0$ always, therefore ∇n is in direction of $\vec{e_n}$

$$\Rightarrow \quad \frac{d}{dz}(n \sin\theta_x) = \frac{dn}{dx} = \text{~~~} \quad (=-1)$$

$$\frac{d}{dz}(n \sin\theta_x) = 0$$

$$n \sin\theta_x = \text{const} \qquad = n \cos\theta_z$$

$$n(0) \sin\theta_x(0) = n(0) \cos\theta_z(0)$$

$$= \sqrt{n^2(0) - \sin^2\theta_a}$$

$$\cancel{n \cos\theta_x = f(z)}$$

$$\cancel{n \cos\theta_x = }$$

$$\Rightarrow \quad n(x) \sin\theta_x(x) = \sqrt{n^2(0) - \sin^2\theta_a}$$

$$\sin\theta_x(x) = \frac{\sqrt{n^2(0) - \sin^2\theta_a}}{n(x)}$$

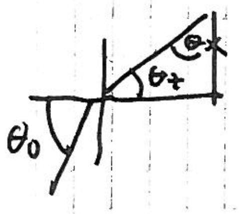
i.e. in correspondence with the eqn. !!

Find expression for $\sin\theta_x(x)$ from ray eqn.

From first argument

$$n(x+dx)\sin\theta_x(x+dx) = n(x)\sin\theta_x(x) \cdots n(0)\sin\theta(0)$$

$$\sin\theta_x(0) = \cos\theta_z(0) = \sqrt{1-\frac{1}{n(0)}\sin^2\theta_0}$$



$$= \frac{1}{n(0)}\sqrt{n(0) - \sin^2\theta_0}$$

$$\Rightarrow n(x)\sin\theta_x(x) = \sqrt{n(0)-\sin^2\theta_0}$$

$$\sin\theta_x(x) = \frac{\sqrt{n(0)-\sin^2\theta_0}}{n(x)}$$

$$= \frac{\sqrt{n(0)-\sin^2\theta_0}}{1.3-|x|}$$

✱ May be proven, based on the ray eqn., then

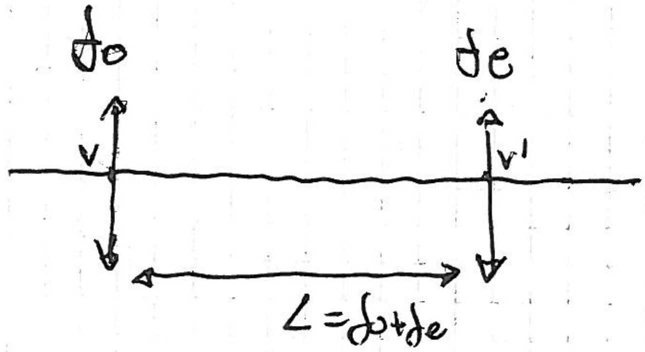
let
$$\frac{dn}{dx}\hat{x} = \frac{d}{d\sigma}\left(n s_x \hat{x} + n s_z \hat{z}\right)$$

$$= \frac{d}{d\sigma}\left(n \sin\theta_z \hat{x} + n\cos\theta_z \hat{z}\right)$$

$$\frac{dn}{dx} \Rightarrow = \frac{d}{d\sigma}\left(n\cos\theta_x\hat{x} + n\sin\theta_x\hat{z}\right)$$

A2

a)



δ min

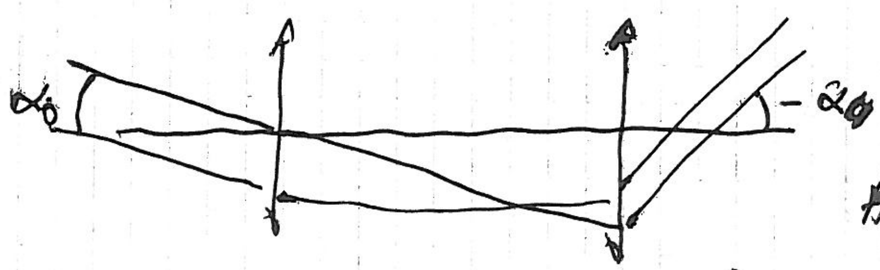
$$M_{vv'} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_e} & 1 \end{bmatrix} \begin{bmatrix} 1 & f_e + f_o \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_o} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_e} & 1 \end{bmatrix} \begin{bmatrix} -f_e/f_o & f_e + f_o \\ -\frac{1}{f_o} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -f_e/f_o & f_e + f_o \\ +\frac{1}{f_o} - \frac{1}{f_o} & -\frac{f_o}{f_e} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

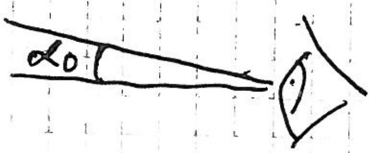
1p the total refraction power $P = -C = -\frac{1}{f_o} + \frac{1}{f_o} =$ ~~$\frac{f_e + f_o}{f_o f_e}$~~

4p $$\begin{bmatrix} y \\ \alpha \end{bmatrix} = \begin{bmatrix} -\frac{f_e}{f_o} & f_e + f_o \\ 0 & -\frac{f_o}{f_e} \end{bmatrix} \begin{bmatrix} y_0 \\ \alpha_0 \end{bmatrix}$$



2p b) Angular magnification is $\frac{\alpha}{\alpha_0} = -\frac{f_o}{f_e}$

b) contd. this is relative to the eye which observes without the telescope



c)

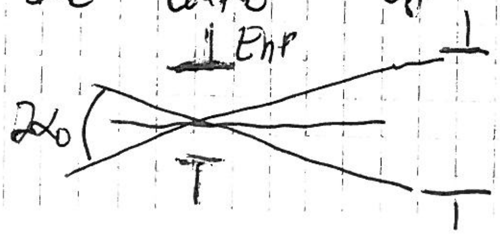
Aperture Stop:

2p The Aperture that limits bundle of rays from object point through the system

2p The entrance pupil is the image of the AS on the object side

2p The exit pupil is the image of the AS on the image side

2p The field stop is the aperture that limits ~~rays~~ the angular field of view, seen from the centre of the EnP



$2\alpha_0$ — Angular field of view.

c) contd.

$D = 30$ cm

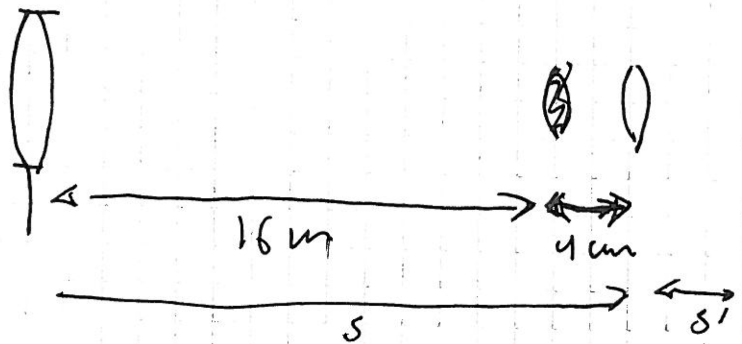


Image of ~~EnP~~ = EnP to image side

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f_e}$$

$$\frac{1}{s'} = \frac{1}{f_e} - \left(\frac{1}{f_o + f_e}\right)$$

$$= \frac{f_o + f_e - f_e}{(f_o + f_e) f_e} = \frac{f_o}{(f_o + f_e) f_e}$$

2p $$s' = \frac{f_e (f_e + f_o)}{f_o} = f_e + \frac{f_e^2}{f_o} = 4\text{ cm} + \frac{16}{1600}\text{ cm}$$

$$\approx 4.01 \text{ cm}$$

$$\cancel{= \frac{1600}{4} = 400 \text{ cm}}$$

2p magnification $m = -\frac{s'}{s} = \approx -\frac{4}{1600}$

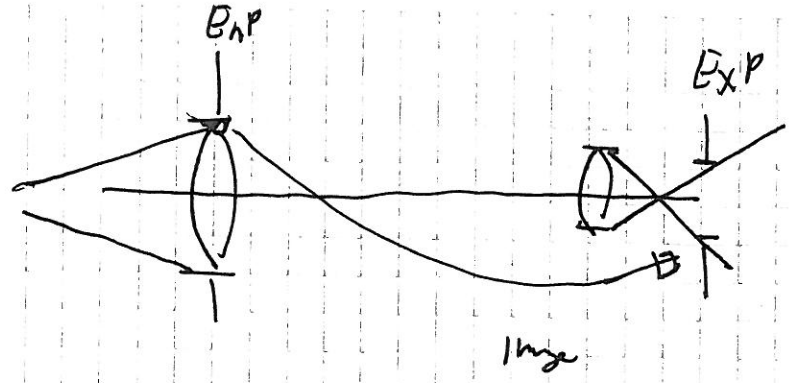
$$\approx -2.5 \times 10^{-3}$$

$$\cancel{\frac{1}{s'} = \frac{1}{f_e} - \left(\frac{1}{f_o + f_e}\right)}$$

$$\cancel{= \quad f_o + f_e}$$

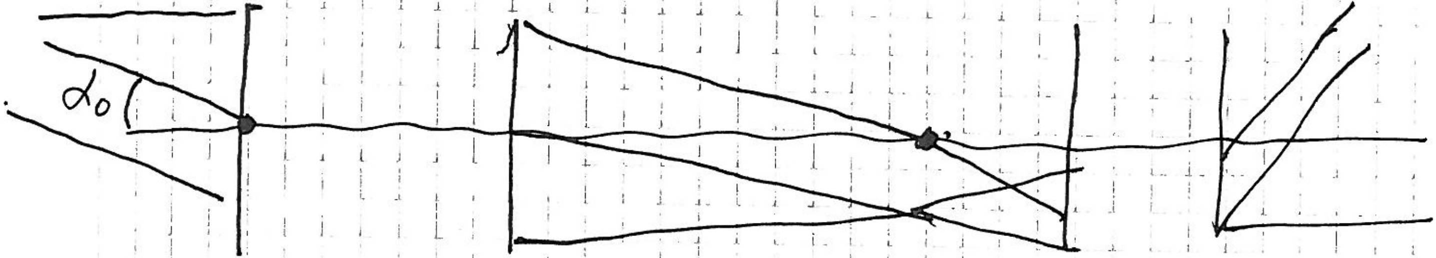
size of Exit Pupil $= m \cdot D = 0.2$ cm

$= \underline{2 \text{ mm}}$



The eye shall be placed at the Exit Pupil of the system. The pupil of the eye ~~will~~ ~~see~~
2p which is 2 mm diameter in day time and ~ 6mm in night time, will thus allow all lights to be received by the ~~the~~ eye, i.e. maximise luminosity.

d)



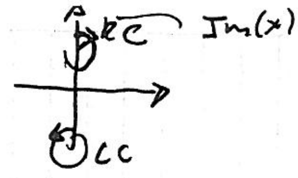
2p The nodal point is identical to the principal plane once submerged in air.

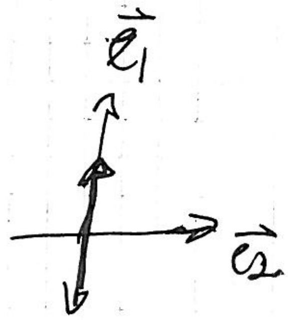
2p parallel rays meet at the focal point

2p Rays from focal plane → parallel

a)

2p $$J = \begin{bmatrix} E_{01} \\ E_{02} e^{i\delta} \end{bmatrix} e^{ikz} e^{-i\omega t}$$

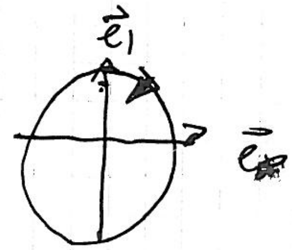




1p $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ — Linearly polarized light

1p $\begin{bmatrix} 1 \\ i \end{bmatrix}$ — Left circular pol. light (Pedrotti)

direction of pol.



1p $\begin{bmatrix} 1 \\ -i \end{bmatrix}$ circ. pol. light

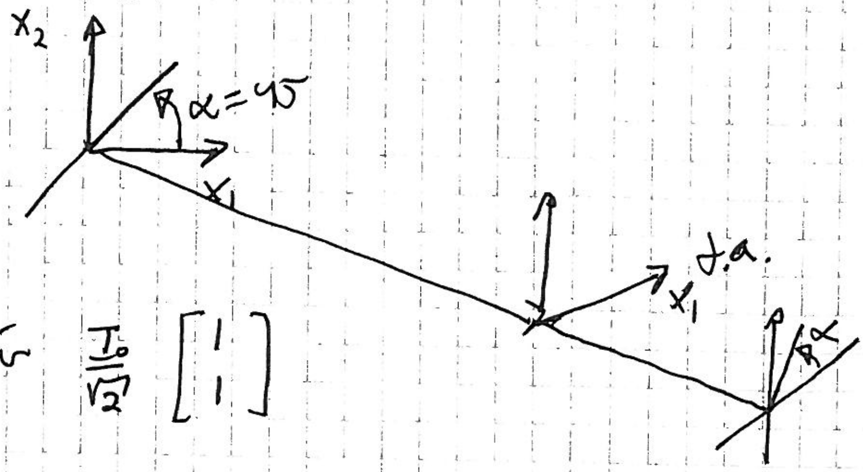
opposite ~~polar~~ rotation (right - Pedrotti)

b) The Jones vector describes only deal with fully polarized light, such as after a perfect polarizer or from a laser source.

3p

The Stokes vector may describe partially polarized light or natural unpolarized light (such as from the sun or a light bulb.

c)



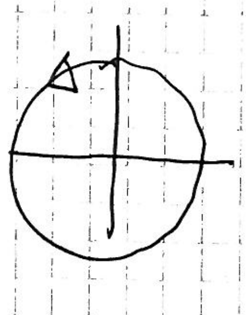
2p After Polarizer $\frac{I_0}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$

QW
~~Half wave~~ plate $\begin{bmatrix}1 & 0\\0 & e^{i\delta}\end{bmatrix} \xrightarrow{\delta=\frac{\pi}{2}} \begin{bmatrix}1 & 0\\0 & i\end{bmatrix}$ along slow axis

1p $\boxed{\delta = \frac{\pi}{2}}$

HW plate $\delta = \pi$

2p $\begin{bmatrix}1 & 0\\0 & i\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}1\\i\end{bmatrix}$ – Left Circular Light



2p $E(\alpha) = R(-\alpha)\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}R(\alpha)\begin{bmatrix}1\\i\end{bmatrix}$

2p $I = E^{\dagger}(\alpha)\,E(\alpha) = \frac{I_0}{\sqrt{2}}\,(3)$ possibly some constants

calc or shortcut 3p However, it is not necessary to do the calculation because Intensity should be indep. of the rotation of the polarizer

B2 Interference from thin film:

b) $\delta = 0$ for $d \to 0$ (no thickness of film)

$\Rightarrow \quad r = r_{01} + \cancel{r_{12}} z_1 e^{i\delta} + z_2 e^{2i\delta}$

$\Rightarrow \quad r = r_{01} + z_1 + z_2 \ldots$

must be identical to the substrate without film.

2p

~~The argum~~

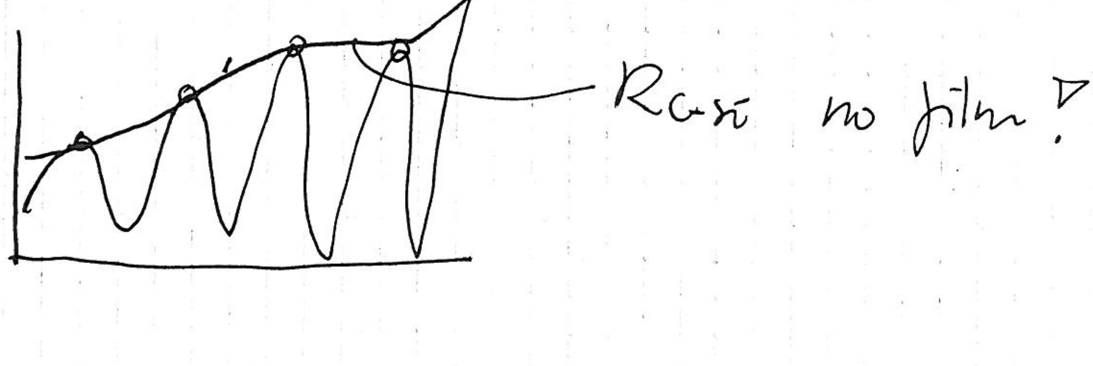
If there is a film $\delta \neq 0$, hence

$r_{12}\, e^{i\delta} = (\cos\delta + i\sin\delta)\, |r_{12}^{s,p}|\, e^{i\delta_{B,p}}$

Must assume that the reflectance is lower with film ~ AR properties

The maximum in the figure must thus correspond to $\delta = 0, 2\pi, 4\pi \ldots$

2p



b) Having assumed that $\delta = 0, 2\pi, 4\pi$ correspond to maxima

$\Rightarrow$ difference between two maxima must be separated by 2π

$$\Rightarrow \frac{4\pi d_1}{\lambda}\sqrt{n_1^2 - n_0^2 m^2 q_{i+1}} - \frac{4\pi d_1}{\lambda}\sqrt{n_1^2 - n_0^2 n^2 q_i} = 2\pi$$

3p
$$d_1 = \frac{\lambda}{2}\left(\frac{1}{\sqrt{n_1^2 - n_0^2 n^2 q_{i+1}} - \sqrt{n_1^2 - n_0^2 n^2 q_i}}\right)$$

$$n(600, SiO_2) = 1.448 + \frac{3642}{(600)^2} = \underline{1.4581}$$

2p
$$d_1 = \cancel{}\,300 \cdot \overset{16.63}{\cancel{0.016}} = \cancel{4}\ 4989 \text{ nm}$$

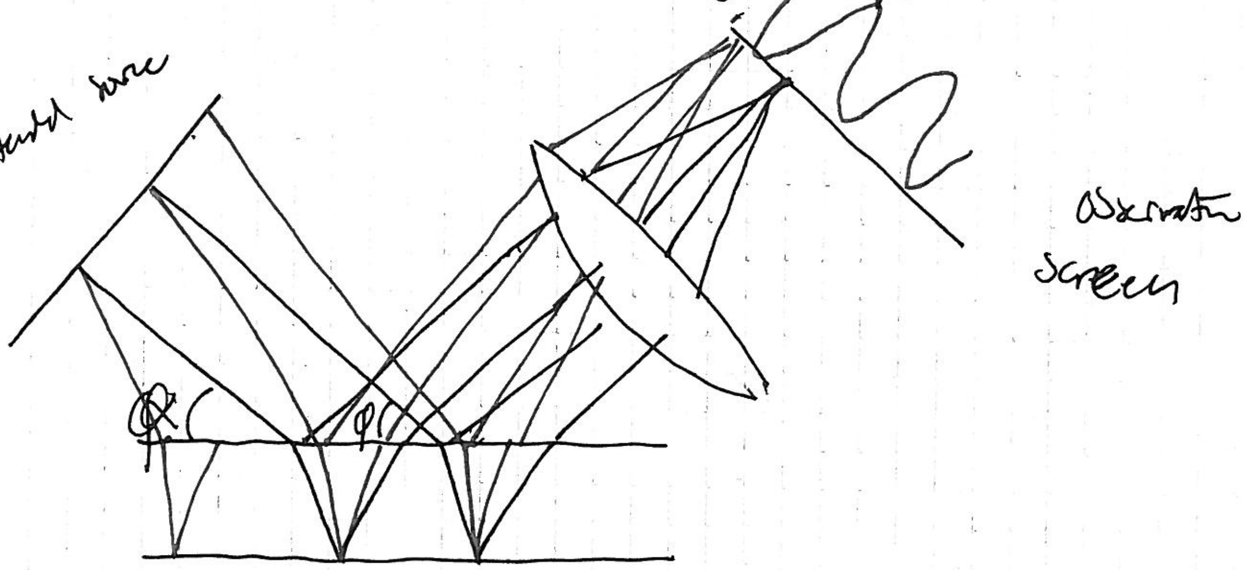
$$\approx \underline{5\ \mu m}$$

$$\sqrt{n_1^2 - \sin^2 47.25} = \cancel{0.98858042}\ 1.2597712$$

$$\sqrt{n_1^2 - \sin^2 38.3} = 1.31984$$

c) This must be the Haidinger experiment:

5p



Fringes are observed as a function of φ on the screen!

a)

$$r = \sqrt{(X - x)^2 + (Y - y)^2 + z^2}$$

expand

$$X^2 + x^2 - 2Xx + Y^2 + y^2 - 2Yy + z^2$$

choose to expand as $R = X^2 + Y^2 + z^2$

small

25p $$r = R\left(1 - \frac{(2Xx + 2Yy)}{R^2} + \frac{(\cancel{X}^2 + \cancel{y}^2)}{R^2}\right)^{\frac{1}{2}}$$

perform binomial expansion $(1+b)^{\frac{1}{2}} \approx 1 + \frac{1}{2}b$

$$r = \left(R - \frac{(Xx + Yy)}{\cancel{R}} + \frac{1}{2R}(X^2 + Y^2)\right)$$

The Fresnel refraction [illegible] can thus be written as:

25p $$U(X,y,z) = \frac{1}{i\lambda} e^{ikR} \cancel{\frac{z}{R^2}} \iint U(x,y,0) e^{\frac{ik}{2R}(X^2+Y^2)} \times e^{-ik(Xx + Yy)} \, dx\,dy$$

$\approx z$

$z \approx R$ $\qquad \frac{1}{R} \sim \frac{1}{z}$

The phase function of a lens

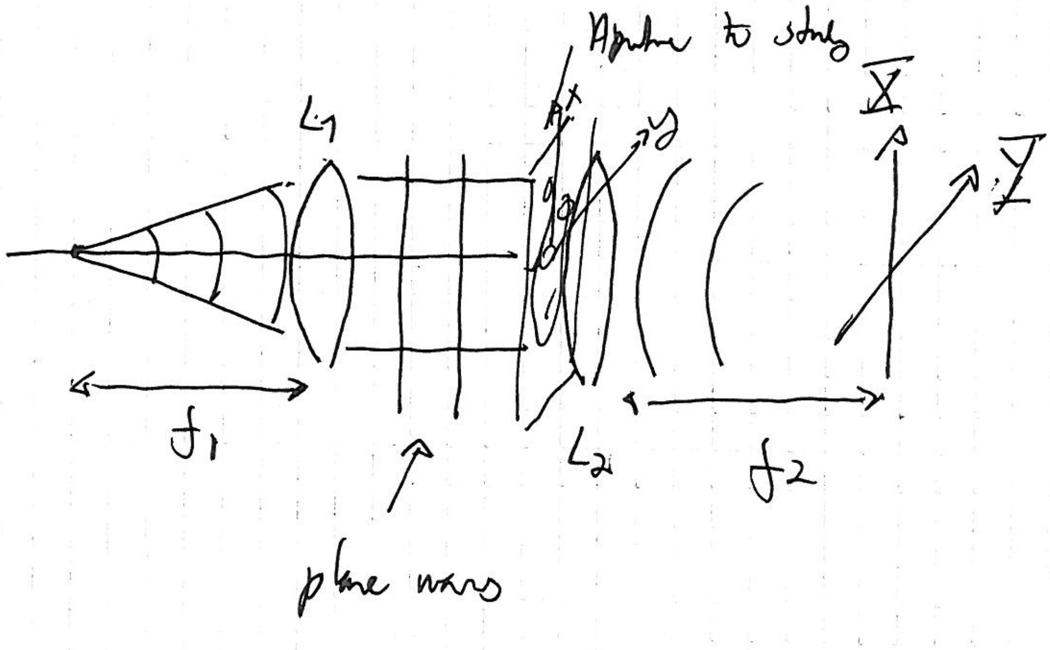
3ρ when inserted just after the aperture will cancel the term : $e^{\frac{ik}{2R}(X^2+Y^2)}$ in the

paraxial approximation $(R = z = f)$

$$\Rightarrow \quad U(X,Y,z) = \frac{1}{i\lambda z} e^{ikz} \iint U(x,y,0)\, t_{lens}(x,y)\, \cancel{e^{-\frac{ik}{2f}(x^2+y^2)}}\, e^{\frac{ik}{2f}(X^2+\dots}$$

$\Rightarrow$ Fraunhofer diffraction formula

3ρ+



c) In this type of problem one uses the convolution theorem in order to get an easy system to handle

1p $t(x,y) \triangleq \dfrac{U^{+}(x,y,0)}{U^{-}(x,y,0)}$ ← Amplitude transmission function

1p Let each hole be represented by a Dirac delta function

~~$\delta(x-a, y-b)$~~

$\delta(x+a, y+b)$ $\qquad$ $\delta(x-a, y-b)$

$\qquad$ $\delta(x+a, y-b)$

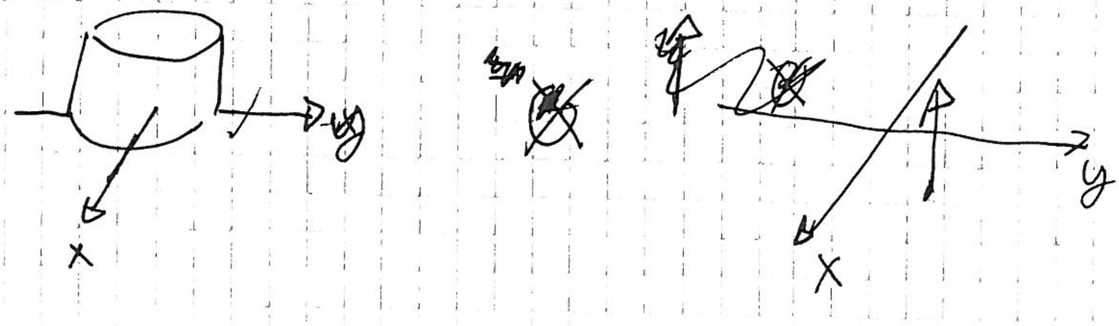
$\delta(x+a, y+b)$

Each hole may be written as a convolution

7p such as

$$\delta(x-a, y-b) \otimes \mathrm{circ}\left(\frac{\rho}{D/2}\right),$$

where $\rho = \sqrt{x^2+y^2}$



The Final [illegible] [illegible] is then

1p $t(x,y) = \operatorname{circ}\left(\frac{s}{D/2}\right) \otimes \Big[\delta(x-a, y-b) + \delta(x+a, y-b)$
$+ \delta(x-a, y+b) + \delta(x+a, y+b)$

C

d)
The [illegible] [illegible] field

3p $$U(x,y,z) = \frac{e^{ikz}}{i\lambda z} \cdot \mathcal{F}\{t(x,y)\}$$

$$\mathcal{F}.\{t(x,y)\} = \mathcal{F}\{\operatorname{circ}(s/(D/2))\}$$
$$\times \Big[e^{ik_X a} e^{ik_Y b} + e^{-ik_X a} e^{ik_Y b} + e^{ik_X a} e^{-ik_Y b} + e^{-ik_X a} e^{-ik_Y b} \Big\}$$

$$e^{ik_X a}\left(e^{ik_Y b} + e^{-ik_Y b}\right) + e^{-ik_X a}\left(e^{ik_Y b} + e^{-ik_Y b}\right)$$

$$= 2 \cos k_X a \, 2 \cos k_Y b$$

The Fourier transform of

$$\mathcal{F}\left\{ \mathrm{circ}\left(\frac{r}{D/2} \right) \right\} = 2\pi \left(\frac{D}{2}\right)^2 \frac{J_1(Kd)}{Kd}$$

$$U(x,y,z) = \frac{e^{ikz}}{i\lambda z} \cancel{U(x_0, y_0, 0)}\, \mathcal{F}\{ t(x,y) \}$$

15p

$$= \frac{e^{ikz}}{i\lambda z} \bar{U}(x,y,0) \left\{ 2\pi \left(\frac{D}{2}\right)^2 \frac{J_1(Kd)}{Kd} \right\} \cdot 4 \cos(k_X a) \cos(k_Y$$

Observable intensity

15p

$$\langle I \rangle = U(x,y,z)\, U^*(x,y,z)$$

$$= \frac{\langle \bar{I}_0 \rangle}{\lambda^2 z^2} \cancel{\quad} \,\text{hole}^2 \left(\frac{2 J_1(Kd)}{Kd} \right)^2 \cdot 16 \cos^2(k_X a) \cos^2(k_Y$$

where $k_X = k\frac{X}{z}$, $k_Y = k\frac{Y}{z}$

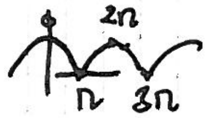
e)

Maxima in intensity given by

op $$k_X a = 2\pi m \wedge k_Y b = 2\pi n$$

$$k_X a = \frac{2\pi}{\lambda}\frac{X}{z}, \quad k_Y b = \frac{2\pi}{\lambda}\frac{Y}{z}$$

$$X = \frac{\lambda z}{a} m \wedge Y = \frac{\lambda z}{b} n$$



Minima

$$k_X a = \pi + m\pi = 2\pi\left(m+\frac{1}{2}\right)$$

$$k_Y b = 2\pi\left(n+\frac{1}{2}\right)$$

$$X = \frac{\lambda z}{a}\left(m+\frac{1}{2}\right)$$

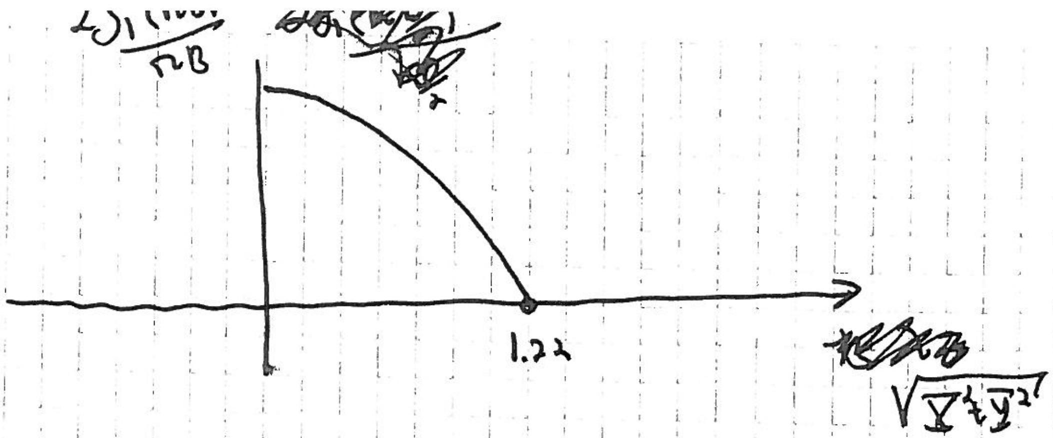
$$Y = \frac{\lambda z}{b}\left(n+\frac{1}{2}\right)$$

If the holes become smaller, then

$$\frac{2J_1(KD/2)}{KD/2} \doteq \frac{2J_1\left(\pi\left(\frac{2}{\lambda z}\sqrt{X^2+Y^2}\right)D/2\right)}{\pi\left(\frac{2}{\lambda z}\sqrt{X^2+Y^2}\right)(D/2)}$$

$$K = \sqrt{k_X^2 + k_Y^2} = k\sqrt{\left(\frac{X}{z}\right)^2+\left(\frac{Y}{z}\right)^2}$$

$$= \frac{2\pi}{\lambda z}\sqrt{X^2+Y^2}$$

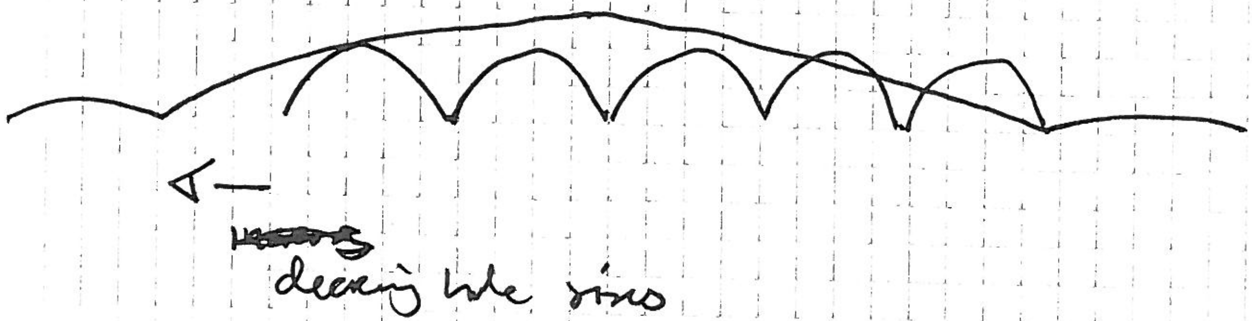


$$B = \frac{\sqrt{X^2+Y^2}}{\lambda z} D$$

If D becomes smaller, then $\sqrt{X^2+Y^2}$ becomes a factor larger to reach 1.22 (first minimum)

2p

Hence more interference fringes should be observed!

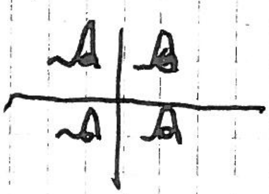


identical D becomes larger

If $\lambda \to 0$, then becomes

2p the geometric optics condition is reached,

and rays follow straight lines

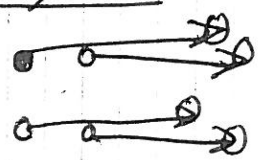


What happens when $\lambda \to 0$

$$\cos^2\left(k_x\frac{a}{2}\right)\cos^2\left(k_y\frac{b}{2}\right) \times \frac{2J_1\left(\frac{kD}{2}\right)}{\frac{kD}{2}}$$

Very sharp peaks as in X-ray's

However, it is known that $\lambda \to 0$, then geometric optics is reached $\Rightarrow$



Must then introduce coherence of source

VERY Difficult!!

spatial coherence $\Rightarrow$

– $\Rightarrow$ Add intensities,

From each hole