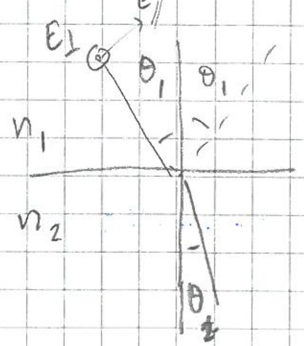
1. SUGGESTED SOLUTIONS OPTIKK TFY4195 - 8/6-2013

a) TO CALCULATE TRANSMISSION WE NEED

FRESNEL t-COEFF ; ALSO REFRACTED ANGLE θ_2



$$T = \frac{n_2}{n_1} \left(\frac{\cos\theta_2}{\cos\theta_1} \right) t^2$$

SNELL $\rightarrow$ $n_1 \sin\theta_1 = n_2 \sin\theta_2 \Rightarrow \theta_2 = \sin^{-1}\left[\frac{n_1}{n_2} (\sin\theta_1) \right]$

$$\theta_2 = \sin^{-1}\left[\left(\frac{1}{1.54} \right) \cdot \sin 57^\circ \right] = \underline{33.0^\circ}$$

WE NOTE $\underline{\theta_1 + \theta_2 = 57.0^\circ + 33.0^\circ = 90.0^\circ \Rightarrow \text{BREWSTER ANGLE}}$

$\Rightarrow$ NO P-POLARIZED $(\parallel)$ REFLECTED

FOR CIRCULAR LIGHT $\bar{E}_{in} = \frac{1}{\sqrt{2}} \left(E_0 \cdot \hat{x}_\perp + E_0 \cdot \hat{x}_\parallel \cdot e^{i\frac{\pi}{2}} \right)$ INCOMING

FRESNEL : $E_{t\perp} = \frac{1}{\sqrt{2}} E_0 \cdot t_\perp = \frac{E_0}{\sqrt{2}} \cdot \frac{2 n_1 \cos\theta_1}{(n_1 \cos\theta_1 + n_2 \cos\theta_2)} = \frac{E_0}{\sqrt{2}} \cdot 0.5932$

$$E_{t\parallel} = \frac{1}{\sqrt{2}} E_0 \cdot e^{i\frac{\pi}{2}} \cdot t_\parallel = \frac{E_0 e^{i\frac{\pi}{2}}}{\sqrt{2}} \cdot \frac{2 n_1 \cos\theta_1}{(n_2 \cos\theta_1 + n_1 \cos\theta_2)} = \frac{E_0}{\sqrt{2}} \cdot 0.6493 \, e^{i\frac{\pi}{2}}$$

$$P_{in} = \bar{E}_{in} \cdot \bar{E}_{in}^* = \frac{1}{2} \left(E_0^2 + E_0^2 \right) = E_0^2$$

$$P_{trans} = \frac{n_2}{n_1} \left(\frac{\cos\theta_2}{\cos\theta_1} \right) \cdot t^2 = \frac{1.54}{1.0} \cdot \frac{\cos 33^\circ}{\cos 57^\circ} \cdot \frac{E_0^2}{2} \left(0.5932^2 + 0.6493^2 \right)$$

$$= 0.9171 \cdot E_0^2$$

$\therefore$ $\underline{91.7\,\% \text{ IS TRANSFERRED}}$

1.

(+)

b) ALL REFLECTED LIGHT IS S-POLARIZED SINCE AT BREWSTER ANGLE. $\theta_1 + \theta_2 = 90.0°$

c) FRESNEL GIVES

$$r_\perp = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2} = -0.4068$$

π-phase shift - ok!

$$E_{r\perp} = \frac{E_0}{\sqrt{2}} \cdot \left(-0.4068\right) = -E_0 \cdot 0.2877$$

$\Rightarrow$ REFLECTED POWER $P_{refl} = \left(-E_0 \cdot 0{,}2877\right)^2 = 0.0827\, E_0^2$

$$P_{refl} + P_{transm} = 0.0827\, E_0^2 + 0.9171\, E_0^2 = 1.00 \cdot E_0^2$$

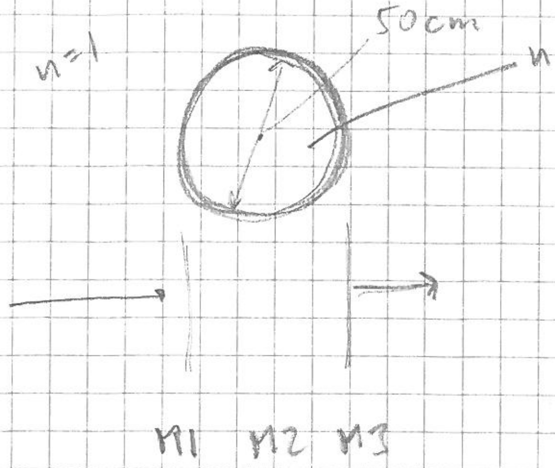
QED

d) AT NORMAL INCIDENCE

$$|r_{\parallel}|^2 = |r_\perp|^2 = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2 = \left(\frac{0.54}{2.54}\right)^2 = 0{,}0452$$

THUS, APPROX 4.5 % IS REFLECTED
95.5 % IS TRANSMITTED

2.



$n = 4/3$

$2R = 50 \Rightarrow R = 25\,cm = \frac{1}{4}\,m$

a)

$$M1 = \begin{pmatrix} 1 & 0 \\ \frac{1-\frac{4}{3}}{\frac{1}{4}\cdot\frac{4}{3}} & \frac{3}{4} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{-\frac{1}{3}}{\frac{1}{3}} & \frac{3}{4} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1 & \frac{3}{4} \end{pmatrix}$$

$$M_2 = \begin{pmatrix} 1 & 1/2 \\ 0 & 1 \end{pmatrix}$$

$-\frac{2}{3} + \frac{4}{3} = \frac{2}{3}$

$$M_3 = \begin{pmatrix} 1 & 0 \\ \frac{\frac{4}{3}-\frac{3}{3}}{-\frac{1}{4}\cdot 1} & \frac{4}{3} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{4}{3} & \frac{4}{3} \end{pmatrix}$$

$$M_{tot} = \begin{pmatrix} 1 & 0 \\ -\frac{4}{3} & \frac{4}{3} \end{pmatrix} \begin{pmatrix} 1 & 1/2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & \frac{3}{4} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1/2 \\ -\frac{4}{3} & 2/3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & \frac{3}{4} \end{pmatrix} = \begin{pmatrix} 1/2 & \frac{3}{8} \\ -2 & 1/2 \end{pmatrix}$$

$-\frac{4}{3} - \frac{2}{3}$

2. CONT

$$M_{tot} = \begin{bmatrix} \frac{1}{2} & \frac{3}{8} \\ -2 & 1/2 \end{bmatrix}$$

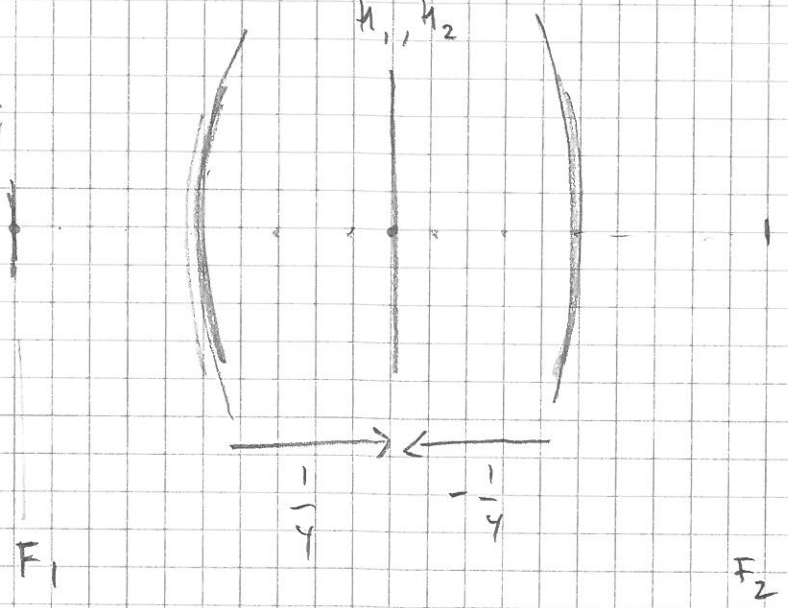
b)

$$H_1 : \frac{D - n_0/n_f}{C} = \frac{1/2 - 1}{-2} = \frac{1}{4}$$

$$H_2 : \frac{1 - A}{C} = \frac{1 - \frac{1}{2}}{-2} = -\frac{1}{4}$$

$$F_1 = \frac{D}{C} = -\frac{1}{4}$$

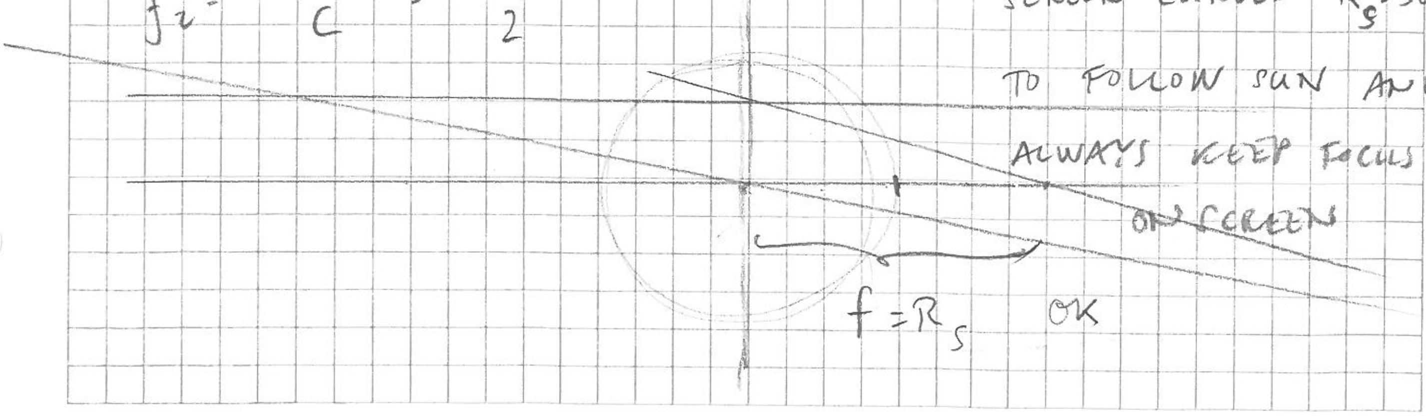
$$F_2 = -\frac{A}{C} = -\frac{1/2}{-2} = \frac{1}{4}$$



$$f_1 = \frac{n_0/n_f}{C} = -\frac{1}{2}$$

$$f_2 = -\frac{1}{C} = \frac{1}{2}$$

c) FOCAL POINT IS 50 cm FROM CENTER. PLACE SCREEN HERE. SCREEN CURVED $R_S = 50$ cm TO FOLLOW SUN AND ALWAYS KEEP FOCUS ON SCREEN



2 : CM SCALE - CORRECT MATRICES

$$M_1 = \begin{pmatrix} 1 & 0 \\ \dfrac{-\frac{1}{3}}{25 \cdot \frac{4}{3}} & \frac{3}{4} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{100} & 3/4 \end{pmatrix}$$

$$M_2 = \begin{pmatrix} 1 & 50 \\ 0 & 1 \end{pmatrix} =$$

$$M_3 = \begin{pmatrix} 1 & 0 \\ \dfrac{1/3}{-25 \cdot 1} & \frac{4}{3} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{75} & \frac{4}{3} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ -\frac{1}{75} & \frac{4}{3} \end{pmatrix} \begin{pmatrix} 1 & 50 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{100} & \frac{3}{4} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 50 \\ -\frac{1}{75} & \overset{\frac{2}{3}}{-\frac{50}{75} + \frac{4}{3}} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{100} & \frac{3}{4} \end{pmatrix}$$

$$\begin{pmatrix} 1 - \frac{50}{100} & \frac{150}{4} \\ -\frac{4}{300} - \frac{2}{300} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1/2 & \frac{300}{8} \\ -\frac{1}{50} & 1/2 \end{pmatrix}$$

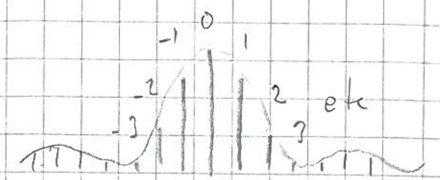
$$-\frac{6}{300} = -\frac{1}{50}$$

3.

Diffraction of N slits $\quad I = I_0 \underbrace{\left(\frac{\sin\beta}{\beta}\right)^2}_{\text{Diffraction from one slit 'b'}} \underbrace{\left(\frac{\sin N\alpha}{\sin\alpha}\right)^2}_{\text{Interfer from slits separate 'a'}}$

$\beta = \frac{1}{2} kb \sin\theta$ SLIT WIDTH

$\alpha = \frac{1}{2} ka \sin\theta$



i) Diffraction

$\left(\frac{\sin\beta}{\beta}\right)^2$ has zero for $\beta = m\cdot\pi$

$\Rightarrow \frac{1}{2} kb\sin\theta = m\cdot\pi \Rightarrow \frac{b}{\lambda}\sin\theta = m$.

1st ZERO $m=1 \Rightarrow b\sin\theta = \lambda$

Interference $\quad N=2 \Rightarrow \frac{\sin 2\alpha}{\sin\alpha} = \frac{2\cancel{\sin\alpha}\cos\alpha}{\cancel{\sin\alpha}} = 2\cos\alpha$

$\cos\alpha$ max for $\alpha = m\cdot\pi = \frac{1}{2} ka\sin\theta$

$\Rightarrow m\cdot\lambda = a\sin\theta$

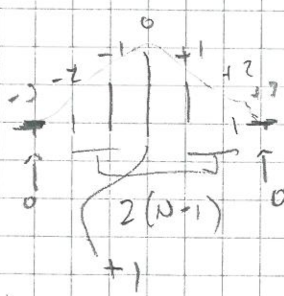
$\lambda = b\sin\theta$

$m\cdot b\cdot\cancel{\sin\theta} = a\cdot\cancel{\sin\theta}$

$\Rightarrow m = \frac{a}{b} \Rightarrow N_{BF} = 2(N-1)+1 = 2\left(\left(\frac{a}{b}\right)-1\right)+1$

ORDER OF INTERFERENCE

NUMBER BRIGHT FRINGES



$= 2\left(\frac{a}{b}\right) - 1$

QED

ii) $b = 0.3$ mm $\quad 2\left(\frac{a}{b}\right) - 1 = 15 \Rightarrow 2\left(\frac{a}{0.3}\right) = 16$

ans. $\underline{a = 2.4 \text{ mm}}$

4.

$$\frac{2\pi b}{\lambda}\sqrt{n_1^2-N^2} - 2\tan^{-1}\sqrt{\frac{N^2-n_2^2}{n_1^2-N^2}} = m\cdot\pi$$

$n_1 = 1.6$

$n_2 = 1.48$

$b = 1.2\ \mu m$

$\lambda = 632\ nm$

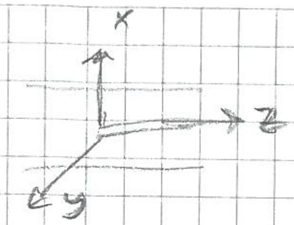
a) At cut-off $N \simeq n_2 \Rightarrow 2\tan^{-1}(0) = 0$

$$\frac{2\cancel{\pi}\ 1200}{632}\sqrt{1.6^2-1.48^2} = m\cdot\cancel{\pi}$$

$$\Rightarrow m = \frac{2\cdot 1200}{632}\cdot\sqrt{1.6^2-1.48^2} = 2.3$$

$\therefore$ MAX 3 modes $(0,1,2)$

4 cont.



$$\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

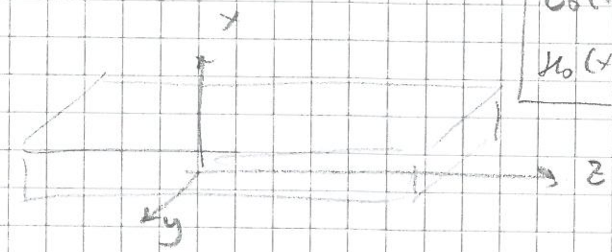
$$\bar{\nabla} \times \bar{H} = +\frac{\partial \bar{D}}{\partial t}$$

$B = \mu H \quad ; \quad D = n^2 E$

$$\begin{cases} \bar{\nabla} \times \bar{H} = \varepsilon_0 n^2 \dfrac{\partial \bar{E}}{\partial t} \\ \bar{\nabla} \times \bar{E} = -\mu_0 \dfrac{\partial \bar{H}}{\partial t} \end{cases}$$

$E \quad E_0 e^{i(\beta z - \omega t)}$

$$\boxed{\begin{array}{l} E_0(x)\, e^{i(\beta z - \omega t)} \\ H_0(x)\, e^{i(\beta z - \omega t)} \end{array}}$$



$$\frac{\partial}{\partial y} = 0$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \hat{x}\left(-\frac{\partial H_y}{\partial z}\right) + \hat{y}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right) + \hat{z}\left(\frac{\partial H_y}{\partial x}\right)$$

TE TM

$$-i\omega \varepsilon_0 n^2 E_x = -i\beta H_y$$

$$-i\omega \varepsilon_0 n^2 E_y = i\beta H_x - \frac{\partial H_z}{\partial x}$$

$$-i\omega \varepsilon_0 n^2 E_z = \frac{\partial H_y}{\partial x}$$

$$+i\omega\mu_0 H_x = -i\beta E_y$$

$$+i\omega\mu_0 H_y = i\beta E_x - \frac{\partial E_z}{\partial x}$$

$$+i\omega\mu_0 H_z = \frac{\partial E_y}{\partial x}$$

4. cont

b) TE-mode: E_y, H_x & H_z

c) TM-mode: H_y, E_x & E_z

$$\bar{\nabla} \times \bar{E} = -\mu_0 \frac{\partial \bar{H}}{\partial t}$$

$$\bar{E} = \bar{E}_0 e^{i(\beta z - \omega t)}$$

$$\bar{H} = \bar{H}_0 e^{i(\beta z - \omega t)}$$

$$\Rightarrow i\omega\mu_0 H_{0x} = -i\beta E_{0y}$$

$$i\omega\mu_0 H_{0y} = i\beta E_{0x} - \frac{\partial E_{0z}}{\partial x}$$

$$i\omega\mu_0 H_{0z} = \frac{\partial E_{0y}}{\partial x}$$

$$H_{0x} = -\frac{\beta}{\omega\mu_0} E_{0y}$$

GIVEN E_{0y} TE

$$H_{0z} = -\frac{i}{\omega\mu_0} \frac{\partial E_{0y}}{\partial x}$$

P.S.S. $\bar{\nabla} \times \bar{H} = \varepsilon_0 n^2 \frac{\partial \bar{E}}{\partial t}$

$$-i\beta H_{0y} = -i\omega\varepsilon_0 n^2 E_{0x}$$

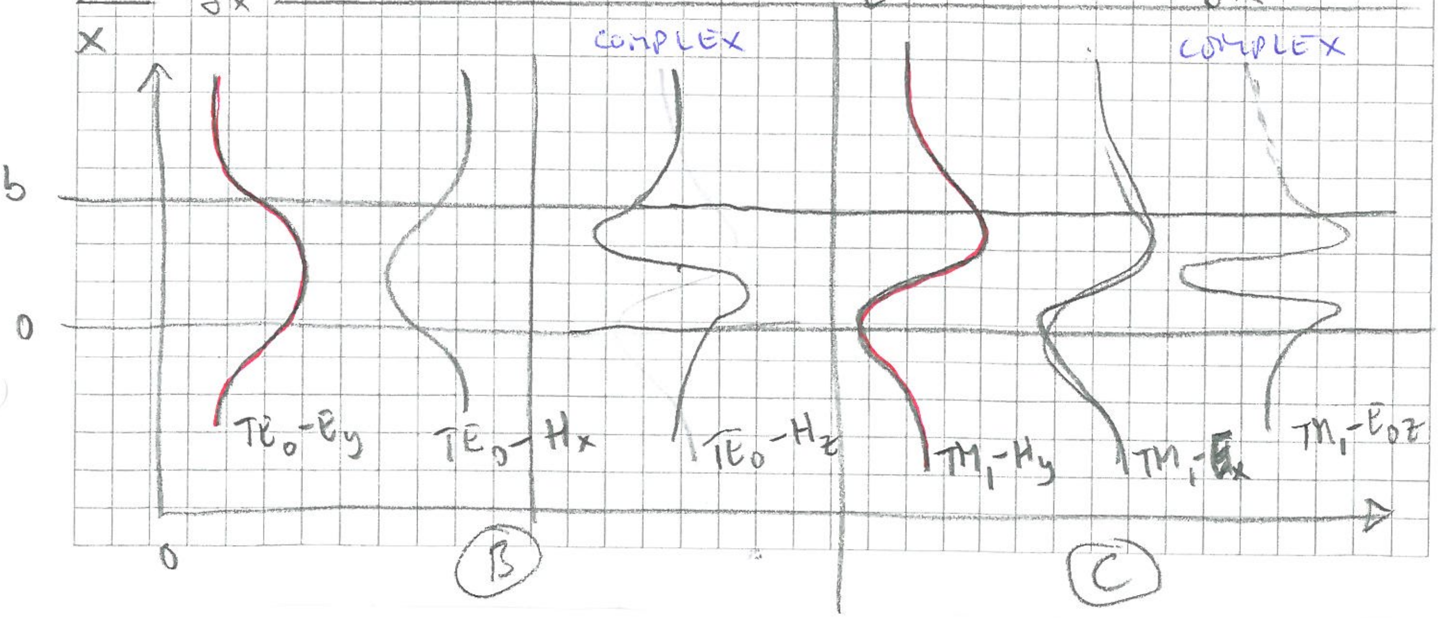
$$i\beta H_{0x} - \frac{\partial H_{0z}}{\partial x} = -i\omega\varepsilon_0 n^2 E_{0y}$$

$$\frac{\partial H_{0y}}{\partial x} = -i\omega\varepsilon_0 n^2 E_{0z}$$

$$E_{0x} = \frac{\beta}{\omega\varepsilon_0 n^2} H_y$$

Given H_{0y} TM

$$E_{0z} = \frac{i}{\omega\varepsilon_0 n^2} \frac{\partial H_{0y}}{\partial x}$$



5. a) IMAGE $\frac{1}{14} + \frac{1}{S_i} = \frac{1}{6} \Rightarrow S_i = \frac{21}{2}$; $M_T = -\frac{S_i}{S_o} = -\frac{21}{2 \cdot 14} = -\frac{3}{4}$; SIZE: $\left|-\frac{3 \cdot 2}{4}\right| = 1.5cm$

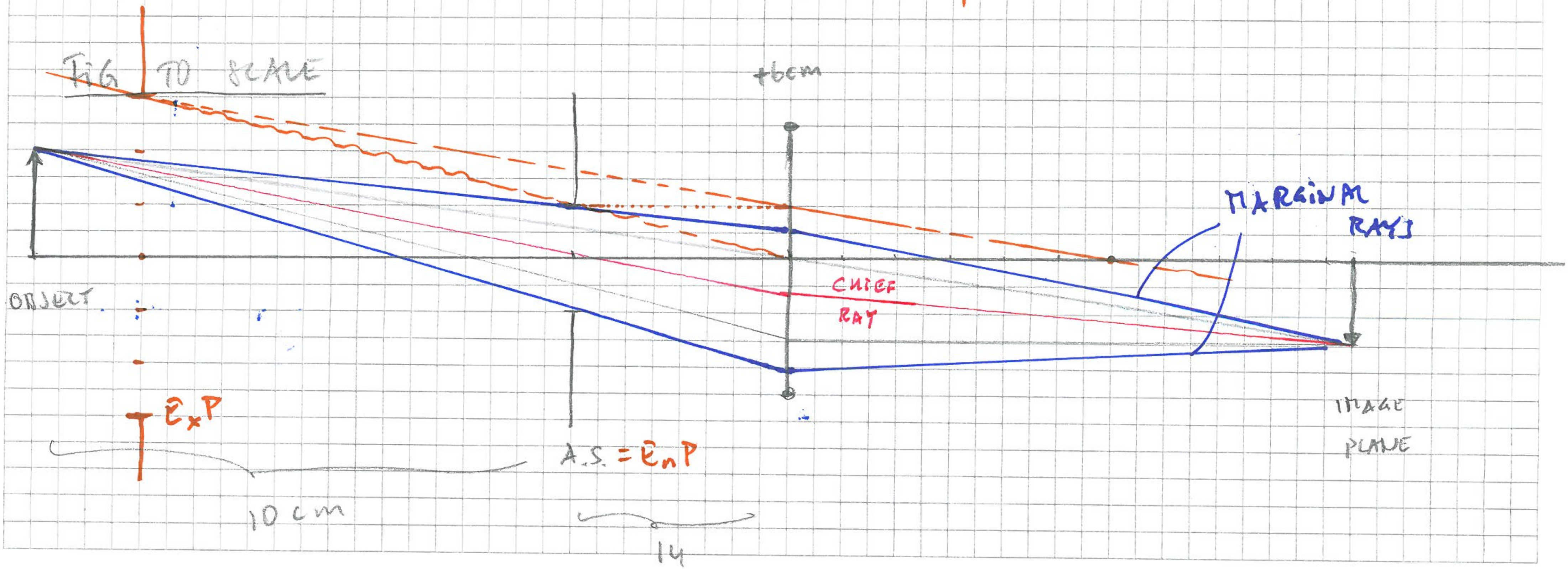
b) ENTRANCE PUPIL - NO PRECEEDING ELEMENT $\Rightarrow E_nP = A.S$

EXIT - PUPIL - IMAGE A.S. THROUGH LENS. $\frac{1}{4} + \frac{1}{S_i} = \frac{1}{6} \Rightarrow S_i = -12$; $M_T = -\frac{-12}{4} = 3$ $\Rightarrow$ SIZE = 6

c) RAY - TRACE IMAGE CONDITION - SEE FIG

d) SHOW E_xP & E_nP - SEE FIG

e) CHIEF & MARGINAL RAYS - SEE FIG



6.)

$$l_t = c \cdot \tau_0$$

a)

$$\Delta \nu = \frac{1}{\tau_0} \Rightarrow \Delta \nu = \frac{c}{l_t}$$

NOW $\nu = \frac{c}{\lambda} \Rightarrow \Delta \nu = -\frac{\Delta \lambda}{\lambda^2} c$

THUS $|\Delta \nu| = \frac{\Delta \lambda}{\lambda^2} c$

SO THAT $\frac{c}{l_t} = \frac{\Delta \lambda}{\lambda^2} \cdot c$

$$\Rightarrow \boxed{\Delta \lambda = \frac{\lambda^2}{l_t}}$$

b) WITH $\left.\begin{array}{l} \lambda = 643.8 \text{ nm} \\ l_t = 0.3 \text{ m} \end{array}\right\} \Rightarrow \Delta \lambda = \frac{(643.8 \text{ nm})^2}{30 \cdot 10^7 \text{ nm}} = 0.00138$ nm

$$\tau_0 = \frac{l_t}{c} = \frac{0.3 \text{ m}}{3 \cdot 10^8 \text{ m/s}} = 10^{-9} \text{ s}$$

Ans. $\Delta \lambda \approx 1.4$ pm

$\tau_0 \approx 1$ ns