LF

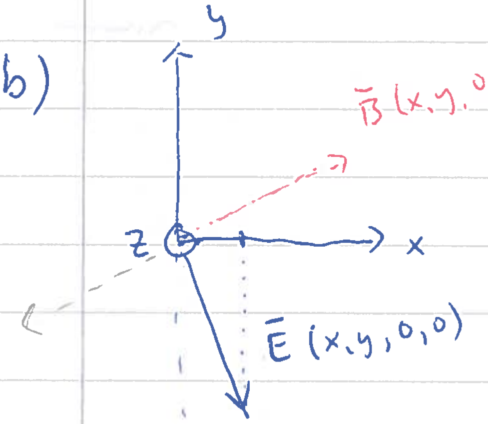
# EXAM TFY 4195 H2016

P 1a): propagation direction $+z$

wavelength from $k = \frac{2\pi}{\lambda} = 1.26 \cdot 10^7$
$\rightarrow \lambda = 499$ nm

frequency from $\omega = 2\pi f = 3.77 \cdot 10^{15}$
$\rightarrow f = 6.00 \cdot 10^{14}$ Hz

b)




WE KNOW $\bar{k}, \bar{E}, \bar{B}$ FORM RIGHT-HANDED SYSTEM AND

$\frac{|\bar{E}|}{c} = |\bar{B}|$ ← SPEED OF LIGHT

AS IN E-FIELD

$\Rightarrow \bar{B}(x,y,z,t) = \frac{1}{c\sqrt{10}}(3\hat{x} + \hat{y})\cos(kz - \omega t)$

c) THE JONES' VECTOR IS DEFINED FROM $\bar{J} = \begin{bmatrix} E_x \\ E_y e^{i\Delta\varphi} \end{bmatrix}$

$\Delta\varphi = \varphi_y - \varphi_x$

SINCE $E_x$ & $E_y$ IN PHASE

$\bar{J} = \frac{1}{\sqrt{10}}\begin{bmatrix} 1 \\ -3 \end{bmatrix}$ LINEARLY POL. AS SHOWN IN (1b)

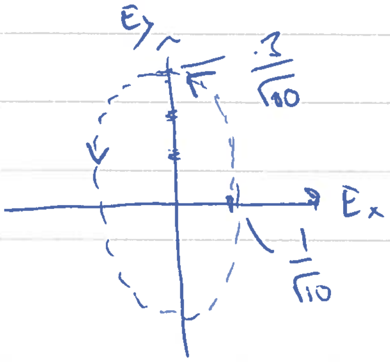
d) QWP SA HORIZONTAL

$$e^{i\frac{\pi}{4}}\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix} \frac{1}{\sqrt{10}}\begin{bmatrix} 1 \\ -3 \end{bmatrix} = \frac{e^{i\frac{\pi}{4}}}{\sqrt{10}}\begin{bmatrix} 1 \\ -3e^{-i\frac{\pi}{2}} \end{bmatrix} = \frac{e^{i\frac{\pi}{4}}}{\sqrt{10}}\begin{bmatrix} 1 \\ 3e^{i\frac{\pi}{2}} \end{bmatrix}$$

INPUT; $3 \cdot e^{i\pi}$

LEFT HANDED SINCE $+90°$

∴ ELLIPTICALLY POLARIZED LEFT-HANDED




↑ SINCE $|E_{0x}| \neq |E_{0y}|$ & $\Delta\varphi \neq 0, \pi$

LF

CASE I

P2.

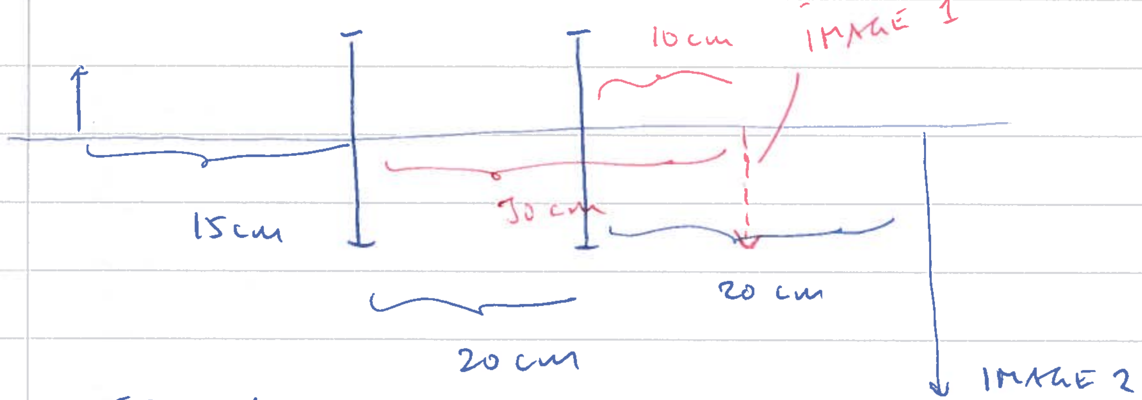

THIN LENS:

$$\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f_1}$$

$$\frac{1}{30} + \frac{2 \cdot 1}{2 \cdot 15} = \frac{1}{f_1} \Rightarrow \frac{1}{f_1} = \frac{3}{30} = \frac{1}{10} \Rightarrow f_1 = 10 \text{ cm}$$

$$M_1 = M_T = -\frac{s_i}{s_o} = -\frac{30}{15} = -2 \quad \text{OK!}$$

CASE II




THIN LENS FORMULA FOR IMAGE TWO

$$-\frac{2 \cdot 1}{2 \cdot 10} + \frac{1}{20} = \frac{1}{f_2} \Rightarrow -\frac{1}{20} = \frac{1}{f_2} \Rightarrow f_2 = -20 \text{ cm}$$

NEGATIVE LENS

$$M_2 = -\frac{s_i}{s_o} = -\frac{20}{(-10)} = 2$$

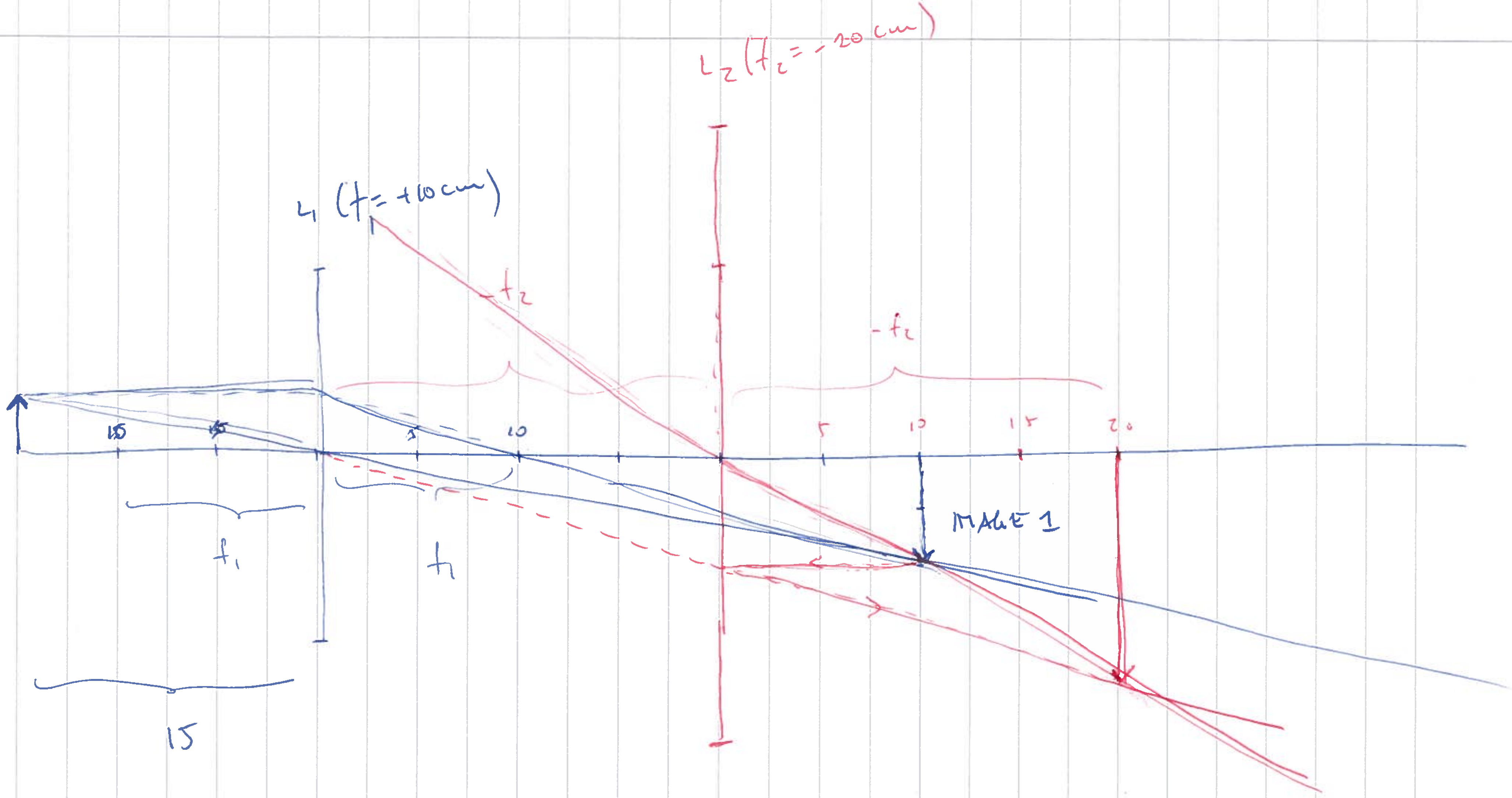
$\therefore$ $L_1$ $f_1 = 10$ cm

$L_2$ $f_2 = -20$ cm

$$M_{tot} = M_1 \cdot M_2 = -2 \cdot 2 = -4 \quad \text{OK}$$

⟶ RAY-TRACE

P2 : RAY-TRACE



CHECK

P2. USING RAY-TRANSFER MATRIX

TWO LENSES SEPARATED d - CHECK....

$$\begin{pmatrix} 1-\frac{d}{f_2} & -\frac{1}{f_1}-\frac{1}{f_2}+\frac{d}{f_1 f_2} \\ d & 1-\frac{d}{f_1} \end{pmatrix} = \quad d=20,\ f_1=10,\ f_2=-20 \quad =$$

$$= \begin{pmatrix} 1-\frac{20}{-20} & -\frac{1}{10}\left(-\frac{1}{-20}\right)+\frac{20}{10(-20)} \\ 20 & 1-\frac{20}{10} \end{pmatrix}$$

$-\frac{1}{10}+\frac{1}{20}-\frac{1}{10}$

$-\frac{2\cdot 2}{2\cdot 10}+\frac{1}{20}$

$$= \begin{pmatrix} 2 & -\frac{3}{20} \\ 20 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 15 & 1 \end{pmatrix} = \begin{pmatrix} 2-\frac{3\cdot 15}{20} & -\frac{3}{20} \\ 20-15 & -1 \end{pmatrix} =$$

OBJECT

$\frac{40-45}{20}$

$$\begin{pmatrix} 1 & 0 \\ S_i & 1 \end{pmatrix} \begin{pmatrix} -\frac{1}{4} & -\frac{3}{20} \\ 5 & -1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{4} & -\frac{3}{20} \\ -\frac{S_i}{4}+5 & -\frac{3S_i}{20}-1 \end{pmatrix}$$

IMAGE

=0 GIVES IMAGE

$M_T$

$M_T = -\frac{3\cdot 20}{20}-1 = -4$

OK

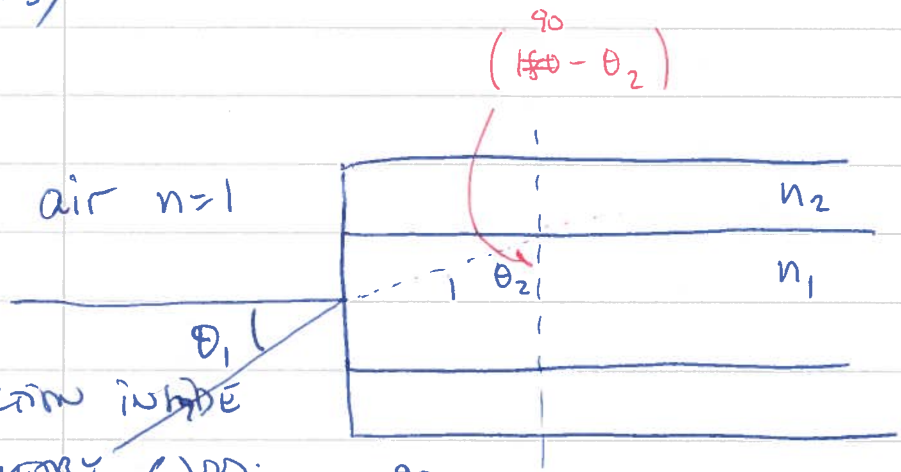
$$-\frac{S_i}{4}+5=0 \Rightarrow S_i=20$$

OK

P3)

$n_1 > n_2$

a) air $n=1$




$n_1 = 1.7$

$n_2 = 1.6$

REFLECTION INSIDE

GEOMETRY GIVES:

$$n_1 \cdot \sin(\cancel{180}\,90 - \theta_2) = n_2 \cdot \sin 90^\circ$$

@ CRITICAL ANGLE

$\sin(90 - \theta_2) = \cos\theta_2$

$$n_1 \cos\theta_2 = n_2 \quad \Rightarrow \quad \cos\theta_2 = \frac{n_2}{n_1}$$

INCIDENT REFR.

$$1 \cdot \sin\theta_1 = n_1 \sin\theta_2 = n_1 \cdot \sqrt{1 - \cos^2\theta_2}$$

$$= n_1 \sqrt{1 - \frac{n_2^2}{n_1^2}} = \sqrt{n_1^2 - n_2^2}$$

MAX $\theta_1$ is $\theta_1 = \arcsin\sqrt{n_1^2 - n_2^2}$

with $n_1 = 1.7$; $n_2 = 1.5$ $\Rightarrow$ $\theta_{max} = 53.1^\circ$

b) CHOSE POLARIZATION IN THE PLANE OF INCIDENCE. THEN CHOOSE INCIDENT ANGLE ≈ BREWSTER ANGLE, I.E.,

$$1 \cdot \sin\theta_1 = 1.7 \cdot \sin\theta_2 \quad \text{with} \quad \theta_1 + \theta_2 = 90^\circ$$

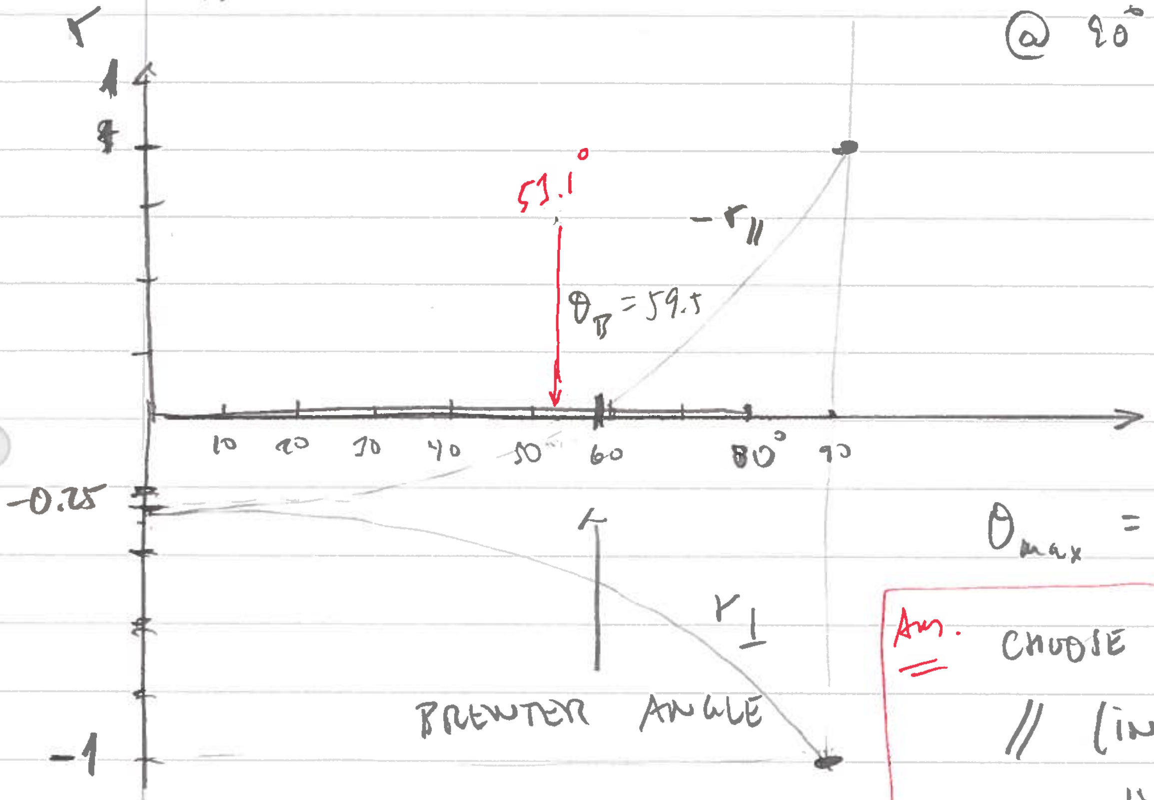
$$\Rightarrow \tan\theta_1 = 1.7 \quad = \quad \theta_1 = 59.5$$

→

P3) b) CONT

$$\left.\begin{array}{l}\theta_1 = 0 \\ \Rightarrow \theta_2 = 0\end{array}\right\} \Rightarrow -r_{\parallel} = r_{\perp} = \frac{n_1 - n_2}{n_2 + n_1} = -\frac{0.7}{2.7} = -0.25$$

RECALL REFLECTION AS FUNCTION OF INCIDENT ANGLE




@ 90° $|r| = 1$

$\theta_{max} = 53.1°$

Ans. CHOOSE POLARIZATION $\parallel$ (IN PLANE OF INCIDENCE) @ 53.1° TO BE AS CLOSE TO $\theta_B$ AS POSSIBLE

CHECK!

$$r_{\perp}(\theta = 53.1°) = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

$$= \frac{1 \cdot \cos 53.1 - 1.7 \cdot \cos 28.1}{1 \cdot \cos 53.1 + 1.7 \cdot \cos 28.1}$$

$$= \frac{0.6 - 1.7 \cdot 0.882}{0.6 + 1.7 \cdot 0.882} = \frac{-0.899}{2.099} = -0.428$$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$1 \cdot \sin 53.1 = 1.7 \cdot \sin\theta_2$

$\Rightarrow \theta_2 = 28.1$

$$r_{\parallel}(\theta = 53.1°) = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2} =$$

$$= \frac{1.7 \cdot \cos 53.1 - 1 \cdot \cos 28.1}{1.7 \cdot \cos 53.1 + 1 \cdot \cos 28.1} = \frac{0.1386}{1.9028} = 0.0728$$

$\therefore |r_{\parallel}| \ll |r_{\perp}|$

CHOOSE $\parallel$ POL

P4. WE HAVE A THICK LENS COMPOSED OF TWO SURFACES AND FREE PROPAGATION BETWEEN THE DISTANCE 2 cm


a) FIND $M_{sys}$

$$M_{sys} = \underbrace{\begin{pmatrix} 1 & -D_2 \\ 0 & 1 \end{pmatrix}}_{\text{SURFACE II}} \underbrace{\begin{pmatrix} 1 & 0 \\ \frac{d}{n} & 1 \end{pmatrix}}_{\substack{d=2 \\ n=1.5}} \underbrace{\begin{pmatrix} 1 & -D_1 \\ 0 & 1 \end{pmatrix}}_{\text{SURFACE 1}} =$$

$$D_1 = \frac{n_t - n_i}{R_1} = \frac{1.5-1.0}{5} = \frac{1}{10}$$

$$D_2 = \frac{n_t - n_i}{R_2} = 0 \qquad R_2 \to \infty$$

$$\frac{2}{1.5} = \frac{2 \cdot 2}{3}$$

$$\frac{30}{30} - \frac{4}{30} = \frac{26}{30}$$

$$M_{sys} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{4}{3} & 1 \end{pmatrix} \begin{pmatrix} 1 & -\frac{1}{10} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -\frac{1}{10} \\ \frac{4}{3} & -\frac{4}{10 \cdot 3} + 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & -\frac{1}{10} \\ \frac{4}{3} & \frac{13}{15} \end{pmatrix}$$

CHECK !!

$$\det M_{sys} = \frac{13}{15} - \left(-\frac{1}{10} \cdot \frac{4}{3}\right) = \frac{13}{15} + \frac{4}{30} = \frac{2 \cdot 13 + 4}{30} = 1 \quad \text{OK!!}$$

$F_1$: $p = \frac{a_{11}}{a_{12}} = -10$ 10 cm left of 1st SURFACE

$F_2$: $q = -\frac{a_{22}}{a_{12}} = \frac{+13 \cdot 10}{15} = +\frac{130}{15} = 8\frac{2}{3} \simeq 8.67$ cm BEHIND 2nd

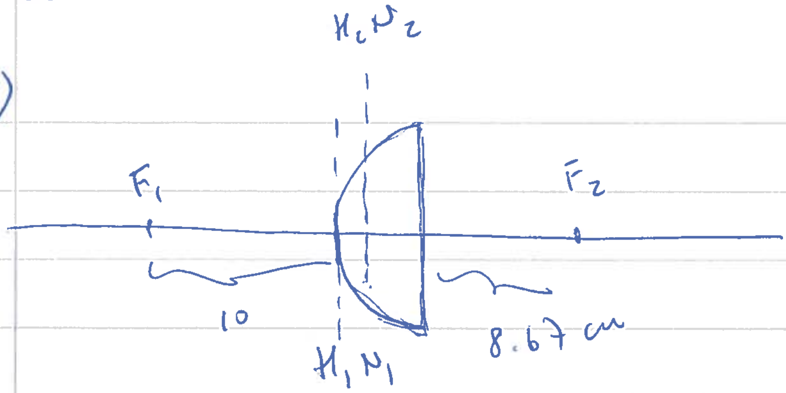
$H_1, N_1$: $r = v = \left(\frac{a_{11} - 1}{a_{12}}\right) = 0$ AT 1st SURFACE

$H_2, N_2$: $s = w = \left(\frac{1 - a_{22}}{a_{12}}\right) = \frac{1 - \frac{13}{15}}{\left(-\frac{1}{10}\right)} = \frac{\frac{2}{15}}{-1} \cdot 10 = -\frac{20}{15} = -\frac{4}{3}$

$\simeq -1.33$ cm LEFT OF 2nd SURFACE

P4 cont

a)




b)

$$\begin{pmatrix}1 & 0\\ S_i & 1\end{pmatrix}\begin{pmatrix}1 & -\frac{1}{10}\\ \frac{4}{3} & \frac{13}{15}\end{pmatrix}\begin{pmatrix}1 & 0\\ S_o & 1\end{pmatrix} =$$

IMAGE — $S_i < 0$ FOR MAGN. GLASS.

OBJECT — $S_o < F_1$ FOR MAGNIFYING GLASS

$$= \begin{pmatrix}1 & -\frac{1}{10}\\ S_i + \frac{4}{3} & -\frac{S_i}{10} + \frac{13}{15}\end{pmatrix}\begin{pmatrix}1 & 0\\ S_o & 1\end{pmatrix} = \begin{pmatrix}1 - \frac{S_o}{10} & -\frac{1}{10}\\ S_i + \frac{4}{3} + S_o\left(-\frac{S_i}{10} + \frac{13}{15}\right) & -\frac{S_i}{10} + \frac{13}{15}\end{pmatrix}$$

$S_i + \frac{4}{3} + S_o\left(-\frac{S_i}{10} + \frac{13}{15}\right) = 0 \rightarrow$ IMAGING CONDITION

$-\frac{S_i}{10} + \frac{13}{15} \rightarrow M_T$

P4 cont.

b)

VIRTUAL IMAGE

$\Rightarrow s_i = -25$ cm (neglect thickness of lens)

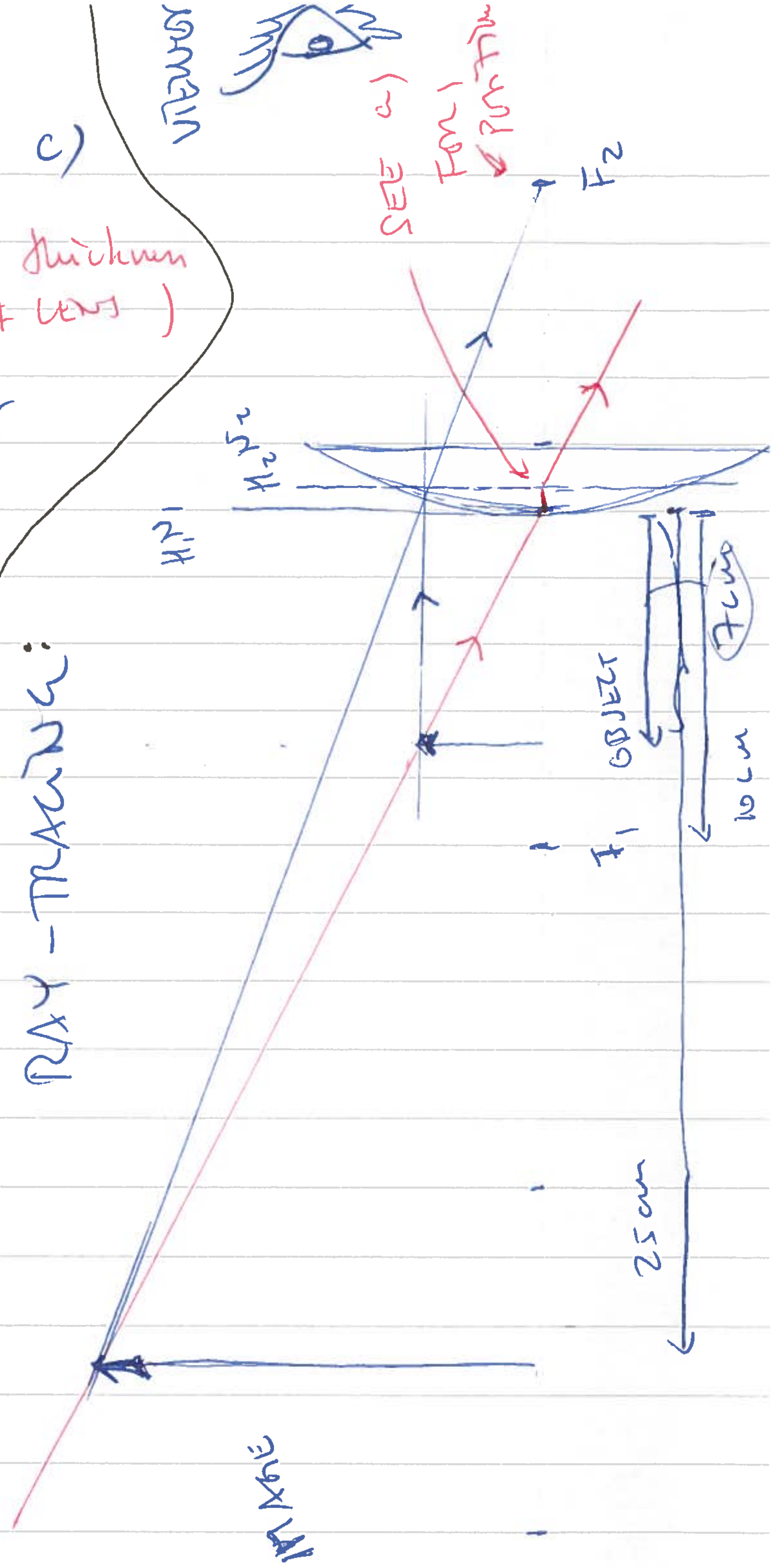
$$M_T = -\left(\frac{-25}{10}\right) + \frac{13}{15} = \frac{101}{30} \simeq 3.3 \text{ ggr}$$

$s_o$?

$$-25 + \frac{4}{3} + s_o\left(-\frac{(-25)}{10} + \frac{13}{15}\right) = 0$$

$$\Rightarrow s_o = \frac{2130}{303} \simeq 7.03 \text{ cm}$$

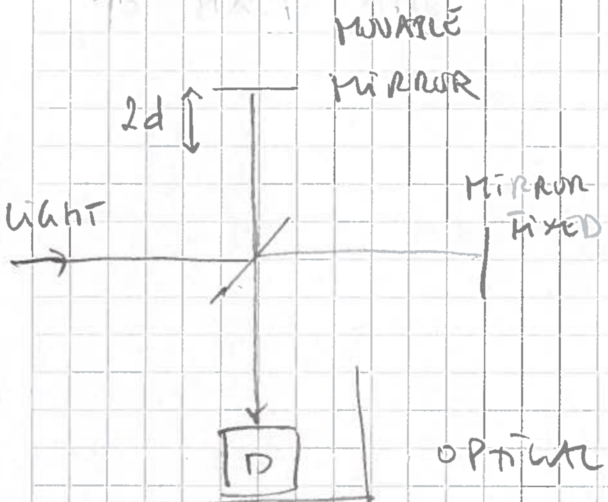
c)



P5) $\lambda = 546.1\ \text{nm}$ ; $\Delta\lambda = 0.050\ \text{nm}$

a) $l_c = c \cdot \tau_0$ WHERE $\Delta\nu = \frac{1}{\tau_0}$ SPECTRAL WIDTH

IN THE MICHELSON INTERFEROMETER, FIND THE RANGE WHERE FRINGES ARE VISIBLE (ASSUME WELL ALIGNED ARMS). SET THE MIRROR AT THE PLACE WHERE FRINGES ARE BARELY SEEN, SCAN THE MIRROR THROUGH THE WHOLE RANGE (OF FRINGES) AND READ THE DISTANCE (d) UNTIL FRINGES DISAPPEAR AGAIN. THE OPTICAL PATH $s = 2 \cdot d$ = COHERENCE LENGTH.




SPEED OF LIGHT

$$c = \lambda \cdot \nu \Rightarrow \nu = \frac{c}{\lambda} \Rightarrow \frac{d\nu}{d\lambda} = -\frac{c}{\lambda^2} \Rightarrow |d\nu| = \frac{c \cdot d\lambda}{\lambda^2}$$

($\lambda$: WAVE-LENGTH; $\nu$: FREQ.)

FIND SPECTRAL WIDTH IN $\Delta\lambda$

LET $d\nu \to \Delta\nu$, $d\lambda \to \Delta\lambda$

$$\Rightarrow l_c = c \cdot \frac{1}{\Delta\nu} = c \cdot \frac{\lambda^2}{c \cdot \Delta\lambda} = \frac{\lambda^2}{\Delta\lambda}$$

NUMBERS:

$$l_c = \frac{(546.1 \cdot 10^{-9})^2}{0.050 \cdot 10^{-9}} = 5.96 \cdot 10^{-3}\ \text{m}$$

SO, EXPECTED $s = 2d = 5.96\ \text{mm} \Rightarrow d \simeq 3.0\ \text{cm}$

(THIS CORRESPONDS TO $\frac{l_c}{\lambda_0} \simeq 11000$ FRINGES)

P5_b) FOR FAR-FIELD DIFFRACTION, THE QUADRATIC EXPONENTIAL IN (FRESNEL APPROX.) DIFFRACTION INTEGRAL MUST BE NEGLIGIBLE, i.e.;

$$u(x,y) = \frac{e^{ikz}}{i\lambda z} e^{\frac{i k}{2z}(x^2+y^2)} \iint_{\text{APERTURE}} \left[ u(\xi,y) e^{i\frac{\pi}{\lambda z}(\xi^2+y^2)} \right] e^{-i\frac{2\pi}{\lambda z}(\xi x + y y)} \, d\xi \, dy$$

x, y COORD OF DIFFRACTION PATTERN

FIELD DISTRIBUTION AT APERTURE PLANE

COORD OF APERTURE PLANE

ONLY CONSIDER ONE-D (SLIT)

$$\frac{\pi}{\lambda z} \cdot \xi^2 \ll \pi \quad \text{FOR FAR-FIELD CONDITION}$$

HERE: $\xi_{max}$ = SLIT WIDTH + SLIT SEPARATION $\simeq 50 \mu m$

$\lambda = 546.1 \cdot 10^{-9} m$ ; $z = 1 m$

$$\Rightarrow \frac{\pi \cdot (50 \mu m)^2}{(546.1 \, nm)} \simeq 0.014 \ll \pi \quad \text{BY FACTOR } 225$$

OK !!

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c) WE CAN USE THE FORMULA FOR N-SLIT

$$I = I_0 \cdot \left(\frac{\sin \beta}{\beta}\right)^2 \cdot \left(\frac{\sin N\alpha}{\sin \alpha}\right)^2$$

$\alpha = \frac{1}{2} k a \sin\theta$ ; a slit separation

$\beta = \frac{1}{2} k b \sin\theta$ ; b slit width

DIFFRACTION TERM

INTERFERENCE TERM

$$\frac{\sin 2\alpha}{\sin \alpha} = \frac{2 \sin\alpha \cos\alpha}{\sin\alpha} = 2 \cos\alpha$$

$$\therefore \quad I = 2 I_0 \left(\frac{\sin\beta}{\beta}\right)^2 \cos^2\alpha$$

→

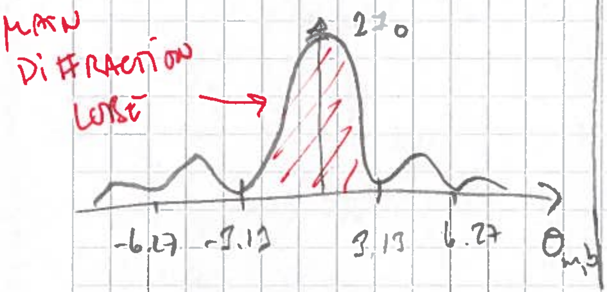
P5 c) CONT.

i) EXAMINE DIFFRACTION TERM : $\frac{\sin\beta}{\beta} = 0$ IF $\beta = \pm m\pi$ MINIMUM

$m = 1, 2, 3, \ldots$

$\beta = \frac{1}{2} k \cdot b \cdot \sin\theta = \pm m \cdot \pi \Rightarrow \frac{1}{2} \frac{2\pi}{\lambda} \cdot b \cdot \sin\theta = \pm m \cdot \pi \Rightarrow$

$\sin\theta_{m,b} = \pm m \cdot \frac{\lambda}{b}$




DIFFRACTION PART

| m | $\sin\theta_{m,b}$ | $\theta_{m,b}$(rad) | $\theta_{m,b}$(deg) |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| ±1 | ±0.05461 | ±0.05461 | 3.13° |
| ±2 | ±0.10922 | ±0.10922 | 6.27° |

ETC

ii) EXAMINE INTERFERENCE PART

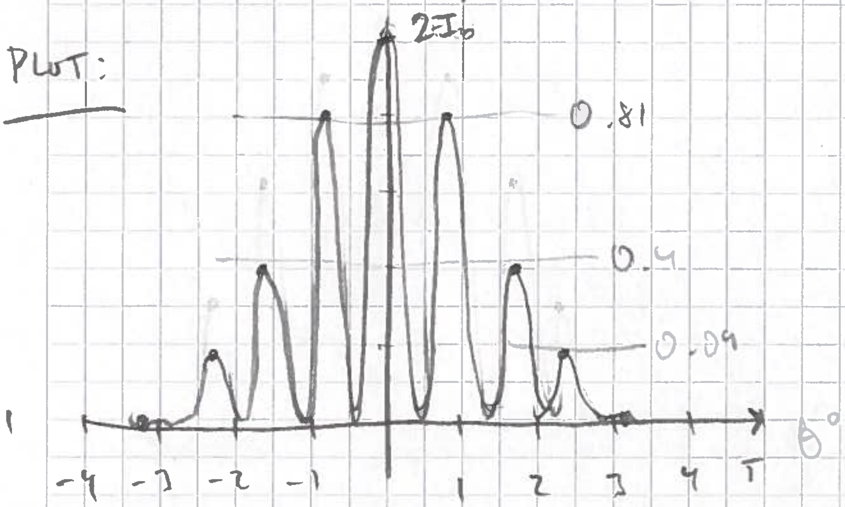
$\alpha = \pm m \cdot \pi \rightarrow$ MAXIMA ! $m = 0, 1, 2, 3 \ldots$

$\frac{1}{2} k a \sin\theta = \pm m \cdot \pi \rightarrow \sin\theta_{m,a} = \pm m \frac{\lambda}{a}$

GIVES RELATIVE INTENSITY

| m | $\sin\theta_{m,a}$ | $\theta_{m,a}$(rad) | $\theta_{m,a}$(deg) | $2I_0\left(\frac{\sin\beta}{\beta}\right)^2$ | $\beta$ @ $\theta_{m,a}$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | $2I_0 \cdot 1$ | 0 |
| ±1 | ±0.01365 | ±0.01365 | 0.782 | $2I_0 \cdot 0.81$ | 0.785 |
| ±2 | ±0.027305 | ±0.027305 | 1.565 | $2I_0 \cdot 0.40$ | 1.571 |
| ±3 | ±0.04096 | ±0.04096 | 2.347 | $2I_0 \cdot 0.09$ | 2.356 |
| ±4 | ±0.05461 | → ~SAME | 3.13 | $2I_0 \cdot 0$ | 3.14159 |

PLOT:




MINIMA COMES IN BETWEEN THE MAXIMA

$\frac{1}{2} k a \sin\theta_m = \pm(2m+1)\frac{\pi}{2}$

$m = 0, 1, 2$