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Department of Physics

Examination paper for TFY4195 Optics

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Examination date: Nov. 28, 2019
Examination time (from-to): 09-13
Permitted examination support material: Level C, Rottman mathematical tables.
Other information: Selected formulas are included as resources herein. Answers to two exam problems shall be scanned in.

Students will find the examination results in Studentweb. Please contact the department if you have questions about your results. The Examinations Office will not be able to answer this.

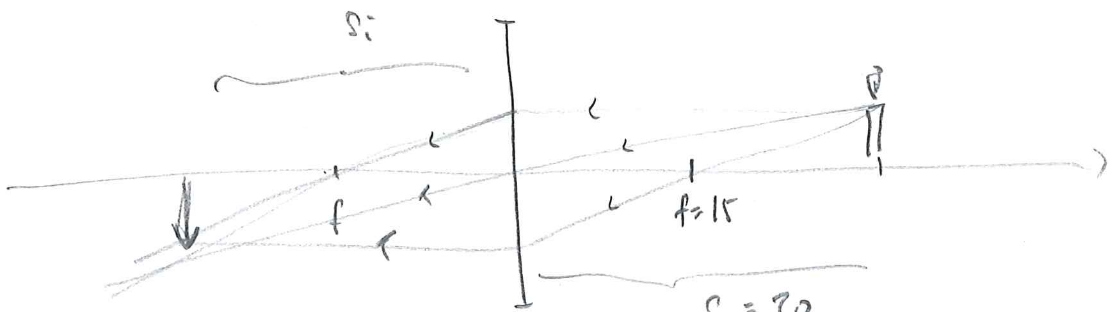
1 MC1 TFY4195 H19

A candle-light is placed 30 cm to the right of a 5 cm diameter positive lens of focal length 15 cm. Using a screen Paul is attempting to obtain an image of the burning candle. What is true about the image?

Select one alternative:

- X There is an inverted, real, non-magnified image 30 cm to the left of the lens.
- ○ There is no image
- ○ There is a non-magnified, real image at the focal plane, to the left of the lens.
- ○ There is a virtual, enlarged image to the right of the lens.
- ○ There is a minimized, real image 30 cm to the left of the lens.

Maximum marks: 5



GAUSS:

$$\frac{1}{30} + \frac{1}{S_i} = \frac{1}{15} \Rightarrow \frac{1}{S_i} = \frac{1}{15} - \frac{1}{30} = \frac{1}{30} \Rightarrow S_i = 30$$

$$M_T = -\frac{S_i}{S_o} = -1$$ ⇒ INVERTED, REAL, NON-MAGNIFIED AT $S_i = 30$ cm

2

MC2 TFY4195 H19

An electromagnetic wave is described by the following expression:

POS. SIGN => NEG. DIRECTION

$$\bar{E}(z,t) = E_0 \cos\left(\frac{\pi}{3}\cdot 10^7 \cdot z + 10\pi \cdot 10^{14} \cdot t\right)\cdot \hat{x} \quad [\text{V/m}],$$

$= E_0 \cdot \cos(k \cdot z + \omega t)$

$v = \frac{\omega}{k} = 3 \cdot 10^8 \frac{m}{s}$

where the position z has the unit meter [m] and t is the time in seconds [s]. What is the propagation direction, speed and wavelength of the wave?

Select one alternative:

$k = \frac{2\pi}{\lambda} = \frac{\pi}{3} \cdot 10^7 \Rightarrow \lambda = \frac{6}{10^7} = 600\ nm$

$\begin{bmatrix} +z \\ \frac{\pi}{3}10^7\left[\frac{m}{s}\right] \\ 10\cdot10^{14}[m] \end{bmatrix}$ ○

$\begin{bmatrix} -z \\ \frac{10}{3}\cdot10^8\left[\frac{m}{s}\right] \\ \frac{1}{3}\cdot10^7[m] \end{bmatrix}$ ○

$\begin{bmatrix} -z \\ 3.0\cdot10^8\left[\frac{m}{s}\right] \\ 600\cdot10^7[Hz] \end{bmatrix}$ ○

$\begin{bmatrix} -z \\ 3.0\cdot10^8\left[\frac{m}{s}\right] \\ 600\cdot10^{-9}[m] \end{bmatrix}$ X

$\begin{bmatrix} +z \\ 5.0\cdot10^{14}\left[\frac{m}{s}\right] \\ 600\ [nm] \end{bmatrix}$ ○

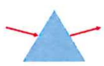
Maximum marks: 5

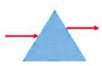
3

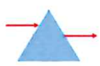
MC3 TFY4195 H19

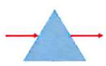
Light from a HeNe-laser (632.8 nm) passes through a prism made of glass (refractive index *n* around 1.5). The prism is placed in air. What ray-path is possible before and after the light passed the glass?

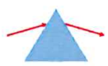
Select one alternative:

 ○

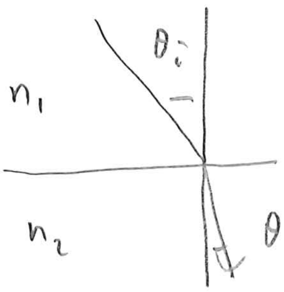
 ○

 ○

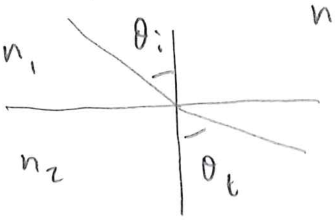
 ○

 X

Maximum marks: 5



$n_1 < n_2 \Rightarrow \theta_t < \theta_i$ REFRACTING TOWARDS NORMAL.



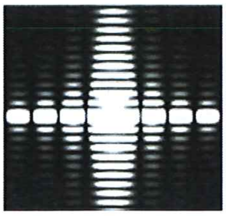
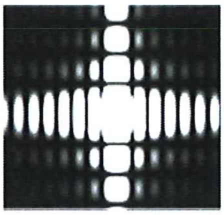
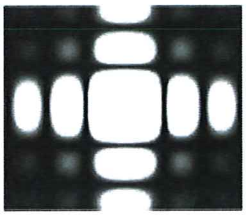
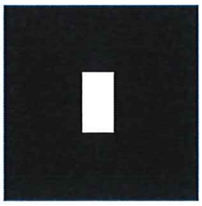
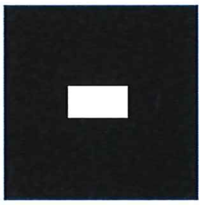
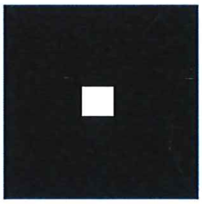
$n_1 > n_2 \Rightarrow \theta_t > \theta_i$ REFRACTING FROM THE NORMAL

ONLY ONE ALTERNATIVE POSSIBLE.

4 MC4 TFY4195 H19

Consider the three far-field diffraction patterns in the upper row. Which of the apertures could have made these diffraction patterns?

Please match the apertures with the diffraction pappterns:

	 2	 3	 1
 One	☒	○	○
 Three	○	☒	○
 Two	○	○	☒

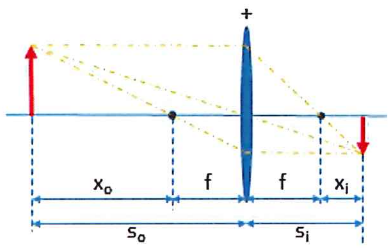
Maximum marks: 5

SMALLER SLIT WIDTH $\Rightarrow$ WIDER DIFFRACTION PATTERN
AND VICE VERSA

5

MC5 TFY4195 H19

The object and the image formed using a positive lens of focal length f is depicted in the image below along with some distances that define their positions relative to the lens.



Which of the following equations does NOT describe their positions?

Select one alternative:

- ○ $\frac{f}{s_o - f} = \frac{s_i - f}{f}$
- ○ $\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i}$ ← GAUSS' THIN LENS LAW
- ○ $x_i x_o = f^2$ ← NEWTON'S THIN LENS LAW
- X $\frac{1}{x_o x_i} = f^2$ MAKES NO SENSE (CHECK DIMENSION)
- ○ $\frac{f}{x_o} = \frac{x_i}{f}$

VARIATIONS OF GAUSS' AND NEWTON'S LENS LAWS

Maximum marks: 5

6

MC6 TFY4195 H19

Paul has a white light source and wants to use it in a Michelson interferometer experiment set-up using yellow light ($\lambda = 580nm$). He finds a 10 nm pass-band i.e., a filter that transmits light between 575 and 585 nm. What is the approximate coherence length of such a wavelength filtered light source?

Select one alternative:

WITH MICHELSON'S INTERFEROMETER WE MEASURE TEMPORAL COHERENCE.

- ○ $l_c = 340\mu m$
- X $l_c = 34\mu m$
- ○ $l_c = 3.4\mu m$
- ○ $l_c = 3.4mm$
- ○ $l_c = 340nm$

$$l_c = c \cdot \Delta t = \frac{c}{\Delta f} = \frac{\lambda \cdot f}{\Delta f}$$

SIGN HAS NO MEANING HERE

$$f = \frac{c}{\lambda} \quad \frac{df}{d\lambda} = -\frac{c}{\lambda^2} \Rightarrow \Delta f = -\frac{c \cdot \Delta\lambda}{\lambda^2} \Rightarrow l_c = \left| -\frac{\lambda^2}{\Delta\lambda} \right| = \frac{(580 \cdot 10^{-9})^2}{10 \cdot 10^{-9}} = 33.6\,\mu m$$

Maximum marks: 5

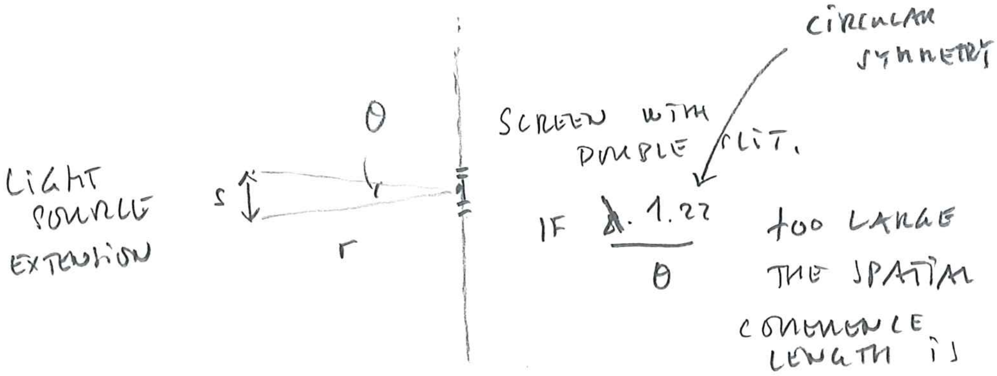
7

MC7 TFY4195 H19

Using the yellow light source in problem MC6 ($\lambda = 580nm$), Paul also wants to test the classical double-slit experiment. As he knows about spatial coherence he puts a lens to focus the light onto a small pinhole (a thin circular aperture) which is $500\mu m$ in diameter. The light passing through the pinhole is then used for the interference experiment using a sensitive photon counting detector. The double slit is placed 50 cm from the pinhole. What is the maximum width between the slits he can examine without running into problems related to the spatial coherence?

Select one alternative:

- $70\mu m$
- X $0.7mm$
- $35\mu m$
- $3.5mm$
- $7mm$



IF $\frac{\lambda \cdot 1.22}{\theta}$ too LARGE THE SPATIAL COHERENCE LENGTH IS REACHED

Maximum marks: 5

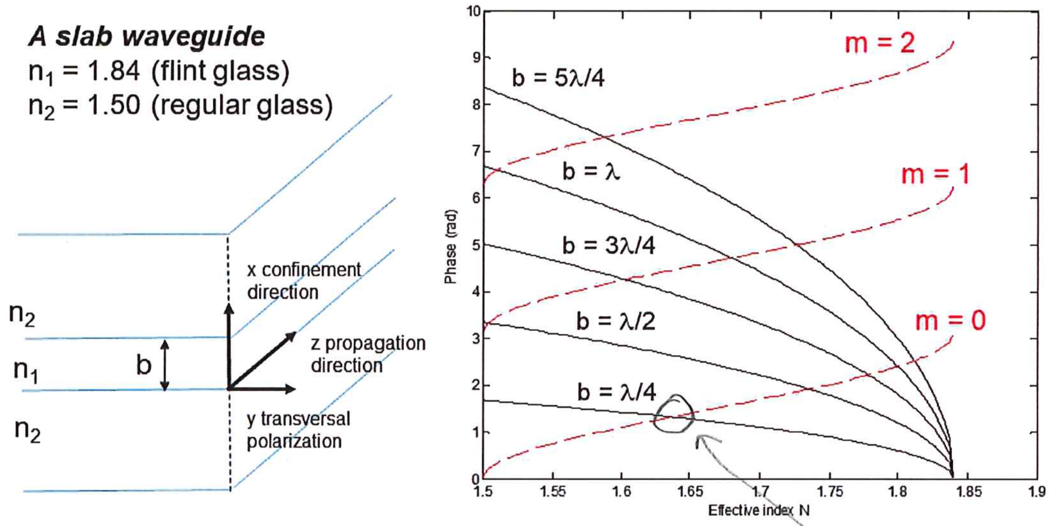
$$l_c = \frac{\lambda}{\theta} \cdot 1.22 = \frac{580 \cdot 10^{-9} \cdot 1.22}{500 \cdot 10^{-6}} \cdot 0.5 = 0.7 \text{ mm}$$

$$\theta = \frac{s}{r}$$

8

MC8 TFY4195 H19

The view graph below shows the principle and essential parameters for a slab wave-guide and the calculated TE mode dispersion for several design options. If the telecommunication wavelength to be used is $1.55\mu m$ and the slab waveguide thickness is 400 nm, how many modes can the wave-guiding structure promote?



The propagating TE-mode:

$$\bar{E}(x,z,t) = \hat{y}\cdot E_0(x)\,e^{i(\beta z-\omega t)} = \hat{y}\cdot E_0(x)\,e^{i(kNz-\omega t)}$$

$k = \dfrac{2\pi}{\lambda}$; $\hat{z}$ *propagation direction*

TE Mode dispersion:

$$2\cdot\tan^{-1}\sqrt{\frac{N^2-n_2^2}{n_1^2-N^2}} + m\cdot\pi = \frac{2\pi b}{\lambda}\sqrt{n_1^2-N^2}$$

$\lambda = 1.55\cdot 10^{-6}$

$b = 0.4\cdot 10^{-6}$

$\Rightarrow \dfrac{b}{\lambda} = \sim 0.26$

ONLY POSSIBLE SOLUTION

Select one alternative:

- >3
- 3
- 2
- 0
- X 1

Maximum marks: 5

9 **MC9 TFY4195 H19**

A left-circularly polarized light-wave is impinging on a glass window of refractive index 1.60 with the incident angle 58°. What can you say about the reflected light?

Select one alternative:

BREWSTER ANGLE
(POLARIZATION ANGLE)
$\tan \theta_i = 1.6 \Rightarrow \theta_i = 58^\circ$
ONLY S-POL LIGHT IS REFLECTED (TE-MODE) ⊥-POL

- ○ The reflected light is right circularly polarized.
- ○ No light is reflected
- ○ All light is reflected.
- ○ The reflected light is left circularly polarized.
- X All reflected light is linearly polarized.

Maximum marks: 5

10 **MC10 TFY4195 H19**

IF REFRACTIVE INDEX OF TRANSM. MEDIUM IS SMALLER ⇒ TOTAL REFLECTION ABOVE CRITICAL ANGLE

A left-circularly polarized light-wave is propagating within a glass of refractive index 1.60 and hits a planar inner surface (with air on the outside). The angle of incidence is 58°. What can you say about the reflected light?

Select one alternative:

CRITICAL ANGLE
$\sin \theta_c = \frac{1}{1.6} \Rightarrow \theta_c = 38.7^\circ$
$\therefore 58^\circ \rightarrow$ TOTAL INTERNAL REFLECTION

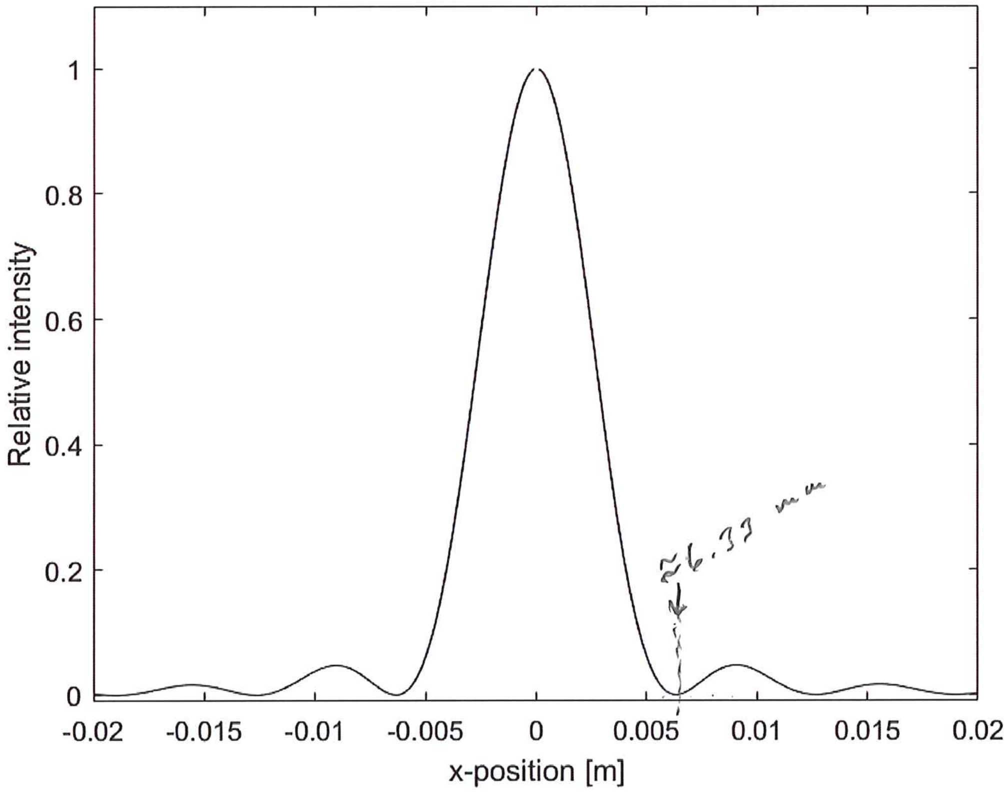
- ○ All reflected light is linearly polarized.
- ○ The reflected light is right circularly polarized.
- X All light is reflected.
- ○ No light is reflected
- ○ The reflected light is left circularly polarized.

FRESNEL REFLECTION COEFF. GIVES PHASE-SHIFTS THAT RUIN CONDITION FOR CIRCULAR LIGHT.

Maximum marks: 5

11 MC11 TFY4195 H19

Lars-Martin is carrying out a diffraction experiment in the Optics lab with a HeNe laser ($\lambda = 632.8nm$) using a single slit positioned with the long extension vertical. A lens of focal length 1.0 m is placed right after the slit to assure an appropriate far-field diffraction pattern. He records the intensity distribution in the horizontal direction by scanning the detector through the diffracted beam 1 m from the slit/lens and the following intensity pattern is recorded.



What is the width of the slit?

Select one alternative:

$$\mathrm{rect}\left(\frac{t}{T}\right) \rightarrow T\,\frac{\sin\left(\omega\frac{T}{2}\right)}{\left(\omega\frac{T}{2}\right)} \qquad T \text{ WIDTH OF SLIT}$$

- ○ $63\mu m$
- ○ $632.8nm$
- ○ $3.14\mu m$
- ○ $3.1mm$
- ☒ $100\mu m$

$\omega = 2\pi f$ WE RECALL $f = \frac{x}{\lambda z}$

$\Rightarrow$ MINIMUM FOR $2\pi \cdot \frac{x}{\lambda z} \cdot \frac{T}{2} = \pm\pi$

Maximum marks: 5

$$\Rightarrow x = \frac{\lambda z}{T} = \frac{632.8 \cdot 10^{-9} \cdot 1.0}{100\,\mu m} = 6.33 \cdot 10^{-3}\ m$$

12

MC12 TFY4195 H19

Paul continues to explore diffraction experiments with the scanning equipment described in MC11. To analyze the results, he wants to derive expressions for the intensity distribution of the diffraction patterns of multiple slits. He learned during the optics lecture that he can calculate the multiple slit diffraction by taking the Fourier transform of *N* slits represented by equally spaced Dirac delta-functions, and then multiply with the Fourier transform of the single slit, according to the convolution theorem of Fourier optics. He arrives at the following expression for the intensity distribution: $I(x) = [a \cdot \frac{\sin(2\pi f_x s)}{\sin(\pi f_x s)} \cdot \frac{\sin(\pi f_x a)}{(\pi f_x a)}]^2$ where $f_x = \frac{x}{\lambda f}$: *f* is the focal length of the lens, λ is the wavelength of the laser, *a* is the with of each single slit and finally, *s* is the separation between the slits. How many slits did he intend to analyse with this expression?

Select one alternative:

SLIT CONTRIBUTION

- 4
- 5
- 3
- 1
- X 2

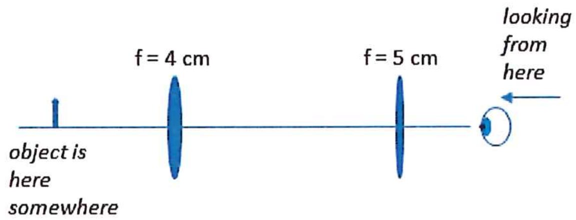
INTERFERENCE CONTRIBUTION ON THE FORM

$\frac{\sin N\alpha}{\sin \alpha}$ WITH $N = 2$

Maximum marks: 5

13 **EQ1 TFY4195 H19**

Victoria is preparing for the optics lab and is figuring out how to set up two (thin) lenses with 4 and 5 cm focal length to realize and test the principle of a microscope. She decides to use the short focal length as objective placed on the rail to the left, and then use the 5 cm focal length lens as eye-piece placed to the right. The object is placed to the very left as shown in the scheme below. Looking from the right she then wants to see the virtual image of the object 20 cm behind the 5 cm focal length lens as shown below, magnified 10 times.



a) Where shall the object be placed and what must be the distance between the lenses?

b) Ray-trace from the object to the image including intermediate images if apparent.

c) Is this a good design? Please comment...

Fill in your answer here

Format

THESE WHO RECALLED THE PRINCIPLES OF THE MICROSCOPE EASILY GOT THE CORRECT SOLUTION BY EITHER USING THE THIN LENS EQN TWICE, STARTING WITH LENS 2; OR USE THE RAY-TRANSFER MATRIX WITH THE PROPER SIGN CONVENTIONS.

TWO OTHER "SOLUTIONS" WERE OFTEN GIVEN AS ANSWER

1) MERGING THE TWO LENSES TOGETHER (d=0), ESSENTIALLY FORMING A MAGNIFYING GLASS

2) LETTING THE 2ND LENS MAKE A "REAL" IMAGE.

BOTH THESE ARE NOT "MICROSCOPES"; BUT I GAVE FULL POINTS (-5) IF CALCULATED AND RAY-TRACED CORRECTLY.

Words: 0

Maximum marks: 20

THE MICROSCOPE WAS A CENTRAL TOPIC IN PENSUM

SEE SUGGESTED SOLUTION =>

13)

a) $f_1 = 4$ cm $\qquad s_o = ?$

$f_2 = 5$ cm $\qquad s_i = -20$

TWO LENSES SEPARATED $d = ?$

$M_T = -10$ FOR CLASSICAL MICROSCOPE

THE "MAGNIFICATION" IS ALWAYS GIVEN AS 10x, 20x, 60x ETC ALTHOUGH IT IS ALWAYS "NEGATIVE".

$$\begin{pmatrix} 1 & 0 \\ s_i & 0 \end{pmatrix}\begin{pmatrix} 1-\frac{d}{f_2} & -\frac{1}{f_1}-\frac{1}{f_2}+\frac{d}{f_1 f_2} \\ d & 1-\frac{d}{f_1} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ s_o & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ -20 & 1 \end{pmatrix}\begin{pmatrix} 1-\frac{d}{5} & -\frac{5\cdot 1}{5\cdot 4}-\frac{4\cdot 1}{4\cdot 5}+\frac{d}{f_1 f_2} \\ d & 1-\frac{d}{4} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ s_o & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ -20 & 1 \end{pmatrix}\begin{pmatrix} 1-\frac{d}{5} & \frac{d-9}{20} \\ d & 1-\frac{d}{4} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ s_o & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1-\frac{d}{5} & \frac{d-9}{20} \\ -20+4d+d & \frac{-20d+180}{20}+1-\frac{d}{4} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ s_o & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1-\frac{d}{5} & \frac{d-9}{20} \\ -20+5d & -\frac{5d}{4}+10 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ s_o & 1 \end{pmatrix}$$

17a) CONT

$$\begin{pmatrix} 1 - \frac{d}{5} + \frac{s_0 d - s_0 9}{20} & \frac{d-9}{20} \\ -20 + 5d - \frac{5 d s_0}{4} + 10 s_0 & -\frac{5d}{4} + 10 \end{pmatrix} \quad M_T$$

$= 0$ as it's imaging

$$M_T = -10 \Rightarrow -10 = -\frac{5d}{4} + 10 \Rightarrow -20 = -\frac{5d}{4}$$

$$\Rightarrow \underline{d} = \frac{4 \cdot 20}{5} = \frac{80}{5} = \underline{16}$$

$$-20 + 5 \cdot 16 - \frac{5 \cdot 16 \, s_0}{4} + 10 s_0 = 0$$

$$-20 + 80 - \frac{80 s_0}{4} + 10 s_0 = 0$$

$$60 - 20 s_0 + 10 s_0 = 0$$

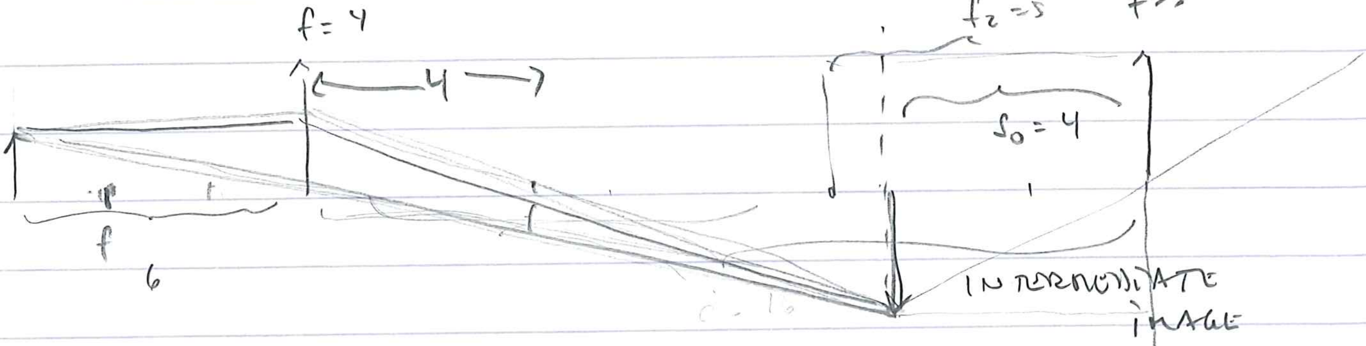
$$60 - 10 s_0 = 0$$

$$\underline{\underline{s_0 = 6}}$$

LENSES SEPARATED 16 cm

OBJECT 6 cm BEFORE L1

13 b) CHECK WITH THIN LENS LAW.



LENS 1: $\frac{1}{6} + \frac{1}{s_i} = \frac{1}{4} \Rightarrow \frac{1}{s_i} = \frac{1}{4} - \frac{1}{6} = \frac{3-2}{12} = \frac{1}{12} \Rightarrow s_i = 12$

$\Rightarrow M_T = -\frac{12}{6}$

$= -2$

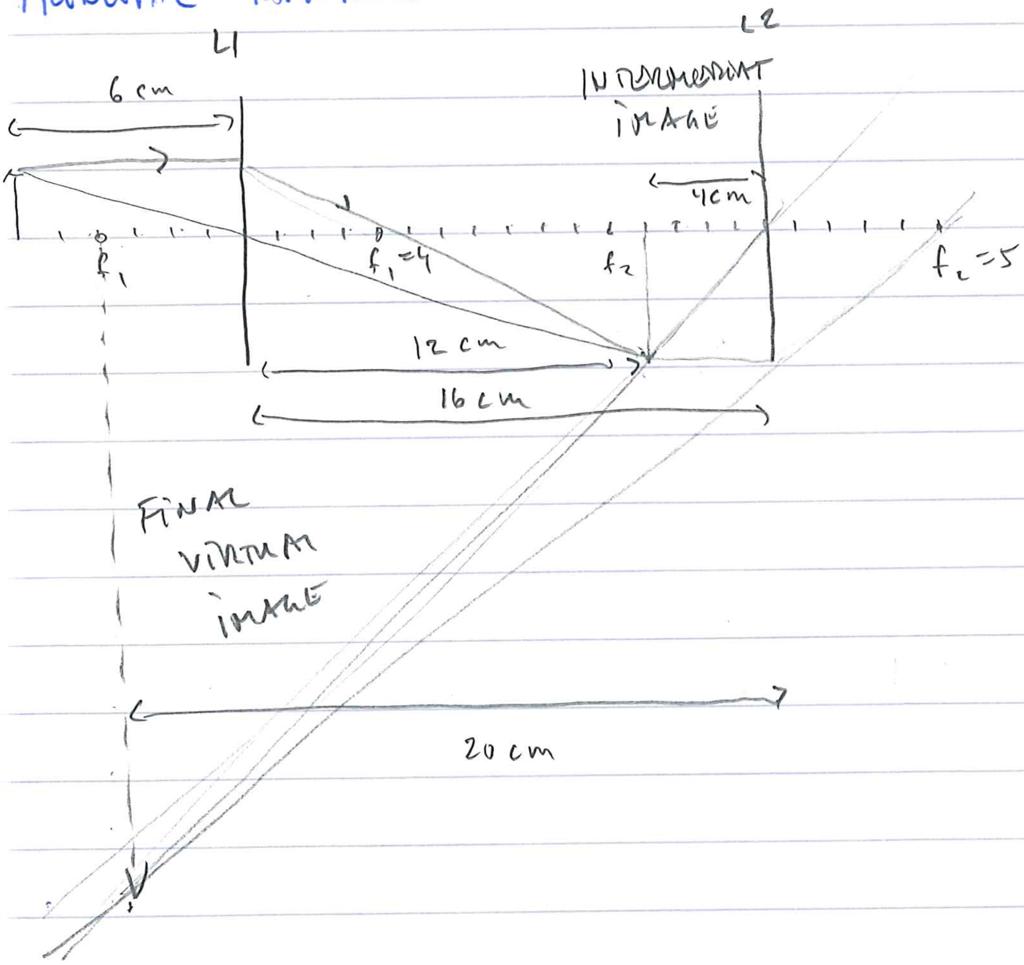
LENS 2: $s_o = 4$ $\quad \frac{1}{4} + \frac{1}{s_i} = \frac{1}{5} \Rightarrow \frac{1}{s_i} = \frac{1}{5} - \frac{1}{4} = -\frac{1}{20} \Rightarrow s_i = -20$

OK!

$M_T = -\frac{-20}{4} = +5$

$\Rightarrow M_{tot} = (-2)(+5) = -10$ OK!

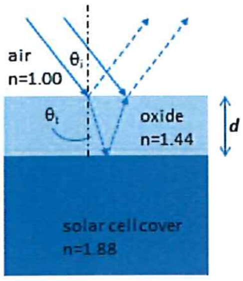
PRINCIPAL RAY-TRACE



13 c) TAKEN OUT IN VIEW OF THE MANY SYSTEMS PROPOSED....

14 EQ2 TFY4195 H19

Oddbjørn got a job at IFE as a technical developer and since he had an **A** in Optics his first task was to design an anti-reflective coating for a new type of solar cell cover material made from a very hard and sustainable transparent glass of high refractive index. Such a coating is made by sputtering a thin layer of SiO_2 ($n = 1.44$) on top of the cover as shown in the scheme below. As the bottom material has a relatively high refractive index only the two depicted reflections need to be considered.



a) Show that the optical path length difference of the two indicated reflected rays can be written as:
$\Delta = 2d \cdot n_{oxide} \cdot \cos\theta_t$

b) What is the thinnest possible anti-reflective coating for visible light ($\lambda = 550nm$)? It is anticipated the anti-reflection coating should be operational for small incident angles (i.e. set $\theta_i = 0$).

c) Estimate and compare the reflected light intensity for the coated and non-coated surface at perpendicular incidence ($\theta_i = 0$).

State clearly any (reasonable) approximations you use.

Fill in your answer here

Format

THIS WAS THE MOST DIFFICULT PROBLEM, BUT THOSE WHO SUCCEEDED TO SET UP Δ(OPL) COULD ALSO SOLVE IT BY USING THE OPL IN THE OXIDE LAYER AND ADOPT SNELL'S LAW TO COMPENSATE FOR THE OPL IN THE AIR.

b) & c) COULD BE SOLVED INDEPENDENT FROM A AS $\Delta = 2d\, n_{oxide} \cos\theta_t$ WAS GIVEN ($\theta_t = 0$)

NOT ALL GOT THE INTERFERENCE CONTRIBUTION RIGHT IN c) BUT GOT BONUS POINTS FOR GOOD ATTEMPTS...

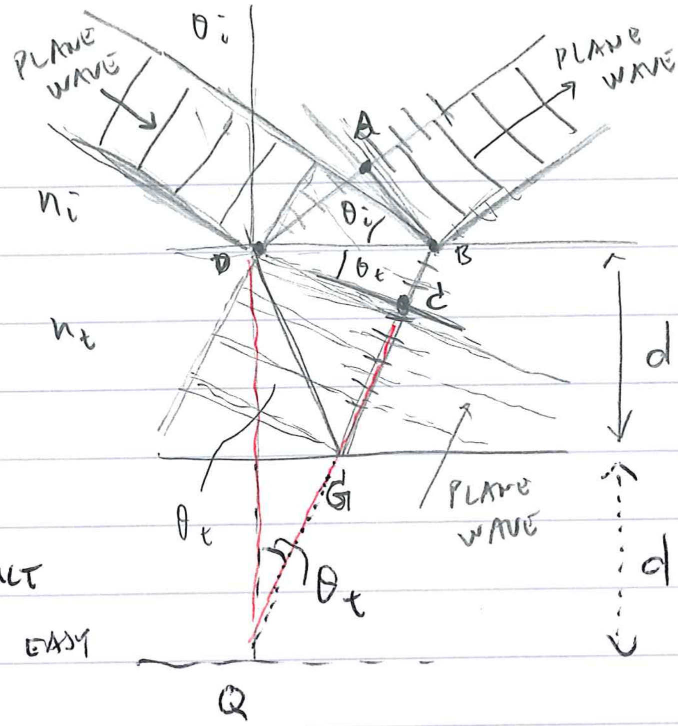
Words: 0

Maximum marks: 20

LF

$\Rightarrow$

14 a)



HELP CONSTRUCT TO GET $2d\cos\theta_t$ EASY

OPL - SNELL

OPTICAL PATH LENGTH $\overline{DA}$ & $\overline{CB}$ MUST BE EQUAL (THAT'S HOW SNELL'S LAW IS PROVED)

HENCE, OPL-DIFFERENCE IS $D\to G\to C\to B - D\to A$

$= D\to G\to C$ AS $\overline{DA}$ CANCELS $\overline{CB}$

OPL: $D\to G\to C = Q\to G\to C = 2\cdot d\cdot n_t\cdot\cos\theta_t$

ORIGE

QED

b) AT PERPENDICULAR INCIDENCE

$$2d\cdot n_t = \frac{\lambda}{2} \quad \text{FOR DESTRUCTIVE INTERFERENCE}$$

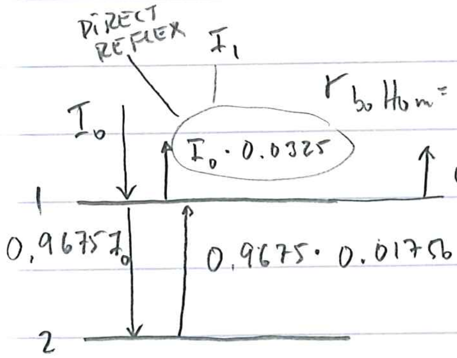
$$\Rightarrow d = \frac{\lambda}{4n_t} = \frac{550\cdot10^{-9}}{4\cdot1.44} \approx \underline{95\,nm}$$

c) NON-COATED ; $r = \frac{n_1-n_2}{n_1+n_2} = \frac{1-1.88}{1+1.88} = \frac{-0.88}{2.88} = -\frac{11}{36}$

$$\Rightarrow r^2 \approx \underline{\underline{9.3\%}}$$

COATED : $r_{top} = \frac{1-1.44}{1+1.44} = \frac{11}{61} \Rightarrow R_{top} = \left(\frac{11}{61}\right)^2 = 0.0325$

$r_{bottom} = \frac{1.44-1.88}{1.44+1.88} = \frac{11}{83} \Rightarrow R_{bottom} = \left(\frac{11}{83}\right)^2 = 0.01756$



$\uparrow 0.9675\cdot0.01756\cdot0.9675\cdot I_0 = I_2$ (REFEX FROM BOTTOM)

$I_1 = 0.0325\, I_0$
$I_2 = 0.01644\, I_0$
USE $I = I_1 + I_2 - 2\sqrt{I_1\cdot I_2}$

$= 0.04894 - 0.04623$

$= 0.00271 \approx \underline{0.27\%}$

TFY4195: Useful formula I

Physical constants

$c = 2{,}998 \times 10^{8}\ m/s$ $\qquad \varepsilon_0 = 8.854 \times 10^{-12}\ F/m$ $\qquad \mu_0 = 4\pi \times 10^{-7}\ H/m$

EM waves and guided light

The Maxwell equations without currents and charges

$$\bar{\nabla} \cdot \bar{E} = 0 \qquad \bar{\nabla} \cdot \bar{B} = 0$$

$$\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \qquad \bar{\nabla} \times \bar{H} = \frac{\partial \bar{D}}{\partial t}$$

With linear transparent (optical) media: $\bar{D} = \varepsilon \bar{E} = \varepsilon_0 n^2 \bar{E}$ and $\bar{B} = \mu_0 \bar{H}$

Fresnel's equations for reflection and transmission

$$r_{//} = r_p = \left(\frac{E_r}{E_i}\right)_p = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2} \qquad r_{\perp} = r_s = \left(\frac{E_r}{E_i}\right)_{\perp} = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

$$t_{//} = t_p = \left(\frac{E_t}{E_i}\right)_p = \frac{2n_1 \cos\theta_1}{n_2 \cos\theta_1 + n_1 \cos\theta_2} \qquad t_{\perp} = t_s = \left(\frac{E_t}{E_i}\right)_{\perp} = \frac{2n_1 \cos\theta_1}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

The energy crossing unit area per unit time in the propagation direction

Poynting vector: $\bar{S} = c^2 \varepsilon_0 \left(\bar{E} \times \bar{B}\right)$; $S = \frac{1}{2} c n \varepsilon_0 E_0^2$ (medium with refractive index n)

The reflectance and transmittance of energy flow across a surface

$R = r^2$; $\qquad T = \frac{n_2}{n_1}\left(\frac{\cos\theta_2}{\cos\theta_1}\right) t^2$; where r and t are reflectance and transmittance coefficients

Thin lenses

$$\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f}$$

Lens maker formula: $\frac{1}{f} = \frac{n_{lens} - n_{medium}}{n_{medium}}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Coherence lengths

$l_c = \frac{1.22\lambda}{\theta}$ spatial coherence length, circular aperture

$l_c = \frac{\lambda_0 f_0}{\Delta f}$ temporal coherence length, rectangular spectrum

Interference of mutually coherent beams

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

TFY4195: Useful formula II

Diffraction

Fresnel approx. $$U(x,y)=\frac{e^{ikz}}{i\lambda z}e^{i\frac{k}{2z}\left(x^2+y^2\right)}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\left(U(\xi,\eta)e^{i\frac{\pi}{\lambda z}\left(\xi^2+\eta^2\right)}\right)e^{-i\frac{2\pi}{\lambda z}(x\xi+y\eta)}d\xi\, d\eta$$

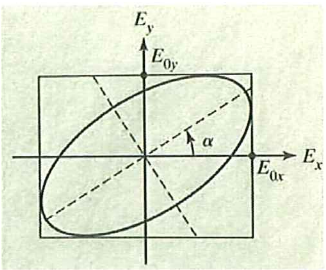
For an array of N slits of width b and separation a, the following far-field diffraction distribution applies.

$$I=I_0\left(\frac{\sin\beta}{\beta}\right)^2\left(\frac{\sin N\alpha}{\sin\alpha}\right)^2 \quad \text{where } \alpha=\frac{1}{2}ak\sin\theta \text{ and } \beta=\frac{1}{2}bk\sin\theta$$

Polarization and Jones formalism

Jones' vector $\bar{J}=\begin{bmatrix}E_{0x}\\ E_{0y}e^{i\Delta\phi}\end{bmatrix}$ has field components: $$E_x=E_{0x}\cos\left(kz-\omega t+\phi_x\right)$$ $$E_y=E_{0y}\cos\left(kz-\omega t+\phi_y\right);\ \Delta\phi=\phi_y-\phi_x$$

The polarization ellipse: $$\left(\frac{E_y}{E_{0y}}\right)^2+\left(\frac{E_x}{E_{0x}}\right)^2-2\left(\frac{E_y}{E_{0y}}\right)\left(\frac{E_x}{E_{0x}}\right)\cos\Delta\phi=\sin^2\Delta\phi$$



where $$\tan 2\alpha=\frac{2E_{0x}E_{0y}\cos\Delta\phi}{\left(E_{0x}^2-E_{0y}^2\right)}$$

Jones matrices:

Polarizers

TA horizontal	TA vertical	TA +45°	TA θ° from horisontal axis
$\begin{bmatrix}1&0\\0&0\end{bmatrix}$	$\begin{bmatrix}0&0\\0&1\end{bmatrix}$	$\frac{1}{2}\begin{bmatrix}1&1\\1&1\end{bmatrix}$	$\begin{bmatrix}\cos^2\theta&\sin\theta\cos\theta\\ \sin\theta\cos\theta&\sin^2\theta\end{bmatrix}$

Phase retarders Wave plates

QWP SA horizontal	QWP SA vertical	HWP SA horizontal	HWP SA vertical	General
$e^{i\frac{\pi}{4}}\begin{bmatrix}1&0\\0&-i\end{bmatrix}$	$e^{-i\frac{\pi}{4}}\begin{bmatrix}1&0\\0&i\end{bmatrix}$	$e^{i\frac{\pi}{2}}\begin{bmatrix}1&0\\0&-1\end{bmatrix}$	$e^{-i\frac{\pi}{2}}\begin{bmatrix}1&0\\0&-1\end{bmatrix}$	$\begin{bmatrix}e^{i\varepsilon_x}&0\\0&e^{i\varepsilon_y}\end{bmatrix}$

Rotator $$\begin{bmatrix}\cos\beta&-\sin\beta\\ \sin\beta&\cos\beta\end{bmatrix} \quad (\theta\rightarrow\theta+\beta)$$

Useful Fourier Transforms and Theorems for Optics

Signal	Signal in Fourier domain
$f(t)$	$\int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$
$\frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega$	$F(\omega)$
$A \cdot \delta(t-t_0)$	$Ae^{-i\omega t_0}$
$rect(t/T)$	$T\frac{\sin(\omega T/2)}{(\omega T/2)}$

Operation	Signal	Signal in Fourier Domain
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(\omega) + a_2 F_2(\omega)$
Shift	$f(t-t_0)$	$F(\omega) e^{-i\omega t_0}$
Scaling	$f(at)$	$\frac{1}{\lvert a \rvert} F\left(\frac{\omega}{a}\right)$
Frequency shift	$f(t) e^{i\omega_0 t}$	$F(\omega - \omega_0)$
Convolution	$f_1(t) \otimes f_2(t)$ $f_1(t) \cdot f_2(t)$	$F_1(\omega) \cdot F_2(\omega)$ $\frac{1}{2\pi} F_1(\omega) \otimes F_2(\omega)$

For simplicity they are given in 1D using time (t) and angular frequency ($\omega = 2\pi f$) representing the Fourier domain

Some ray transfer matrices

$$\begin{pmatrix} n_t\alpha_t \\ y_t \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} n_i\alpha_i \\ y_i \end{pmatrix}$$

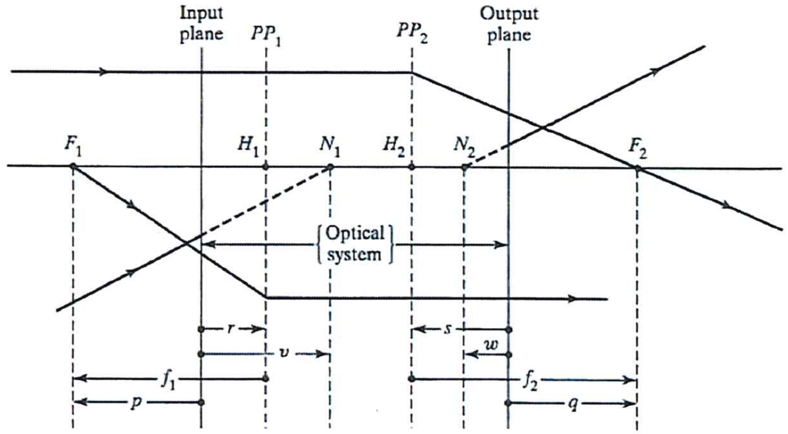
$$\begin{pmatrix} 1 & -(n_l-1)\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right) \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -\dfrac{1}{f} \\ 0 & 1 \end{pmatrix}$$ Thin lens

$$\begin{pmatrix} 1 & -D \\ 0 & 1 \end{pmatrix}; \quad D = \frac{(n_t - n_i)}{R}$$ Curved interface with radius *R*

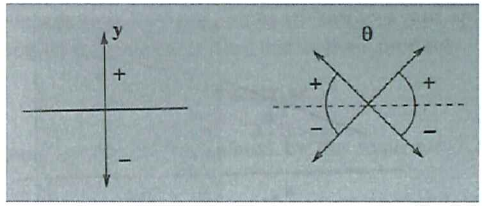
$$\begin{pmatrix} 1 & 0 \\ \dfrac{d}{n_t} & 1 \end{pmatrix}$$ Translation the distance *d* through medium with refractive index n_t

$$\begin{pmatrix} 1 & \dfrac{2}{R} \\ 0 & 1 \end{pmatrix}$$ Spherical mirror with radius of curvature R

$$\begin{pmatrix} 1-\dfrac{d}{f_2} & -\dfrac{1}{f_1}-\dfrac{1}{f_2}+\dfrac{d}{f_1 f_2} \\ d & 1-\dfrac{d}{f_1} \end{pmatrix}$$ Two lenses separated a distance *d*



Conventions: arrows to left are negative values



Cardinal points and principal planes (holds for systems with $n_i = n_t$):

$p = \dfrac{a_{11}}{a_{12}}$	F_1	
$q = -\dfrac{a_{22}}{a_{12}}$	F_2	Defined in relation to input and output planes
$r = v = \dfrac{(a_{11}-1)}{a_{12}}$	H_1, N_1	
$s = w = \dfrac{(1-a_{22})}{a_{12}}$	H_2, N_2	
$f_1 = p - r = \dfrac{1}{a_{12}}$	F_1	Defined in relation to principal planes
$f_2 = q - s = -\dfrac{1}{a_{12}}$	F_2	