Optics TFY4195

Solutions to problems from the exam.

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May-2023

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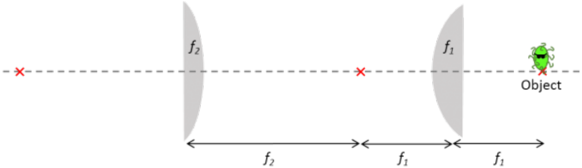
Geometrical optics

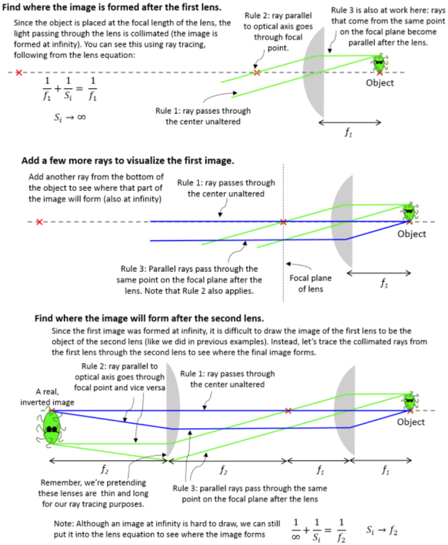
There is a cool bacterium you want to study in closer detail so you want to build a microscope to see it. You are told that you can build a microscope by placing two positive lenses (having focal lengths f_1 and f_2) at a separation distance equal to the sum of their focal lengths as shown below.



A) Trace the rays to show how your microscope works and where the image is formed. Is the image real or virtual? upright or inverted? Note that this time the object is on the right and the image will form at the left.
B) Calculate how much magnification you get from this optical system.
C) Will your microscope still work if we change the distance between the lenses?
D) Does changing the distance between the lenses affect the amount of magnification you achieve?

https://measurebiology.org/

A) Trace the rays to show how your microscope works and where the image is formed. We will see that the image is real and inverted.

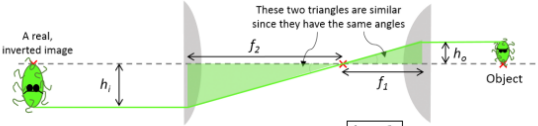


Solution

- By definition, the rays stay parallel and go on to infinity when light from an object originates exactly at the focal point.
- The rays technically shouldn't focus, but practically moving the lens slightly away so we can get an image focused on a far away screen the way a projector does.
- Depending how precise your placement is, you could get a clear, fuzzy, or no image on a screen far away.
- As the screen gets farther away, your image gets larger and dimmer, so it couldn't form a real image at a very far distance anyway let alone infinity.
- It has just become an expression to say an image is ”formed” even though an image is never formed since you can never reach infinity.
- It would similar to saying that parallel lines ”meet” at infinity, but that would contradict the meaning of ”parallel”.
- A better description would be to say that an image if formed very far away as an object approaches the focal point.

B) Calculate how much magnification you get from this optical system.

For the magnification, find a ray that relates the height of the object to the height of the final image so you can calculate their ratio.

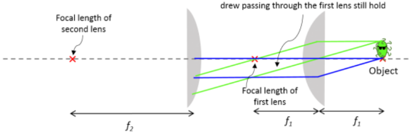


Based on similar triangles, the magnification = $\frac{h_i}{h_o} = \frac{f_2}{f_1}$

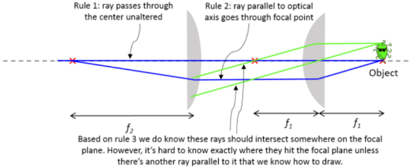
https://measurebiology.org/

C) Will your microscope still work if we change the distance between the lenses?

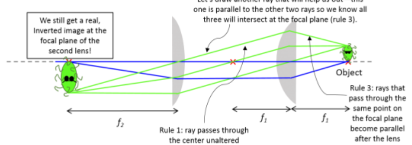
Let's try moving the lenses closer together.
Trace some rays through the first lens.



Trace some rays through the second lens.

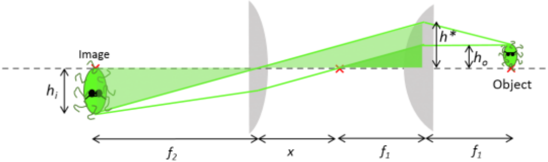


Trace more rays through the second lens.



D) Does changing the distance between the lenses affect the amount of magnification you achieve?

Find rays that relate the height of the object to the height of the final image.



Based on similar triangles:

$$\frac{h_i}{h^*} = \frac{f_2}{x + f_1} \text{ and } \frac{h^*}{h_o} = \frac{x + f_1}{f_1}$$

Thus, the magnification is:

$$\boxed{\frac{h_i}{h_o} = \frac{f_2}{f_1}}$$

This is the same result we had when the lenses were $f_1 + f_2$ apart!

https://measurebiology.org/

Principle of superposition

5-7. At the indicated position,

$$\psi_1 = A_1 \cos(8\,\pi/3 - \omega\,t), \quad A_1 = 4\,\text{cm}, \omega = 20/\text{s}$$
$$\psi_2 = A_2 \cos(3\,\pi/2 - \omega\,t), \quad A_2 = 2\,\text{cm}, \omega = 20/\text{s}$$

Using Eqs (5 9) and (5 10),

$$A = \sqrt{A_1^2 + A_2^2 + 2\,A_1\,A_2 \cos(\alpha_2 - \alpha_1)} = \sqrt{20 + 16\cos(3\,\pi/2 - 8\,\pi/3)}\ \text{cm} = 2.48\ \text{cm}$$
$$\tan\alpha = \frac{4\sin(8\,\pi/3) + 2\sin(3\,\pi/2)}{4\cos(8\,\pi/3) + 2\sin(3\,\pi/2)} \Rightarrow \alpha = 2.51$$
$$\psi_R = (2.48\ \text{cm})\cos(2.51 - (20/\text{s})\,t)$$

F.L. Pedrotti *et al.*, *Introduction to Optics*, (3rd ed., Cambridge University Press, UK, 2018)

Interference

7-9. The fringe separation is,

$$y_{m+1} - y_m \equiv \Delta y = \lambda\, L/a$$

(a) So the slit to screen distance should be,

$$L = \frac{\Delta y\, a}{\lambda} = \frac{(0.001)\,(0.0005)}{6 \times 10^{-7}}\,\mathrm{m} = 0.833\,\mathrm{m}$$

(b) The optical path difference can be written in terms of wavelengths as $\Delta = m\,\lambda$. The with and without the plate of thickness t,

$$\Delta_2 - \Delta_1 = \Delta m\,\lambda \Rightarrow \Delta m = \frac{n\,t - t}{\lambda} = (n-1)\frac{t}{\lambda} = (1.5-1)\frac{10^{-4}}{6 \times 10^{-7}} = 83.3\,\text{fringes}$$

(c) $I = 4\,I_0 \cos^2\left(\frac{\pi\,\Delta}{\lambda}\right)$ where $\Delta = a\,y/L$. At $\Delta = 0, I = I_{\max} = 4\,I_0$. Then for $I = 2\,I_0 = I_{\max}/2$:

$$2\,I_0 = I = 4\,I_0 \cos^2\left(\frac{\pi\,\Delta}{\lambda}\right) \Rightarrow \Delta = \lambda/4 = 150\,\mathrm{nm}$$

F.L. Pedrotti *et al.*, *Introduction to Optics*, (3rd ed., Cambridge University Press, UK, 2018)

Thin film interference

7-15. At normal incidence, $(m+1/2)\,\lambda = 2\,n\,t$. At 45°, $(m+1/2)\,\lambda' = 2\,n\,t\cos\theta_t$.
Here θ_t is the angle the ray makes with the normal in the film which can be found from Snell's law,

$$\sin(45°) = n\sin\theta_t \Rightarrow \sin\theta_t = \frac{1}{\sqrt{2}\,n} = \frac{1}{\sqrt{2}\,(1.38)} = 0.5124 \Rightarrow \theta_t = 30.825° = \cos\theta_t = 0.859$$

Then,

$$\lambda' = \frac{2\,n\,t\cos\theta_t}{m+1/2} = \frac{(m+1/2)\,\lambda\cos\theta_t}{m+1/2} = \lambda\cos\theta_1 = (580\,\text{nm})\,(0.859) = 498\,\text{nm}$$

F.L. Pedrotti *et al.*, *Introduction to Optics*, (3rd ed., Cambridge University Press, UK, 2018)

Diffraction

11-3. See Figure 11 19 that accompanies the problem in the text.

(a) The diffraction minima are located at angles $\theta_m = y_m/L$ where $L = 2$ m is the slit to screen distance, The positions of the minima are given by $m\,\lambda = b\sin\theta_m = by_m/L \Rightarrow y_m = m\lambda L/b$. Then,

$$y_3 - y_{-3} = \Delta y = (3-(-3))\lambda\,L/b \Rightarrow b = \frac{6\,\lambda\,L}{\Delta y} = \frac{6\,(632.8\times10^{-7}\,\text{cm})\,(200\,\text{cm})}{5.625\,\text{cm}} = 0.013\,\text{cm} = 0.13\,\text{mm}$$

(b) $L_{\min} = b^2/2\lambda$, so,

$$\frac{L}{L_{\min}} = \frac{200\,\text{cm}}{(0.0135\,\text{cm})^2/(2\cdot632.8\times10^{-7}\,\text{cm})} = 139$$

The screen is in the far field.

F.L. Pedrotti *et al.*, *Introduction to Optics*, (3rd ed., Cambridge University Press, UK, 2018)

Beam diffraction

Example 2

Imagine a parallel beam of 546-nm light of width $b = 0.5$ mm propagating a distance of 10 m across the laboratory. Estimate the final width W of the beam due to diffraction spreading.

Solution

Using Eq. (15),

$$W = \frac{2L\lambda}{b} = \frac{2(10\text{ m})(546 \times 10^{-9}\text{ m})}{0.5 \times 10^{-3}\text{ m}} = 0.0218\text{ m} = 21.8\text{ mm}$$

F.L. Pedrotti *et al.*, *Introduction to Optics*, (3rd ed., Cambridge University Press, UK, 2018)