NTNU
Norwegian University of Science and Technology
Department of Physics

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TFY4200 Optikk VK (Optics – advanced course)

Examination June 3rd, 2004, Time: 09.00 – 14.00

Permitted aid
Text: All kinds of text-books, supplementing written material, etc, are allowed.
Electronics: Calculator, portable computer (no equipment with internet access, wireless LAN, etc).

Grades to be announced before June 24th, 2004.

The examination problems (and correct solutions)
have been reviewed by Prof Ola Hunderi, May 25th, 2004

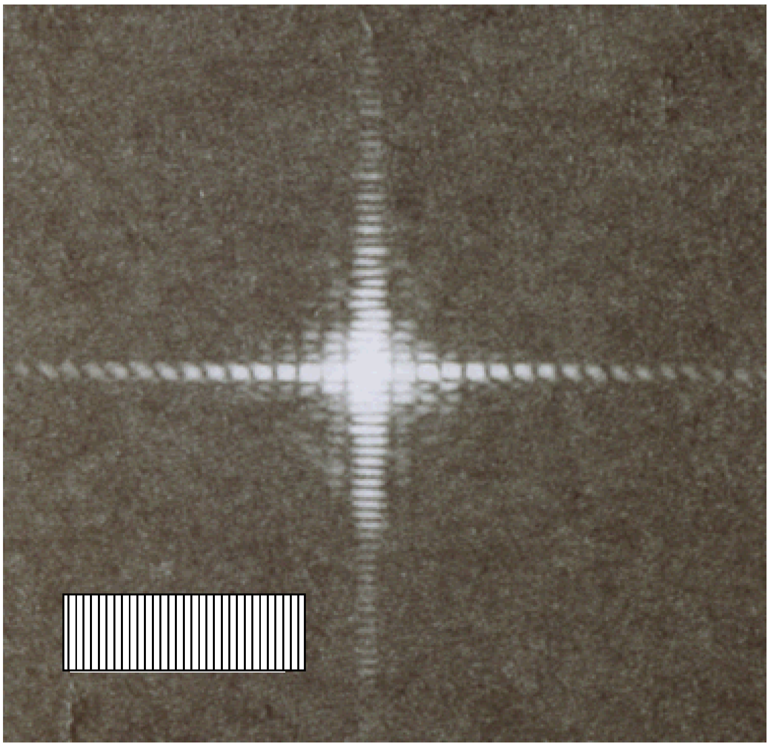
Ola Hunderi

Evaluation/grades
Total number of points of the written examination is 75. These together with lab-report (giving max 25 points) will constitute the basis for evaluation (totally max 100 points). The following table recommended by NTNU will be used for converting to A, B, C-scale.
A: better than 85 points
B: better than 75 points
C: better than 65 points
D: better than 55 points
E: better than 35 points
F: less than 35 points

Section A: Problems to which only answers shall be given (each max 5 points)

1: The Jones vector of completely polarized light was found to be: $\overline{E}_1 = \begin{bmatrix} 1 \\ -3i \end{bmatrix}$ in a certain experiment. Find the Jones vector $\overline{E}_2$ representing an orthogonal polarization state. Sketch the evolution of the electric field vector over time at z = 0 for the two polarization states. x is taken to be horizontal and y vertical polarization, respectively. (Hint: For two orthogonal polarization states $\overline{E}_1^* \cdot \overline{E}_2 = 0$)

2: A plane wave of coherent laser light propagates through a rectangular slit of width w_x and height w_y. A positive thin lens of focal length 50 cm is placed right behind the slit. At the focal plane the following diffraction pattern was recorded:



What is the approximate ratio of the width to height, i.e., $\frac{w_x}{w_y}$. (The striped area can be used as ruler if you do not possess one.)

3: A biaxial crystal is characterized by three refractive indices n_x = 2.50, n_y = 2.00 and n_z = 3.00 along its crystallographic principal directions x, y and z, respectively. What are the directions of the optic axes?

4: The following table of tensor and vector expressions is to be completed:

	A	**B**	**C**
$\varepsilon_{ijk}\varepsilon_{irs}a_r b_k c_j d_s$			
$\varepsilon_{ijk}a_i b_j c_k$ - $c_i \varepsilon_{ijk} a_j b_k$			
$\varepsilon_{ijk}\varepsilon_{rsi}a_j b_k c_s$			

The expressions needed are summarized below:

	A	**B**	**C**
1	$a_i \varepsilon_{ijk} b_j c_k$ - $c_k \varepsilon_{kij} a_i b_j$	$\bar{c}\times(\bar{a}\times\bar{b})$	0
2	$a_j b_i c_i$ - $a_k b_j c_k$	$\bar{a}\cdot(\bar{b}\times\bar{c}) - \bar{c}\cdot(\bar{a}\times\bar{b})$	$(\bar{c}\cdot\bar{a})(\bar{b}\cdot\bar{d}) - (\bar{c}\cdot\bar{d})(\bar{b}\cdot\bar{a})$
3	$c_i a_i b_j d_j$ - $c_k d_k b_l a_l$	$(\bar{c}\times\bar{b})\cdot(\bar{a}\times\bar{d})$	$(\bar{c}\cdot\bar{b})\bar{a} - (\bar{c}\cdot\bar{a})\bar{b}$

Entries from the columns A, B and C of the lower table are to be placed in empty positions of the corresponding A, B and C columns of the upper table in order to have equality along each row. (ε_{ijk} is the permutation symbol with its usual meaning.)

5: The following dielectric constant tensor is given:

$$\tilde{K} = \begin{pmatrix} 1.5^2 & 0 & 0 \\ 0 & 1.5^2 & 0 \\ 0 & 0 & 1.6^2 \end{pmatrix}$$

What is the same tensor expressed in a coordinate system obtained by rotating an angle 30^o about the y-axis (3-4 digits in answer).

6: A static electric field with components: $\bar{E}_{static} = (E_x \quad E_y \quad E_z)$ is imposed on an electrooptic crystal. The corresponding change in the impermeability tensor is:

$$\Delta\tilde{\eta} = \begin{pmatrix} r_{11}E_x + r_{13}E_z & r_{62}E_y & r_{51}E_x + r_{53}E_z \\ r_{62}E_y & r_{21}E_x + r_{23}E_z & r_{42}E_y \\ r_{51}E_x + r_{53}E_z & r_{42}E_y & r_{31}E_x + r_{33}E_z \end{pmatrix}.$$

What is the corresponding r-matrix?

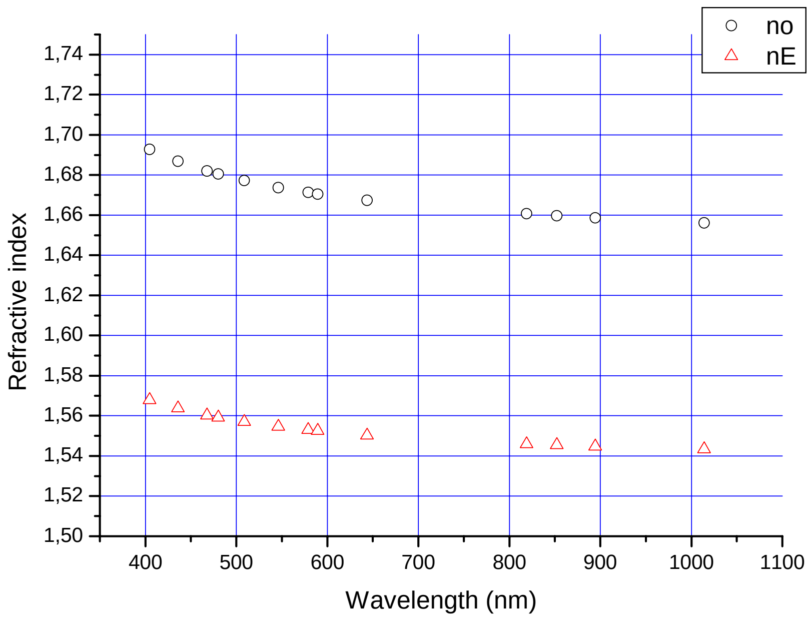
Section B: Problems to which also solutions must be handed in (each max 15 points)

7: An extraordinary electro-magnetic light wave is propagating through an anisotropic (non-magnetic) medium characterized by refractive indices: $n_x = n_y = n_o$ and $n_z = n_e$. The angle between the normal of constant phase (i.e. along $\bar{k}$) and the crystallographic z-axis is . Derive an expression that determines the corresponding angle to the ray vector, i.e. the direction indicating the energy flux with respect to the z-axis?

8: Beta-Barium Borate (BBO) is a nonlinear crystal (3m symmetry) with high transparency in the ultraviolet range. One desires to produce UV-light at 488 nm using a pulsed Ti:Sapphire laser that can be tuned in the approximate range 700 – 1000 nm giving out linearly polarized light. One wants to first try SHG with collinear phase-matching.

A) What wavelength shall be used and what will be the phase-matching angle be.
B) What is the effective nonlinearity expression (in terms of   and) in the phase-matching direction. Kleinman symmetry applies.

Refractive index data:



The d-Matrix for 3m symmetry in general conditions:

$$d = \begin{pmatrix} 0 & 0 & 0 & 0 & d_{15} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix}$$

9: Fresnel did not know about electromagnetic wave theory and the Maxwell equations but he found/made remarkable correct predictions concerning optical phenomena. He derived a formula that explained the propagation properties based on the measurements on anisotropic crystals:

$$\frac{1}{n^2} = \frac{s_x^2}{n^2 - n_x^2} + \frac{s_y^2}{n^2 - n_y^2} + \frac{s_z^2}{n^2 - n_z^2}\ .$$

n_x, n_y, n_z are material parameters describing the refractive index along dielectric principal axes, **s** denotes an arbitrary direction of the wave normal. The solution in terms of n corresponds to the two (only) possible polarization states propagating in an anisotropic crystal. Show that this equation can be derived from the Maxwells “wave equation”, e.g. from:

$$\frac{1}{\ _0}\ \frac{n}{c}\ ^2\ \bar{s}\quad \bar{s}\quad \bar{E}\qquad \bar{D}\ .$$ (eq.6.43 in the compendium)