NTNU
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Department of Physics

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TFY4200 Optikk VK (Optics – advanced course)

Examination June 8th, 2005, Time: 09.00 – 13.00

Allowed aid
Text: All kinds of text-books, supplementing written material, etc, are allowed.
Electronics: Calculator, portable computer (no equipment with internet access, wireless LAN, etc).

Grades to be announced before June 23rd, 2005.

The examination problems (and correct solutions) have been reviewed by: Prof Ola Hunderi, June 1st, 2005

Prof. Ola Hunderi

Evaluation/grades
Total number of points of the written examination is 75. These together with lab-report (giving max 25 points) will constitute the basis for evaluation (totally max 100 points). The following table recommended by NTNU will be used for converting to A, B, C, …-scale.
A: better than 85 points
B: better than 75 points
C: better than 65 points
D: better than 55 points
E: better than 35 points
F: less than 35 points

Section A: Problems to which only answers shall be given (each max 5 points)

1: The orientational average of some function $f(\bar{n})$ is defined as:

$$\left\langle f(\bar{n}) \right\rangle = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} f(\bar{n}) \sin\theta \, d\theta \, d\varphi \, ,$$

The angles θ and φ have their usual meaning as defining polar coordinates. Calculate $\left\langle f(\bar{n}) \right\rangle$ given that $f(\bar{n}) = \bar{n} = \hat{x}\sin\theta\cos\varphi + \hat{y}\sin\theta\sin\varphi + \hat{z}\cos\theta$.

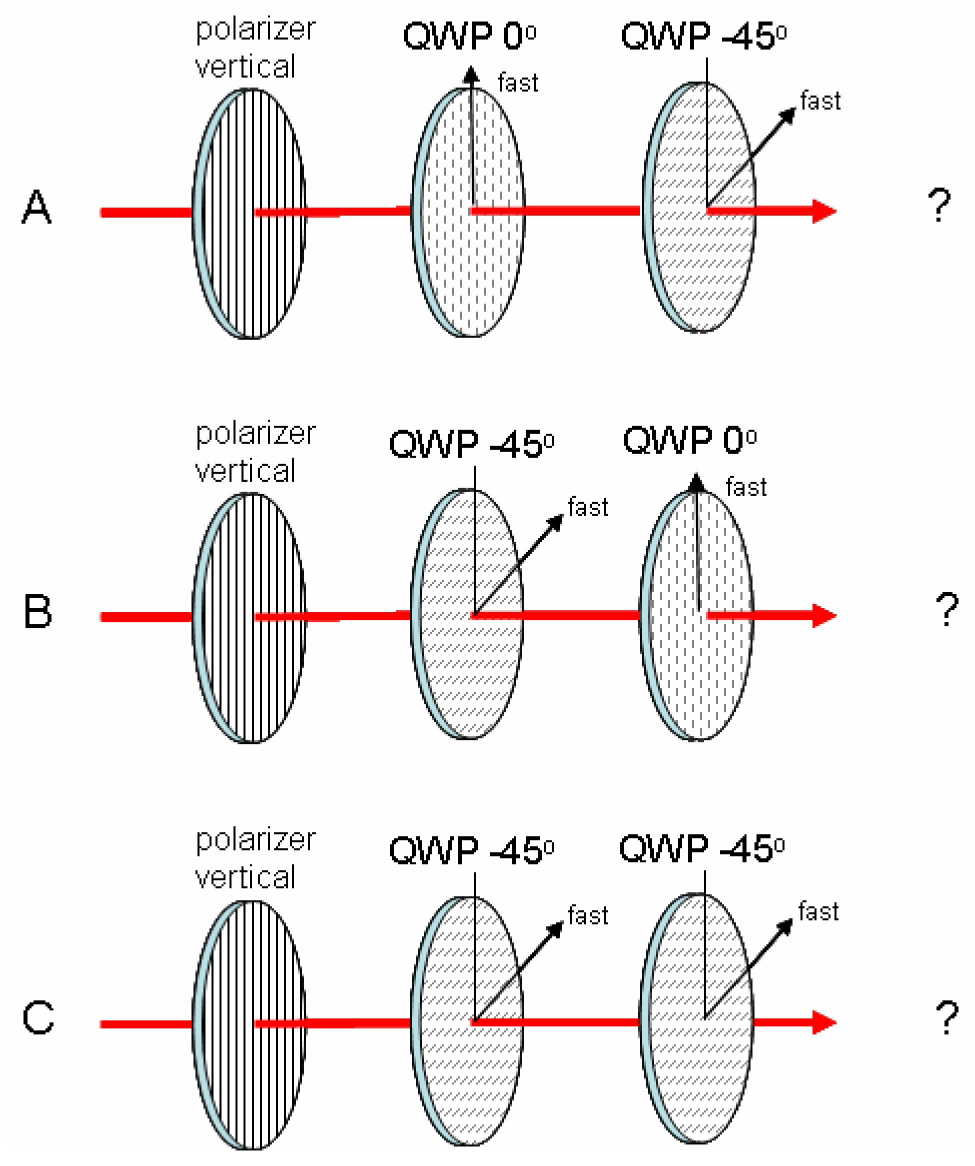
2: The d-matrix for general three-wave interactions in crystals of point group $\bar{4}2m$ is indicated below.

$$d = \begin{pmatrix} 0 & 0 & 0 & d_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{14} & 0 \\ 0 & 0 & 0 & 0 & 0 & d_{36} \end{pmatrix}$$

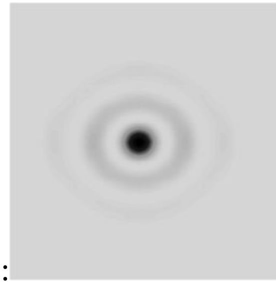
What is the effective nonlinearity expressions for the $o+o \Rightarrow e$ and $e+e \Rightarrow o$ SHG interactions, i.e. d_{eff}^{ooe} and d_{eff}^{eeo} , respectively.

3: A biaxial crystal is characterized by three refractive indices $n_x = 1.50$, $n_y = 3.00$ and $n_z = 3.40$ along its crystallographic principal directions x, y and z, respectively. What are the directions of the optic axes?

4: A polarizer and two quarter-wave plates (QWP) with the fsat axis indicated, are aligned along a laser beam as indicated below. The initial laser beam is unpolarized. What is the resulting polarization state after passing all three optical components oriented as in the cases A, B and C?



5: A student made some optical Fourier transforming (FT) experiments by using a lens. The FT patterns shown below were recorded in the focal plane using a CCD camera.
As a start-function to FT the student used the letters O, V and E (font Arial) taken as a transparent region in a mask else blocking the laser beam. (A plane wave laser-beam was used.) Unfortunately the images got mixed up. Which of the figures 1, 2, and 3 are the FT of the letters O, V and E ?

1:  2:  3: 

The images have been inverted (black → white, and vice versa) to obtain better contrast in the print/copy process.

6: The electrooptic tensor of a monoclinic crystal with a two-fold rotation axis around the z-axis is given by:

$$\begin{bmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{23} \\ 0 & 0 & r_{33} \\ r_{41} & r_{42} & 0 \\ r_{51} & r_{52} & 0 \\ 0 & 0 & r_{63} \end{bmatrix}$$

What is the change in the impermeability tensor $\Delta\eta_{ij}$ upon applying a static field: $\overline{E} = E_0\left(\hat{x} + \sqrt{5}\,\hat{z}\right)$ **?**

Section B: Problems to which also solutions must be handed in (each max 15 points)

7: Assuming a harmonic time-dependency on the form $e^{-i\omega t}$ for the E and B fields, the Maxwell curl equations can be written,

Eq.1 $$\varepsilon_{abc}\frac{\partial E_c}{\partial x_b} = i\omega B_a$$

Eq.2 $$\varepsilon_{abc}\frac{\partial B_c}{\partial x_b} = -i\omega\mu_0\varepsilon_0 K_{ad} E_d$$

μ_0 and ε_0 are the permeability of vacuum and electric permittivity of vacuum, respectively. ε_{ijk} is the permutation symbol with its usual meaning.

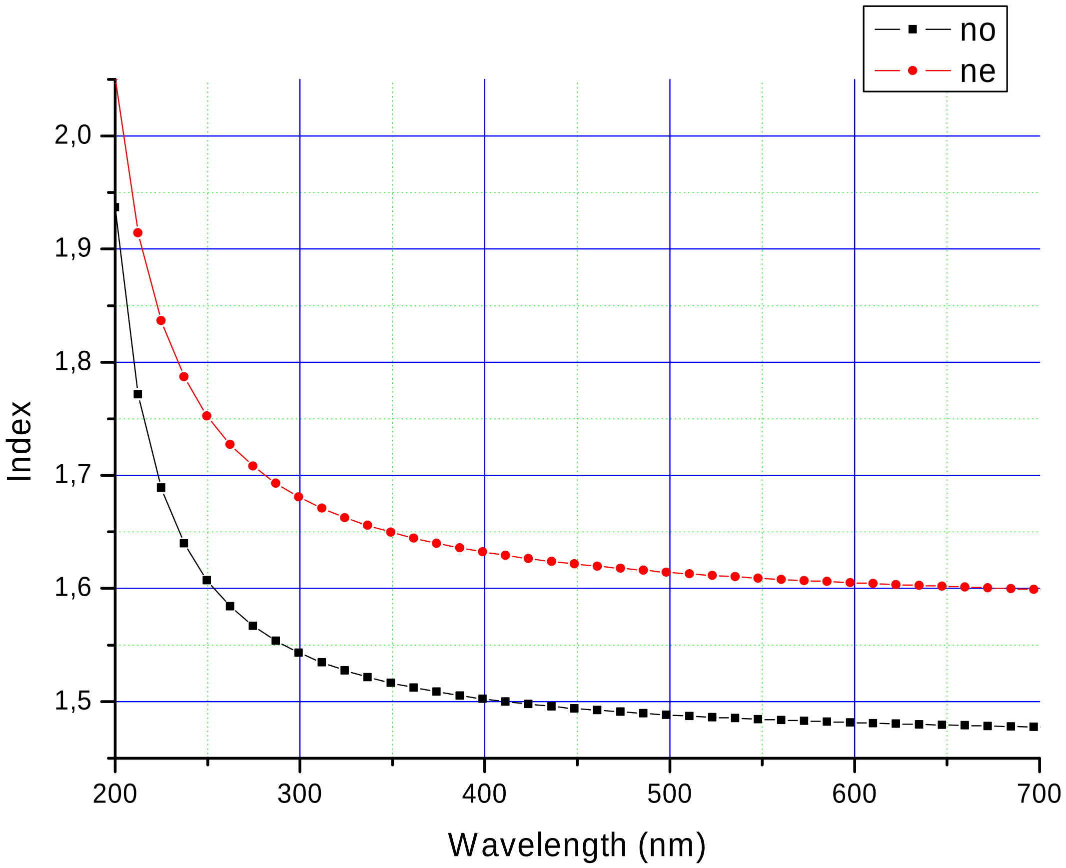
$K_{ij} = \begin{pmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{pmatrix} = \tilde{K}$ is the dimensionless dielectric constant tensor having no time dependence.

Using tensor notation, show that the equations Eq.1 and Eq.2 together give the "vector wave equation", in traditional vector notation taken as

W.eq. $$\overline{\nabla}\left(\overline{\nabla}\cdot\overline{E}\right) - \nabla^2\overline{E} = \frac{\omega^2}{c^2}\tilde{K}\,\overline{E}\,.$$

8: Urea was one of the first “organic” crystals used in nonlinear optics. It is positively uniaxial with point group $\bar{4}2m$ with a transparency range of approximately 200 nm – 1.43 m. The refractive indices in the ultraviolet and visible ranges are plotted below.

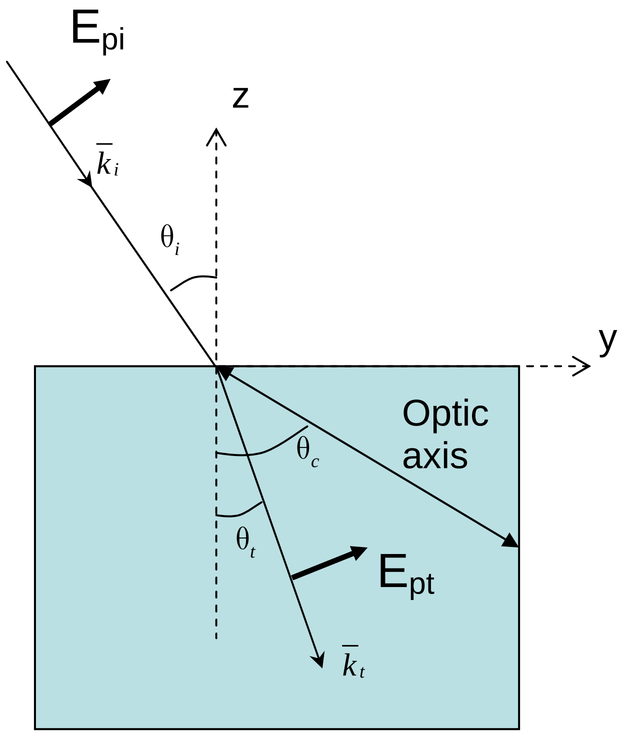
Refractive index data for Urea:



A. What is the approximate wavelength of the fundamental (pump) frequency for critical SHG phase-matching? (With critical phase-matching means the case with light propagating along one of the crystallographic directions.)

B. Draw a scheme of the crystal with the optic axis as the z-direction. Assume critical phase-matching (as in A). Indicate the polarization of the fundamental and SHG in relation to the propagation direction (k) and the optic axis.

9: A lightwave with polarization vector E in the plane of incidence (here yz-plane), with incidence angle θ_i, is refracted through the surface of a uniaxial crystal. The light-wave is refracted at an angle θ_t with respect to the surface normal direction (z-axis). The optic axis of the crystal is also in the plane of incidence however, at an angle θ_c from the surface normal (typically, $0^o < \theta_c < 90^o$). The situation is summarized in the schematic below.



Show that an expression that relates the incident angle θ_i and the refracted angle θ_t can be written as:

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{n_O\left[1+\tan^2\left(\theta_c-\theta_t\right)\right]^{\frac{1}{2}}}{\left[1+\frac{n_O^2}{n_E^2}\tan^2\left(\theta_c-\theta_t\right)\right]^{\frac{1}{2}}},$$

Where n_O and n_E is the ordinary and extraordinary refractive index, respectively, of the uniaxial crystal.