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Department of Physics

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TFY4200 Optikk VK (Optics – advanced course)

Examination May 29, 2007, Time: 09.00 – 13.00

Allowed aid
Text: All kinds of text-books, supplementing written material, etc, are allowed.
Electronics: Calculator, portable computer (no equipment with internet access, wireless LAN, etc).

Grades to be announced before June 18, 2007

The problems were reviewed by Prof. Mikael Lindgren.

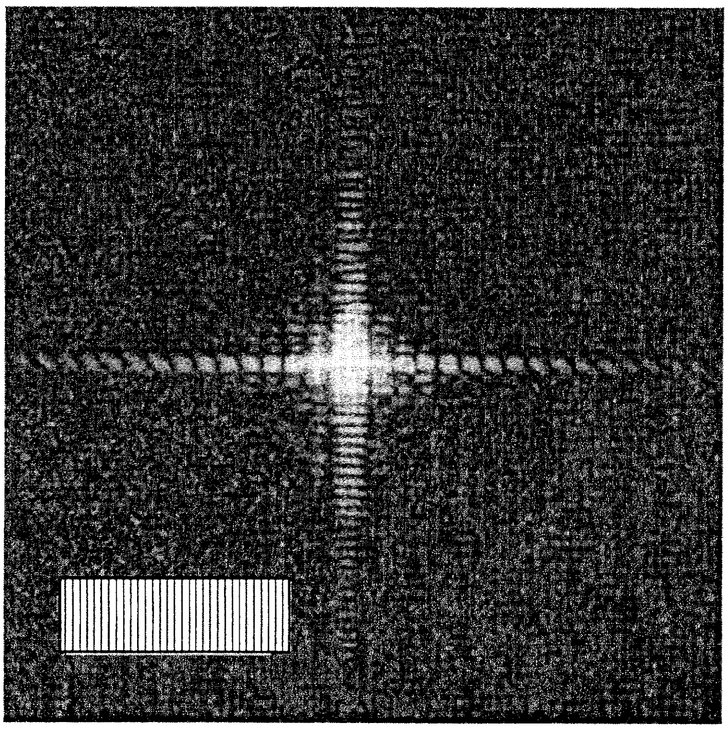
Evaluation/grades
Total number of points of the written examination is 76. These together with lab-report (giving max 25 points) will constitute the basis for evaluation (totally max 100 points after scaling). The following table recommended by NTNU will be used for converting to A, B, C, …-scale.
A: better than 90 points
B: better than 80 points
C: better than 60 points
D: better than 50 points
E: better than 40 points
F: less than 40 points

Section A: Problems to which only answers shall be given (each max 5 points)

1: The Jones vector of completely polarized light was found to be: $\overline{E}_1 = \begin{bmatrix} 1 \\ -3i \end{bmatrix}$ in a certain experiment. Find the Jones vector $\overline{E}_2$ representing an orthogonal polarization state. Sketch the evolution of the electric field vector over time at z = 0 for the two polarization states. x is taken to be horizontal and y vertical polarization, respectively. (Hint: For two orthogonal polarization states $\overline{E}_1^* \cdot \overline{E}_2 = 0$)

Find the corresponding Stokes vector.

2: A plane wave of coherent laser light propagates through a rectangular slit of width w_x and height w_y. A positive thin lens of focal length 50 cm is placed right behind the slit. At the focal plane the following diffraction pattern was recorded:



What is the approximate ratio of the width to height, i.e., $\frac{w_x}{w_y}$. (The striped area can be used as ruler if you do not possess one.)

3: A uniaxial crystal of indicies n_O and n_E is cut so that the optic axis is parallel the surface. Find a formula for the angle of refraction of the extraordinary ray for a light ray incident from the outside at an angle of incidence θ. The optic axis is in the plane of incidence.

4: The dielectric tensor of a trigonal crystal system is given by $\tilde{\varepsilon} = \begin{bmatrix} 2.25 & 0 & 0 \\ 0 & 2.25 & 0 \\ 0 & 0 & 2.56 \end{bmatrix}$

Express the same dielectric tensor in a new coordinate system generated by a negative rotation about the y-axis with 60°. What is the dielectric tensor described in the new coordinate system.

Section B: Problems to which also solutions must be handed in (each max 14 points)

5: *Consider $LiNbO_3$ with the following d-tensor:*

$$d = \begin{pmatrix} 0 & 0 & 0 & 0 & d_{31} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{31} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix}.$$

Kleinman symmetry is valid. Derive d_{eff} for the following three-wave interactions.

eoe and eee

6: A biaxial crystal is characterized by three refractive indices n_x=2.50, n_y = 2. 00 and n_z = 3.00 along its crystallographic principal directions x, y and z respectively. We shall study the wave-vector surface of this material.

a) Prove that the crossings of the wave-vector surfaces with the crystallographic axes occurs at:
n_y and n_z (crossing with the x-axis)
n_x and n_z (crossing with the y-axis)
n_x and n_y (crossing with the z-axis)

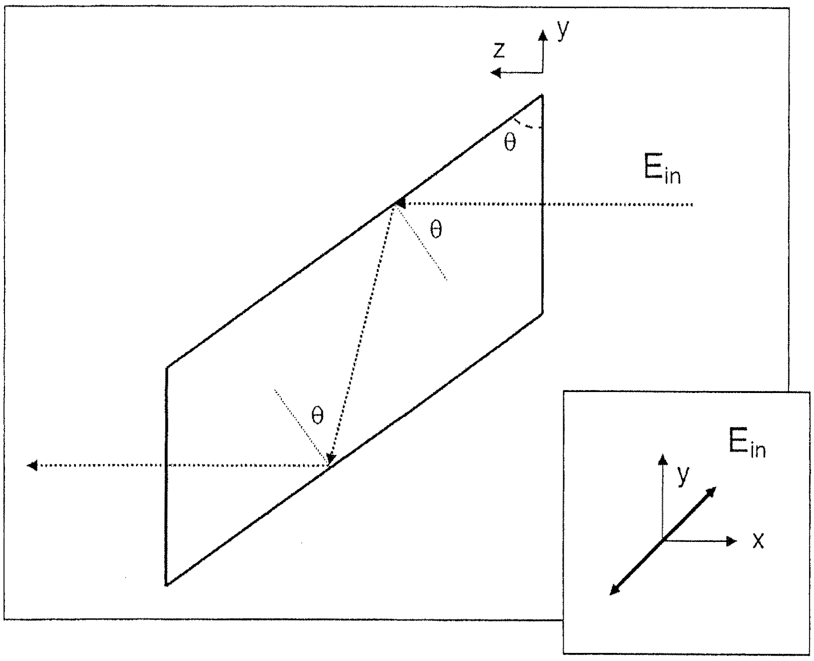
b) Prove that the optic axis will lie in the plane containing the lowest and highest index

c) Find the direction of the optic axis.

7: a) Polarised light is reflected from the surface of a material with dielectric constant ε. Call the ratio of the reflection coefficients of p and s polarized light ρ, ie. $\frac{r_p}{r_s} = \rho$. Show that the dielectric constant of the reflecting surface ε is given by the expression

$$\varepsilon = \sin^2\phi + \sin^2\phi\tan^2\phi\left(\frac{1-\rho}{1+\rho}\right)^2$$

b) Polarized light (+45°) is incident on a Fresnel rhomb as shown in the figure. The angle of the rhomb is 55°. What should the refractive index of the material in the rhomb have to give a phase difference of 90° between the x and y-componets after two reflections as shown in the figure.



Relevant formulas:

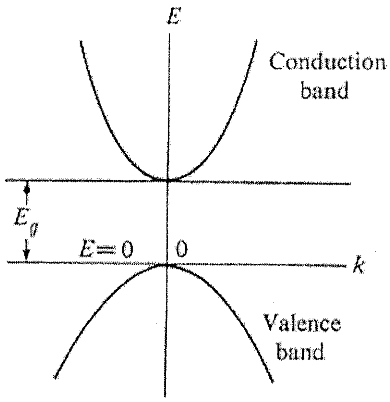
$$r_s = \frac{\cos\theta - \sqrt{\varepsilon - \sin^2\theta}}{\cos\theta + \sqrt{\varepsilon - \sin^2\theta}}; \qquad r_p = \frac{\varepsilon\cos\theta - \sqrt{\varepsilon - \sin^2\theta}}{\varepsilon\cos\theta + \sqrt{\varepsilon - \sin^2\theta}}$$

8: Quantum mechanically the imaginary part of the dielectric constant, $\varepsilon_2(\omega)$, is given by

$$\varepsilon_2 = \frac{2\pi e^2}{m^2 \varepsilon_o \omega^2} \int \frac{d^3k}{(2\pi)^3} |M|^2 \, \delta\,(E_f - E_i - \hbar\omega)$$

where E_f is the energy in the final state and E_i is the energy in the initial state for an optical transition $i \rightarrow f$.

a)



Assume that we have a semiconductor with a direct gap E_g as shown in the figure. The valence band has a parabolic E(k) with hole mass m_h and the conduction band is also of parabolic form with electron mass m_e. Assume that the matrix element M for an optical transition near k=0 is constant. Show that the joint density of states for the system is $\sim \sqrt{\hbar\omega - E_g}$ and that ε_2 thus is of the form

$$\varepsilon_2 \sim \frac{1}{\omega^2}\sqrt{\hbar\omega - E_g} \qquad \textit{for } \hbar\omega > E_g$$

$$\varepsilon_2 = 0 \qquad \textit{for } \hbar\omega < E_g$$

b) Assume now that the transition at k=0 is a so-called forbidden transition so that the matrix element is zero for k=0. In this case we can expand the matrix element in a series around k=0 and write $|M|^2 = |M(0)|^2 + \alpha k^2 = \alpha k^2$. Find ε_2 for this case.