x-tra   ekstra oppgave!

~~3~~6. $\bar{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$

1

Average of any function of $\bar{n}$, $f(\bar{n})$ is

$$\langle f(\bar{n}) \rangle = \frac{1}{4\pi} \cdot \int_0^{2\pi} d\phi \cdot \int_0^{\pi} \sin\theta \cdot d\theta \cdot f(\vec{n})$$

$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

$$\langle \bar{n} \rangle = \hat{x}: \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \cdot d\theta \cdot \sin\theta\cos\phi$$

$$= \frac{1}{4\pi} \int_0^{2\pi} \cos\phi \, d\phi \int_0^{\pi} \frac{1}{2}(1-\cos 2\theta)\, d\theta$$

$$= \frac{1}{8\pi} \cdot \int_0^{2\pi} \cos\phi \, d\phi \left[\theta - \underbrace{\frac{1}{2}\cdot\sin 2\theta}_{0}\right]_0^{\pi}$$

$$= \frac{1}{8} \int_0^{2\pi} \cdot \cos\phi \, d\phi = \frac{1}{8}\left[(\sin\phi)\right]_0^{2\pi} = 0$$

$$\hat{y}: \frac{1}{4\pi} \quad \ldots \text{pss} \quad \frac{1}{8}\left[-\cos\phi\right]_0^{2\pi} = 0$$

$$\hat{z}: \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^{\pi} \cdot \sin\theta \, d\theta \cdot \cos\theta = \frac{1}{2}\left[\cos^2\theta\right]_0^{\pi} = \frac{1}{2}(1-1) = 0$$

$$\langle \vec{n} \rangle = 0 \quad !$$

IF you are smart you do not need to calculate. Average of unit vector over all orientations is of course zero....

2.

$$d = \begin{pmatrix} 0 & 0 & 0 & d_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{14} & 0 \\ 0 & 0 & 0 & 0 & 0 & d_{36} \end{pmatrix} \Rightarrow$$

$$\begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} = \begin{pmatrix} 2 d_{14} E_y E_z \\ 2 d_{14} E_z E_x \\ 2 d_{36} E_x E_y \end{pmatrix} \quad \text{using} \quad \hat{o}: \begin{pmatrix} -\sin\phi \\ \cos\phi \\ 0 \end{pmatrix} \quad \hat{e}: \begin{pmatrix} \cos\phi\cos\theta \\ \sin\phi\cos\theta \\ -\sin\theta \end{pmatrix}$$

we get for

$$d_{ooe}: \left[2d_{14} \cdot 0\hat{x} + 2d_{14} \cdot 0\hat{y} + 2d_{36} \cdot (-\sin\phi\cos\phi)\hat{z} \right] \cdot \hat{e}$$

$$= 2d_{36} \sin\phi\cos\phi\sin\theta = \underline{d_{36} \sin 2\phi \sin\theta}$$

$$d_{eeo}: \left[-2d_{14} \sin\phi \sin\theta\cos\theta \,\hat{x} - 2d_{14} \cos\phi \sin\theta\cos\theta \,\hat{y} + \right.$$

$$\left. + 2d_{36} \sin\phi\cos\phi\sin\theta\cos\theta \,\hat{z} \right] \cdot \hat{o}$$

$$= 2d_{14} \sin^2\phi \sin\theta\cos\theta - 2d_{14} \cos^2\phi \sin\theta\cos\theta$$

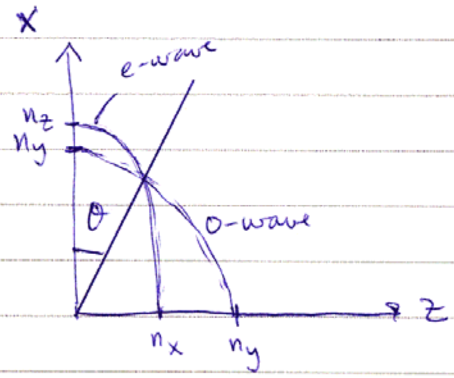
$$= -2 d_{14} \cos 2\phi \sin\theta\cos\theta = \underline{-d_{14} \cdot \cos 2\phi \sin 2\theta}$$

+ also OK due to crystal symmetry

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ $\qquad 2\sin^2\alpha - 1 = -\cos 2\alpha$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

3. Optics axis will occur in the plane of the "highest and lowest" refraction index. In this case xz-plane



o-wave θ independent (pol along $\hat{y}$) $n_{eff} = n_y$

e-wave θ dependent $\frac{1}{n_{eff}^2(\theta)} = \frac{\sin^2\theta}{n_x^2} + \frac{\cos^2\theta}{n_z^2}$

along optic axis o- & e-wave same speed $\Rightarrow$

$\frac{1}{n_y^2} = \frac{\sin^2\theta}{n_x^2} + \frac{\cos^2\theta}{n_z^2}$; a little algebra yields

$$\sin^2\theta = \frac{n_y^2 n_x^2 - n_z^2 n_x^2}{n_y^2 n_x^2 - n_y^2 n_z^2} = \frac{20.25 - 26.01}{20.25 - 104.04} = \frac{5.76}{83.79}$$

$\sin\theta = 0.2622$ $\Rightarrow$ $\underline{\pm 15.2^\circ = \theta}$ angle from x-axis

4. A. first QWP does nothing

second 45° away from polarization axis

$\Rightarrow$ circ polarized (left)

equivalent Jones  $e^{i\pi/4}\underbrace{\begin{pmatrix}1 & 0\\ 0 & -i\end{pmatrix}}_{\text{QWP vert}}\underbrace{\begin{pmatrix}-1\\ 1\end{pmatrix}}_{\text{diag in}} \rightarrow -e^{i\pi/4}\begin{pmatrix}1\\ i\end{pmatrix}$

see Hecht 8.6

B. first QWP $\rightarrow$ circ pol

2nd QWP circ $\rightarrow$ linear, it

cannot be horizontal (C),

therefore ~~vertical~~ LP (+45°)

$\begin{pmatrix}1 & 0\\ 0 & -i\end{pmatrix}\begin{pmatrix}1\\ i\end{pmatrix} = \begin{pmatrix}1\\ 1\end{pmatrix}$ $+45^\circ$

C. $2 \times$ QWP $\Rightarrow$ HWP

HWP 45° from LP $\rightarrow$ LP $\perp$

i.e LP horizontal

5. Symmetry must be retained in FT. Therefore:

1 – E
2 – O
3 – V

6.

$$\Delta\eta_{ij} = \begin{array}{c} \\ \\ \\ yz \\ xz \\ xy \end{array} \begin{pmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{23} \\ 0 & 0 & r_{33} \\ r_{41} & r_{42} & 0 \\ r_{51} & r_{52} & 0 \\ 0 & 0 & r_{63} \end{pmatrix} E_0 \begin{pmatrix} 1 \\ 0 \\ \sqrt{5} \end{pmatrix} =$$

$$= \begin{pmatrix} r_{13}\sqrt{5}\,E_0 & r_{63}\sqrt{5}\,E_0 & r_{51}E_0 \\ r_{63}\sqrt{5}\,E_0 & r_{23}\sqrt{5}\,E_0 & r_{41}E_0 \\ r_{51}E_0 & r_{41}E_0 & r_{33}\sqrt{5}\,E_0 \end{pmatrix} \leftarrow \text{ans}$$

7

(2) $\epsilon_{abc} \frac{\partial B_c}{\partial x_b} = -i\omega \mu_0 \varepsilon_0 \cdot K_{ad} \bar{E}_d$

change index of (1)

$\Rightarrow \epsilon_{ckj} \frac{\partial E_j}{\partial x_k} = i\omega B_c$ or $B_c = \frac{1}{i\omega} \cdot \epsilon_{ckj} \frac{\partial E_j}{\partial x_k}$

$\Rightarrow (2) \; \frac{1}{i\omega} \epsilon_{abc} \frac{\partial}{\partial x_b} \epsilon_{ckj} \frac{\partial E_j}{\partial x_k} = -i\omega \mu_0 \varepsilon_0 K_{ad} \bar{E}_d$

ϵ_{ckj} constant $\qquad \mu_0 \varepsilon_0 = \frac{1}{c^2}$

$\Rightarrow$

(2) $\epsilon_{abc} \epsilon_{ckj} \frac{\partial}{\partial x_b} \frac{\partial E_j}{\partial x_k} = \frac{\omega^2}{c^2} \cdot K_{ad} \bar{E}_d$

$\epsilon_{cab} \epsilon_{ckj}$

$(\delta_{ak}\delta_{bj} - \delta_{aj}\delta_{bk})$

$(\delta_{ak} \delta_{bj} - \delta_{aj} \delta_{bk}) \frac{\partial}{\partial x_b} \frac{\partial E_j}{\partial x_k} = \frac{\omega^2}{c^2} K_{ad} \bar{E}_d$

$\frac{\partial}{\partial x_b} \frac{\partial E_b}{\partial x_a} - \frac{\partial}{\partial x_b} \frac{\partial E_a}{\partial x_b} = \frac{\omega^2}{c^2} K_{ad} E_d$

$\frac{\partial}{\partial x_a} \left(\frac{\partial E_b}{\partial x_b} \right) - \left(\frac{\partial}{\partial x_b} \right)^2 \bar{E}_a = \frac{\omega^2}{c^2} K_{ad} E_d$

identify: $\bar{\nabla} (\bar{\nabla} \cdot \bar{E}) - \nabla^2 \cdot \bar{E} = \frac{\omega^2}{c^2} \tilde{K} \bar{E}$ QED

8. A. SHG $\Rightarrow$ $\omega \to 2\omega$ or $2\lambda \to \lambda$ $\left(\omega = \frac{c\,2\pi}{\lambda}\right)$

critical phasematching $n_o \to n_e$ or $n_e \to n_o$.

From figure we see immediately

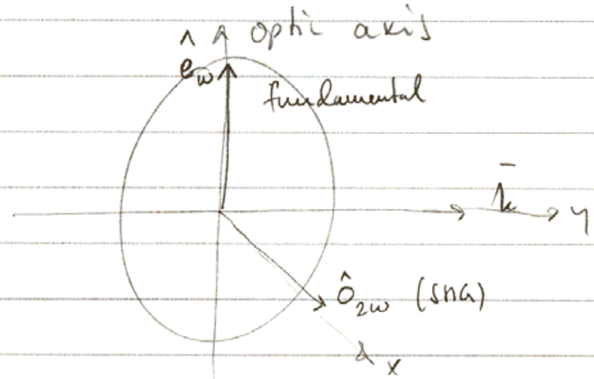
that in the λ range 240 - 250 nm for $n_o =$

n_e at approximately 480 - 500 $\Rightarrow$ SHG for

$\underset{\omega}{n_e} \to \underset{2\omega}{n_o}$

B.

scheme



or

problem 5

Solution:

Snell's law relates θ_i and θ_t;

Specifically, $n_i \sin\theta_i = n_t \sin\theta_t$

since air, $n_i = 1$; n_t depends on θ_t since

the crystal is uniaxial, hence

$$n_t(\theta_t) = \frac{\sin\theta_i}{\sin\theta_t}$$

what is $n_t(\theta_t)$? introduce help-angle

$\alpha = \theta_c - \theta_t$, specifically

$$\frac{1}{n_{eff}^2(\theta_t)} = \frac{\sin^2\alpha}{n_e^2} + \frac{\cos^2\alpha}{n_o^2}$$

$$n_{eff}^2(\theta_t) = \frac{1}{\frac{\sin^2\alpha}{n_e^2} + \frac{\cos^2\alpha}{n_o^2}} = \frac{\sin^2\alpha + \cos^2\alpha}{\frac{\sin^2\alpha}{n_e^2} + \frac{\cos^2\alpha}{n_o^2}} =$$

$$= \frac{n_o^2 \cdot \tan^2\alpha + n_o^2}{\frac{n_o^2}{n_e^2}\tan^2\alpha + 1}$$

$\alpha = \theta_c - \theta_t$

$$\Rightarrow \frac{\sin\theta_i}{\sin\theta_t} = n_t(\theta_t) = \frac{n_o\sqrt{1 + \tan^2(\theta_c - \theta_t)}}{\sqrt{1 + \frac{n_o^2}{n_e^2}\tan^2(\theta_c - \theta_t)}}$$ QED