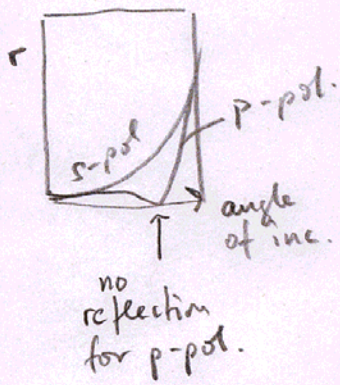
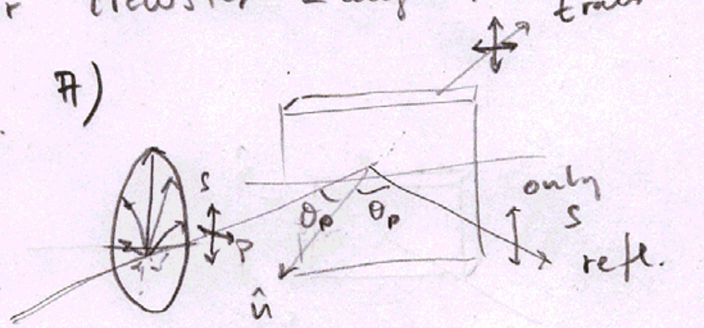
Optikk VK 2006 Solutions (Suggested)

Del A: 1. Use the phenomenon of "polarization" or "Brewster" – angle.



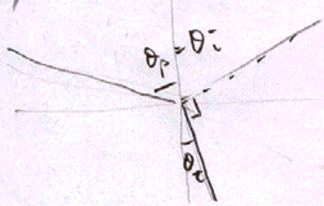
A)



Arrangement.

Hecht eq. 8.25. $\tan\theta_p = \frac{n_t}{n_i} = \frac{4}{1} \Rightarrow \theta_p \simeq 76^\circ$

B) Snell $n_i \sin\theta_i = n_t \sin\theta_t \Rightarrow \sin\theta_t = \sin 76^\circ \cdot \frac{1}{4}$



$\theta_t = 14.04^\circ$

check: $r_{\parallel} = + \frac{\tan(\theta_i - \theta_t)}{\underbrace{\tan(\theta_i + \theta_t)}_{90^\circ}} = 0$ ok.

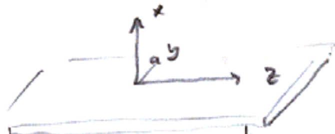
$$r_{\perp} = - \frac{\sin(\theta_i - \theta_t)}{\underbrace{\sin(\theta_i + \theta_t)}_{90^\circ}} = - \sin(76^\circ - 14^\circ) = -0.8829$$

$I_{\perp} \propto r_{\perp}^2 = 0.78$ almost 80% reflect as s-pol.

efficiency: $\frac{I_{\perp}}{I_{\parallel} + I_{\perp}} = \frac{0.5 \cdot 0.78}{0.5 + 0.5} \approx 0.39$

Ans. if unpol light is used, ca 40% is reflected as s-pol @ incident angle ~76°

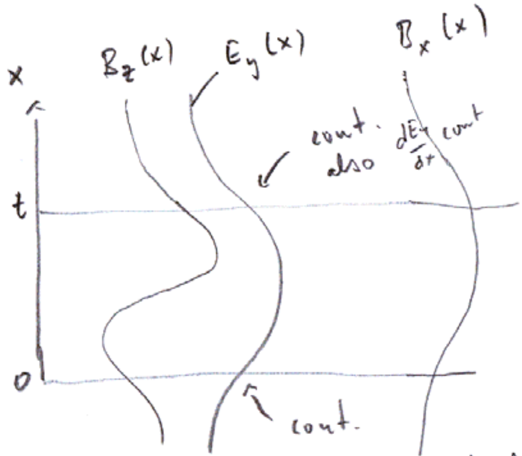
Del 1.2:

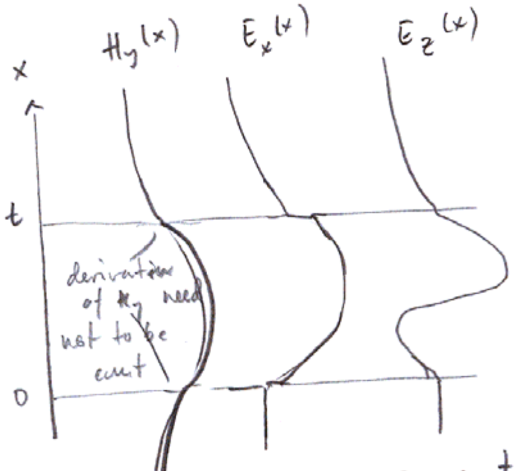


all field prop. $e^{i(\beta z - \omega t)}$

TE: we know E_y

TM: we know H_y





$B_x \propto \frac{-\beta}{\omega} \cdot E_y(x)$ Proportional $E_y(x)$

$B_z \propto \frac{1}{i\omega} \cdot \frac{\partial E_y(x)}{\partial x}$ Proportional $\frac{\partial E_y}{\partial x}$

use: $\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$

$\rightarrow i\beta \quad \rightarrow -i\omega$

$E_x(x) = \frac{\beta}{\omega n^2} H_y(x)$ D_x cont. prop to $H_y(x)$

$E_z(x) = \frac{i}{\omega} \cdot \frac{\partial H_y(x)}{\partial x}$

use $\bar{\nabla} \times \bar{H} = \frac{\partial \bar{D}}{\partial t} = n^2 \cdot \frac{\partial \bar{E}}{\partial t}$

$\rightarrow i\beta \quad \rightarrow -i\omega$

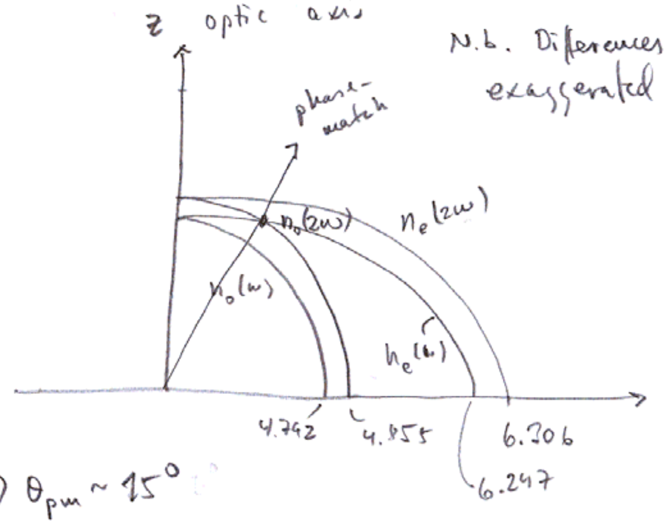
Del A3!

A: $n_E > n_o$

phase-matching when

$$\frac{1}{4.855^2} = \frac{\cos^2\theta}{4.742^2} + \frac{\sin^2\theta}{6.247^2}$$

$\uparrow 2\omega$ (5μm) $\qquad \underbrace{\qquad}_{\uparrow\ \omega\ (10\mu m)}$ $\Rightarrow \underline{\theta_{pm} \sim 15^\circ}$



B: $\underline{\overset{\omega\omega}{ee} \to \overset{2\omega}{o}}$ ans.

A4: Kleinmann does not change anything in this case (!).

$$\begin{pmatrix} d_{11} & -d_{11} & 0 & d_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & -d_{14} & -d_{11} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} E_x^2 \\ E_y^2 \\ E_z^2 \\ 2E_yE_z \\ 2E_xE_z \\ 2E_xE_y \end{pmatrix} \to$$

$$\to \quad d_{11}(E_x^2 - E_y^2) + 2d_{14}\cdot E_yE_z \quad : \hat{x}$$

$$-d_{14}E_xE_z - 2d_{11}E_xE_y \quad : \hat{y}$$

apply ee ↑

$e: \begin{pmatrix} \cos\theta\cos\phi \\ \cos\theta\sin\phi \\ -\sin\theta \end{pmatrix}$ project onto $o: \begin{pmatrix} -\sin\phi \\ \cos\phi \\ 0 \end{pmatrix}$ ......

ans.

$$\Rightarrow \underline{d_{eff}(\theta,\phi) = \pm(d_{11}\cos^2\theta\sin 3\phi + d_{14}\cdot\sin 2\theta)}$$

A5: Apply C_2 around x

$$\Rightarrow \quad \begin{array}{l|l|l} r_{11} \to r_{11} & r_{12} \to -r_{12} \;\; 0 & 0 \\ r_{21} \to r_{21} & r_{22} \to -r_{22} \;\; 0 & 0 \\ r_{41} \to r_{41} & r_{42} \to -r_{42} \;\; 0 & 0 \\ r_{51} \to -r_{51} \;\; 0 & r_{52} \to r_{52} & 0 \\ r_{61} \to -r_{61} \;\; 0 & r_{62} \to r_{62} & 0 \end{array}$$

left is

$$\begin{pmatrix} r_{11} & 0 & 0 \\ -r_{11} & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & -r_{41} & 0 \\ 0 & -r_{11} & 0 \end{pmatrix}$$

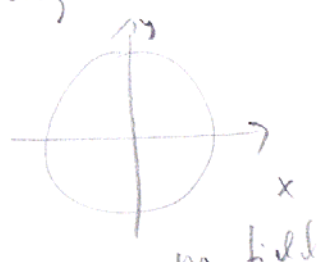
ans.

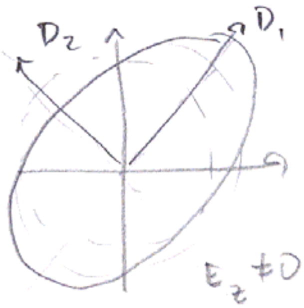
A6:
$$\begin{array}{c} \\ \\ \\ yz \\ xz \\ xy \end{array}\begin{pmatrix} r_{11} & 0 & 0 \\ -r_{11} & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & -r_{41} & 0 \\ 0 & -r_{11} & 0 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \Rightarrow \Delta\eta_{ij} = \begin{pmatrix} r_{11}E_x & -r_{11}E_z & -r_{41}E_y \\ -r_{11}E_z & -r_{11}E_y & r_{41}E_x \\ -r_{41}E_y & r_{41}E_x & 0 \end{pmatrix}$$

arbitrary static field

ans. I would apply static $E_z \Rightarrow \Delta\eta_{ij} = \begin{pmatrix} 0 & -r_{11}E_z \\ -r_{11}E_z & 0 \end{pmatrix}$

propagating along z, then would split the degenerate modes





with input vertical (or horiz) there would be a phase shift along D_1 & D_2

$\to$ pol. rotation depends on $L \cdot V_{ext}$

B7:

$S_{out} = M \cdot S_{in}$

We measure S_{out} comp. S_0 (int)

use Mueller mat in Table 8.6

1 exp.

$$\frac{1}{2}\begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} S_0 - S_1 \\ -S_0 + S_1 \\ 0 \\ 0 \end{pmatrix} \leftarrow$$

2 exp

$$\frac{1}{2}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \frac{1}{2}\left(S_0 + S_2\right) \leftarrow$$

↑ we measure int. S_0

↓

3 exp

$$\frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \frac{1}{2}\left(S_0 + S_1\right)$$

4. exp. 1st QWP.

$$\frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} S_0 \\ S_1 \\ -S_3 \\ S_2 \end{pmatrix}$$

B7 cont.

exp. 4. 2nd.

$$\frac{1}{2}\begin{pmatrix}1 & 0 & 1 & 0\\ 0 & 0 & 0 & 0\\ 1 & 0 & 1 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}\begin{pmatrix}S_0\\ S_1\\ -S_3\\ S_2\end{pmatrix} = \frac{1}{2}\left(S_0 - S_3\right)$$

$$\left.\begin{aligned}\frac{1}{2}\left(S_0 - S_1\right) &= 0.25\\ \frac{1}{2}\left(S_0 + S_1\right) &= 0.75\end{aligned}\right\} \begin{aligned}S_0 &= 1\\ S_1 &= 0.5\end{aligned}$$

$$\frac{1}{2}\left(S_0 + S_2\right) = 0.5 \Rightarrow S_2 = 0$$

$$\frac{1}{2}\left(S_0 - S_3\right) = 0.25 \Rightarrow S_3 = 0.5$$

$$\begin{pmatrix}S_0\\ S_1\\ S_2\\ S_3\end{pmatrix} = \begin{pmatrix}1\\ 0.5\\ 0\\ 0.5\end{pmatrix}$$

Degree of polarization

$$V = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0} \sim 0.707$$

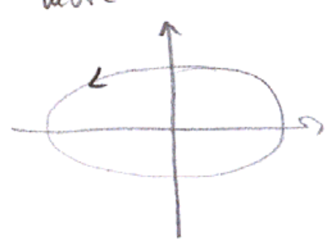
Polarization ellipse for the polarized part.

$S_1 > 0 \Rightarrow$ more horizontal

$S_2 = 0$ not ↗ or ↘

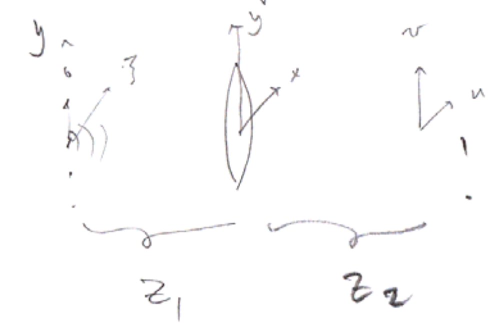
$S_3 > 0$ right circular component larger

$\therefore$ polarized part is elliptical, more horizontal than vertical



No "tilt" since $S_2 = 0$

P.8. A point source emits spherical waves



In the lens plane the emitted wave can be written:

$$u_\ell(x,y) = u_0 \cdot \frac{1}{z_1} \cdot e^{i\frac{k}{2z_1}\cdot\left((x-\xi)^2+(y-\eta)^2\right)}$$

depends on where the point source is located

Here we used the paraxial approximation and neglect an unimportant phase factor

After passing the lens, the field distribution becomes $u_\ell'(x,y) = u(x,y) \cdot e^{-i\frac{k}{2f}(x^2+y^2)}$

again we neglected an unimportant (constant) phase factor.

Finally, Fresnel diffraction will give the field distribution at the plane u,v at a distance z_2 after the lens

$$u(u,v) = \frac{e^{ikz_2}}{i\lambda z_2} \iint u_\ell'(x,y)\, e^{i\frac{k}{2z_2}\left((u-x)^2+(v-y)^2\right)}\, dx\,dy$$

(again paraxial approx)

B8, cont.,

inserting everything in succession we obtain:

$$u(u,v) = \underbrace{\frac{e^{ikz_2}}{i\lambda z_2}\, u_0 \,\frac{1}{z_1}\, e^{i\frac{k}{2z_1}(\xi^2+\eta^2)} \cdot e^{i\frac{k}{2z_2}(u^2+v^2)}}_{\frac{e^{i\gamma}}{i\lambda z_1 z_2}} \cdot \iint \ldots\ldots \, dx\,dy$$

this phase term disappears when we consider intensity

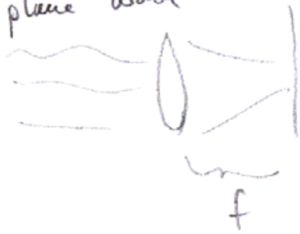
inside integral is left:

$$\ldots \iint_{-\infty}^{\infty} \left[\underbrace{e^{i\frac{k}{2}\left[\left(\frac{1}{z_1}-\frac{1}{f}+\frac{1}{z_2}\right)\right](x^2+y^2)}}_{\text{FT provided this term vanishes}} \right] \underbrace{e^{ik\left[x\underbrace{\left(\frac{\xi}{z_1}+\frac{u}{z_2}\right)}_{f_x}+y\underbrace{\left(\frac{\eta}{z_1}+\frac{v}{z_2}\right)}_{f_y}\right]}}_{\text{FT-operator with freq } f_x, f_y} \, dx\,dy$$

hence $\frac{1}{f} = \frac{1}{z_1} + \frac{1}{z_2}$

Profound conclusion: the FT is found where the (point) source is imaged ☺

note:
cf. plane wave



FT at focal plane

$$\frac{1}{f} = \frac{1}{\infty} + \frac{1}{z_2} \Rightarrow z_2 = \frac{1}{f}$$

8 9.

This DEVICE IS CALLED A FRESNEL RHOMB see Hecht

$\theta = 53^\circ$ what applies

$n_{glass} \cdot \sin 53^\circ = n_{air} \cdot \sin \theta_{out} \Rightarrow$

$1.5 \cdot \sin 53^\circ = 1.19 \Rightarrow \sin \theta_{out}$ does not exist

TOTAL INTERNAL REFLECTION

$$r_\perp = \frac{\cos\theta_i - \sqrt{n_{ti}^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{n_{ti}^2 - \sin^2\theta_i}}$$ 4.70 Hecht

$$r_{\parallel} = \frac{n_{ti}^2\cos\theta_i - \sqrt{n_{ti}^2 - \sin^2\theta_i}}{n_{ti}^2\cos\theta_i + \sqrt{n_{ti}^2 - \sin^2\theta_i}}$$ 4.71 Hecht

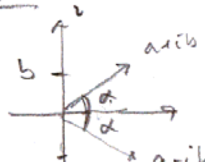
$$n_{ti} = \frac{n_t}{n_i} = \frac{1}{1.5} = 0.66666$$ in this case since $n_i > n_t$

$$\Rightarrow \underset{\substack{TE\\ s\text{-pol}}}{} \quad r_\perp = \frac{0.602 - \sqrt{(\frac{2}{3})^2 - 0.6378}}{(\quad) + \sqrt{\quad}} = \frac{0.602 - 0.44i}{0.602 + 0.44i} = e^{-i\cdot\frac{72.32\cdot\pi}{180}}$$

$$\underset{\substack{TM\\ p\text{-pol}}}{} \quad r_{\parallel} = \frac{0.268 - \sqrt{(\frac{2}{3})^2 - 0.6378}}{(\quad) + (\quad)} = \frac{0.268 - 0.44i}{0.268 + 0.44i} = e^{-i\frac{117.31\cdot\pi}{180}}$$

Phase-shift

$$\frac{a+ib}{a-ib}$$



$\alpha = \tan\frac{b}{a}$

$\Rightarrow \phi = 2\alpha$

$$\left.\begin{aligned}\phi_{TE} &= -72.32\\ \rightarrow \phi_{TM} &= -117.31^\circ\end{aligned}\right\} \phi_{TM} - \phi_{TE} = -45^\circ \; (!)$$

2 reflection $\Rightarrow 2\times\phi \Rightarrow -90^\circ$ ~~as a~~ quarter wave plate

$\Rightarrow E_{in} = \binom{1}{1} \rightarrow$ circularly polarized light out