NTNU

Institutt for fysikk

Contact during the exam:
Hans Joakim Skadsem
Telephone: 73593101

Exam in TFY4205 Quantum Mechanics
February 26, 2008 (Midterm Exam)
14:15–16:00

Allowed help: Alternativ C
Approved calculator (HP30s)
K. Rottman: *Matematisk formelsamling*
Barnett and Cronin: *Mathematical formulae*

Write your student number here : ....

There are five alternative answers to all questions of which only one alternative is correct. You can choose 0, 1, 2, 3, 4, or 5 of these alternatives. Your resulting score becomes:

No alternatives chosen: 0 points.

One alternative chosen: 1 point if this is the correct answer, -1/4 point if this answer is incorrect.

Two alternatives chosen: 3/4 point if one of these is the correct answer, -2/4 point if none of these are the correct answer.

Three alternatives chosen: 2/4 point if one of these is the correct answer, -3/4 point if none of these are the correct answer.

Four alternatives chosen: 1/4 point if one of these is the correct answer, -1 point if none of these are the correct answer.

Five alternatives chosen: 0 points.

There are 20 problems in total. Your maximum score is therefore 20 points. This mid term exam accounts 20% of the total grade in TFY4205 Quantum Mechanics.

This problem set consists of 14 pages.

Problem 1. General Quantum Mechanics

Which of the following statements is true ?

1. The electron spin arises because the electron rotates around the positive core of the hydrogen atom.
2. Schrödinger's formulation of wave mechanics in position space gives different predictions than Heisenberg's matrix mechanics.
3. Quantum mechanics does not play any role in the determination of the aluminum material parameters which e.g. is important for the second largest Norwegian company Norsk Hydro.
4. Relativistic effects do not matter for objects described by quantum mechanics because they can be interpreted as waves.
5. Electrons, protons, and neutrons are all fermions with a half integer spin.

Solution

Electrons, protons, and neutrons are all fermions with a half integer spin.

Problem 2. The Variational Method

We assume that a particle with mass m only can move in one direction. The particle is within a potential that is described by

$$V(x) = Kx^4\,, \tag{1}$$

where K is a constant and x is the position. We use a trial function of the form

$$f(x) = \exp\left(-\alpha x^2\right), \tag{2}$$

where α is a constant to find an approximate expression for the ground state energy E_0. The result is

1. $$E_0 = 2^{2/3}\frac{\hbar^2}{2m}K^2.$$
2. $$\boxed{E_0 = \frac{3^{4/3}}{4}\left(\frac{\hbar^2}{2m}\right)^{2/3}K^{1/3}.}$$
3. $$E_0 = \frac{3}{4}6^{2/3}\left(\frac{\hbar^2}{2m}\right)^{1/3}K^{2/3}.$$
4. $$E_0 = 3^{2/3}\frac{\hbar^2}{2m}K^3.$$
5. $$E_0 = \frac{3}{4}6^{2/3}\left(\frac{\hbar^2}{2m}\right)^{-3/4}K^{1/3}.$$

PS: A detailed computation is not necessary. Dimensional analysis is sufficient to find the answer.

Solution

Dimensional analysis for the kinetic energy determines

$$E = \frac{\hbar^2}{2m}\frac{1}{L^2}, \tag{3}$$

where L is a length scale. We can therefore express the length scale L by Planck's constant $\hbar$, the energy E, and the mass m:

$$L^2 = \frac{\hbar^2}{2mE}. \tag{4}$$

Similarly, we find for the potential energy

$$\begin{aligned} E &= KL^4 && (5)\\ &= K\left(\frac{\hbar^2}{2mE}\right)^2. && (6) \end{aligned}$$

This means that the ground state energy should be of the form

$$E = c\left(\frac{\hbar^2}{2m}\right)^{2/3} K^{1/3}, \tag{7}$$

where c is a dimensionless constant. A direct calculation gives

$$c = \frac{3^{4/3}}{4}.$$

Problem 3. The Pauli Matrices

The electron spin fulfills the commutation relations

$$\begin{aligned} [S_x, S_y] &= i\hbar S_z\,,\\ [S_y, S_z] &= i\hbar S_x\,,\\ [S_z, S_x] &= i\hbar S_y\,. \end{aligned}$$

The electron spin can be expressed by

$$\vec{S} = \frac{\hbar}{2}\vec{\sigma},$$

where $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ consists of the Paul matrices. We use the eigenstates of σ_z as a basis, so that

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Which of the following expressions is the correct one for the Paul matrices σ_x and σ_y?

1.
$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \text{ and } \sigma_y = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}.$$

2.

$$\sigma_x = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \text{ and } \sigma_y = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

3.

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \text{ and } \sigma_y = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}.$$

4.

$$\boxed{\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}.}$$

5.

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \text{ and } \sigma_y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}.$$

Solution

The commutation relations for the spin are

$$\begin{aligned} [S_x, S_y] &= i\hbar S_z\,, \\ [S_y, S_z] &= i\hbar S_x\,, \\ [S_z, S_x] &= i\hbar S_y\,. \end{aligned}$$

The commutation relations for the Pauli matrices is therefore

$$\begin{aligned} [\sigma_x, \sigma_y] &= \sigma_x\sigma_y - \sigma_y\sigma_x = i2\sigma_z\,, \\ [\sigma_y, \sigma_z] &= \sigma_y\sigma_z - \sigma_z\sigma_y = i2\sigma_x\,, \\ [\sigma_z, \sigma_x] &= \sigma_z\sigma_y - \sigma_y\sigma_z = i2\sigma_y\,. \end{aligned}$$

It is given in the text that

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

By inserting this into the commutation relations we find that the only possibility is

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}.$$

Problem 4. Statistics Force

We consider two particles with symmetric spin wave functions in the potential

$$V(\hat{q}) = \frac{1}{2}m\hat{q}^2. \tag{8}$$

We assume that one particle is in the ground state and the other particle is in the first excited state. We want to compute the average relative distance $r = \sqrt{\langle(\hat{q}_1 - \hat{q}_2)^2\rangle}$ between the particles when the particles are different ($r = r_{\mathrm{i}}$), when the particles are bosons ($r = r_{\mathrm{b}}$), and when the particles are fermions ($r = r_{\mathrm{f}}$). What is the relation between the relative difference between the particles in these three cases?

1.
$$r_\mathrm{i} = r_\mathrm{b} = r_\mathrm{f}.$$

2.
$$r_\mathrm{i} > r_\mathrm{f} > r_\mathrm{b}.$$

3.
$$r_\mathrm{i} > r_\mathrm{b} > r_\mathrm{f}.$$

4.
$$\boxed{r_\mathrm{f} > r_\mathrm{i} > r_\mathrm{b}.}$$

5.
$$r_\mathrm{b} > r_\mathrm{f} > r_\mathrm{i}.$$

Solution

Fermions cannot be in the same quantum state, and therefore have an effective repulsive interaction. Bosons can be in the same quantum state and therefore have an effective attractive interaction. We therefore find

$$r_\mathrm{f} > r_\mathrm{i} > r_\mathrm{b}.$$

Problem 5. Dual Vector

The dual vector corresponding to $c|a\rangle$, where c is a complex number and $|a\rangle$ is a state (ket) vector is

1. $c^*|a\rangle$.
2. $c\langle a|$.
3. $\boxed{c^*\langle a|.}$
4. $|c||a\rangle$.
5. $|c|\langle a|$.

Solution

The dual vector to $c|a\rangle$, where c is a complex number and $|a\rangle$ is a state vector is $c^*\langle a|$.

Problem 6. Quantum Operators

We consider a particle with a total spin $S^2 = s(s+1)\hbar^2$ and $s = 1$. What could be a possible matrix representation of the spin operator along the z direction, $\hat{S}_z$?

1.
$$\hat{S}_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{9}$$

2.
$$\boxed{\hat{S}_z = \hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}} \tag{10}$$

3.
$$\hat{S}_z = \hbar \begin{pmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{11}$$

4.
$$\hat{S}_z = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{12}$$

5.
$$\hat{S}_z = \frac{\hbar}{2} \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \tag{13}$$

Solution
A particle with total spin $S^2 = s(s+1)\hbar^2$ and $s = 1$ has three possible eigenvalues of the spin operator along the z direction, $S_z = \hbar$, $S_z = 0$, and $S_z = -\hbar$. The matrix representation must therefore be a 3×3 hermitian matrix with these eigenvalues, e.g. a possible representation is
$$\hat{S}_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{14}$$

Problem 7. General Uncertainty Relation
Let us assume that $\hat{Q}$ and $\hat{P}$ are two hermitian operators with the following commutation relation
$$\left[\hat{Q}, \hat{P}\right] = i\hbar\hat{K}\,, \tag{15}$$

where $\hat{K}$ is also a hermitian operator. We define the uncertainties

$$\Delta Q = \langle(\hat{Q} - \langle\hat{Q}\rangle)^2\rangle^{1/2}, \tag{16}$$
$$\Delta P = \langle(\hat{P} - \langle\hat{P}\rangle)^2\rangle^{1/2}\,. \tag{17}$$

Which statement is correct?

1.
$$\Delta Q \Delta P \leq \frac{\hbar}{2}|\langle\hat{K}\rangle|.$$

2.
$$\boxed{\Delta Q \Delta P \geq \frac{\hbar}{2}|\langle\hat{K}\rangle|.}$$

3.
$$\Delta Q \Delta P = 0.$$

4.
$$\Delta Q \Delta P \geq \frac{\hbar}{2}|\langle\hat{K}\rangle|^2.$$

5.
$$\Delta Q \Delta P \leq \frac{\hbar}{2}|\langle\hat{K}\rangle|^2.$$

Solution

We can check this for the position and momentum operators

$$[\hat{x}, \hat{p}_x] = i\hbar.$$

We know that

$$\Delta x \Delta p_x \geq \frac{\hbar}{2}$$

and therefore recognize the general uncertainty relation

$$\Delta Q \Delta P \geq \frac{\hbar}{2} \left| \left\langle \hat{K} \right\rangle \right|.$$

since there is a linear relation between $\hat{Q}\hat{P} - \hat{P}\hat{Q}$ and $\hat{K}$.

Problem 8. Addition of Angular Momentum

A system consists of two spin 1 particles with spin $\vec{S}_1$ and spin $\vec{S}_2$. What are the eigenstates for the total spin $\vec{S} = \vec{S}_1 + \vec{S}_2$? It is known that

$$\begin{aligned} \left(\hat{S}_x^2 + \hat{S}_y^2 + \hat{S}_z^2\right) |S, M\rangle &= \hbar^2 S(S+1) |S, M\rangle, \\ \hat{S}_z |S, M\rangle &= \hbar M |S, M\rangle. \end{aligned}$$

1. There is one singlet eigenstate ($S = 0; M = 0$), one triplet eigenstate ($S = 1; M = -1, 0, 1$), and one quintet eigenstate ($S = 2; M = -2, -1, 0, 1, 2$).

2. There is one doublet eigenstate ($S = 1/2; M = -1/2, 1/2$), one triplet eigenstate ($S = 1; M = -1, 0, 1$), and one quartet eigenstate $S = 3/2; M = -3/2, -1/2, 1/2, 3/2$).

3. There is not enough information to determine the eigenstates.

4. There are four singlet eigenstates ($S = 0; M = 0$) and one quintet eigenstate ($S = 2; M = -2, -1, 0, 1, 2$).

5. There is one singlet eigenstate ($S = 0; M = 0$) and two quartet eigenstates ($S = 3/2; M = -3/2, -1/2, 1/2, 3/2$).

Solution

The total spin can attain the values $S = 0$ (singlet eigenstate $M = 0$), $S = 1$ (triplet eigenstate $M = -1, 0, 1$) or $S = 2$ (quintet eigenstate $M = -2, -1, 0, 1, 2$).

Problem 9. Quantum Mechanical Inner Product

Let us assume that $\hat{A}$ is a quantum mechanical operator that represents a physical quantitity. Which of the following statements is *true* about the inner product $\left\langle g|\hat{A}|g \right\rangle$?

1. It vanishes when $|g\rangle$ is an eigenstate of $\hat{A}$.

2. It is equal to $A \langle g|g \rangle$.

3. It vanishes if $\left[\hat{H}, \hat{A}\right] = 0$, where $\hat{H}$ is the Hamilton operator of the system.

4. It must be a real number.

5. It must be an imaginary number.

Solution
$\left\langle g|\hat{A}|g\right\rangle$ must be real since $\hat{A}$ is hermitian.

Problem 10. The Momentum Operator in the Position Representation
We assume that $|x_1\rangle$ is an eigenstate for the position operator $\hat{x}$ with eigenvalue x_1. Correpondingly, $|x_2\rangle$ is an eigenstate for the position operator $\hat{x}$ with eigenvalue x_2. What is the matrix element for the momentum operator $\hat{p}$?

1.
$$\langle x_2|\hat{p}|x_1\rangle = -\frac{\hbar}{i}\frac{\partial}{\partial x_2}\delta\left(x_2 - x_1\right).$$

2.
$$\langle x_2|\hat{p}|x_1\rangle = \frac{\hbar}{i}\delta\left(x_2 - x_1\right).$$

3.
$$\langle x_2|\hat{p}|x_1\rangle = -\frac{\hbar}{i}\delta\left(x_2 - x_1\right).$$

4.
$$\langle x_2|\hat{p}|x_1\rangle = \frac{\hbar}{i}.$$

5.
$$\boxed{\langle x_2|\hat{p}|x_1\rangle = \frac{\hbar}{i}\frac{\partial}{\partial x_2}\delta\left(x_2 - x_1\right).}$$

Solution
The matrix element for the momentum operator $\hat{p}$ is

$$\langle x_2|\hat{p}|x_1\rangle = \frac{\hbar}{i}\frac{\partial}{\partial x_2}\delta\left(x_2 - x_1\right).$$

Problem 11. Planck's Constant
Planck's constant determines quantum mechanical effects. It is therefore an important constant in nature. In what units can Planck's constant be expressed?

1. Similar to angular momentum.
2. Similar to energy divided by time.
3. Similar to velocity.
4. Similar to the electron charge.
5. Similar to the electron mass.

Solution
The electron spin is in units of $\hbar$. This means that Planck's constant can be expressed in units of angular momentum.

Problem 12. The Momentum Representation

The momentum operator $\hat{p}_x$ and the position operator $\hat{x}$ are expressed in the momentum representation as

1. $$\hat{p}_x = p_x, \quad \hat{x} = \frac{\hbar}{i}\frac{\partial}{\partial p_x}.$$

2. $$\boxed{\hat{p}_x = p_x, \quad \hat{x} = -\frac{\hbar}{i}\frac{\partial}{\partial p_x}.}$$

3. $$\hat{p}_x = -\frac{\hbar}{i}\frac{\partial}{\partial p_x}, \quad \hat{x} = x.$$

4. $$\hat{p}_x = \frac{\hbar}{i}\frac{\partial}{\partial p_x}, \quad \hat{x} = x.$$

5. $$\hat{p}_x = \frac{\hbar}{i}\frac{\partial}{\partial x}, \quad \hat{x} = x.$$

Solution

In the momentum representation, we have

$$\hat{p}_x = p_x, \quad \hat{x} = -\frac{\hbar}{i}\frac{\partial}{\partial p_x}.$$

Problem 13. The Quantum Mechanical Postulates

Which of the following statements is *not* one of the basic postulates in quantum mechanics?

1. There is a state vector in the Hilbert space to any physical state in a physical system. The norm of the state vector equals 1.
2. There is an operator in the Hilbert space corresponding to any physical observable variable. This operator is linear and hermitian.
3. If the state of a system is $|\alpha\rangle$, then the probability to find the system in state $|\beta\rangle$ is $\langle\beta|\alpha\rangle$.
4. The operator that represents the coordinate and the operator that represents the momentum obey the canonical commutation rules $[\hat{q}_j, \hat{p}_k] = i\hbar\delta_{j,k}$.
5. The time evolution for the state vector $|\Psi(t)\rangle$ describing a system is determined by

$$i\hbar\frac{\partial}{\partial t}|\Psi(t)\rangle = \hat{H}|\Psi(t)\rangle.$$

Solution

The following statement is *not* one of the basic postulates in quantum mechanics: If the state of a system is $|\alpha\rangle$, then the probability to find the system in state $|\beta\rangle$ is $\langle\beta|\alpha\rangle$.

Problem 14. Angular Momentum

$\vec{L}$ is the orbital angular momentum and $\vec{S}$ is the electron spin. We define $\vec{I} = \vec{L} - \vec{S}$. Which of the following statements is true ?

1. The operator that is associated to the quantity $\vec{I} = \vec{L} - \vec{S}$ is an angular momentum operator.

2. $$\boxed{\left[\hat{L}_x, \hat{L}_y\right] = i\hbar\hat{L}_z.}$$

3. $$\left[\hat{I}_y, \hat{I}_z\right] = i\hbar\hat{I}_x.$$

4. We cannot find common eigenstates to the operators associated with $\vec{I}^2$, $\vec{L}^2$ og $\vec{S}^2$.

5. All the other statements are incorrect.

Solution

$$\left[\hat{L}_x, \hat{L}_y\right] = i\hbar\hat{L}_z.$$

Problem 15. The Number Representation for a Harmonic Oscillator

The Hamilton operator for an electron in a one dimensional harmonic oscillator is

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{q}^2,$$

where $\hat{p}$ is the momentum operator and $\hat{q}$ is the position operator. We introduce new operators

$$\begin{aligned} \hat{a} &= \sqrt{\frac{m\omega}{2\hbar}}\hat{q} + \frac{i}{\sqrt{2m\hbar\omega}}\hat{p}, \\ \hat{a}^\dagger &= \sqrt{\frac{m\omega}{2\hbar}}\hat{q} - \frac{i}{\sqrt{2m\hbar\omega}}\hat{p}. \end{aligned}$$

Which of the following statements is *not* true ?

1. The commutation relation is $\left[\hat{a}, \hat{a}^\dagger\right] = 1$.
2. $\hat{a}$ is an annihilation operator and $\hat{a}^\dagger$ is a creation operator.
3. $\boxed{\text{The eigenvalues to the operator } \hat{a} \text{ must be real.}}$
4. $\hat{n} = \hat{a}^\dagger\hat{a}$ is the number operator.
5. The operators $\hat{a}$ og $\hat{a}^\dagger$ do not commute with the Hamilton operator $\hat{H}$.

Solution

The following statement is *not* true. The eigenvalues of the operator $\hat{a}$ must be real.

Problem 16. Stationary Perturbation Theory

We assume that we know the eigenvalues E_n^0 and the eigenstates $|n\rangle$ for the exactly solvable (unperturbed) system

$$\hat{H}_0|n\rangle = E_n^0|n\rangle$$

and seek the eigenvalues and eigenstates for the perturbed Hamilton operator

$$\hat{H} = \hat{H}_0 + \lambda\hat{H}_1,$$

where λ is a real parameter, $\hat{H}_1$ is time independent and we assume that $\lambda\hat{H}_1$ is small. We furthermore assume that the states are non-degenerate. What is the correct expression to the second order in λ for the energy levels?

1.
$$E_n = E_n^0 + \langle n|\lambda\hat{H}_1|n\rangle - \sum_{m(\neq n)} \frac{\left|\langle n|\lambda\hat{H}_1|m\rangle\right|^2}{E_n^0 - E_m^0}.$$

2.
$$\boxed{E_n = E_n^0 + \langle n|\lambda\hat{H}_1|n\rangle + \sum_{m(\neq n)} \frac{\left|\langle n|\lambda\hat{H}_1|m\rangle\right|^2}{E_n^0 - E_m^0}.}$$

3.
$$E_n = E_n^0 + \langle n|\lambda\hat{H}_1|n\rangle - \sum_{m} \frac{\left|\langle n|\lambda\hat{H}_1|m\rangle\right|^2}{E_n^0 - E_m^0}.$$

4.
$$E_n = E_n^0 + \langle n|\lambda\hat{H}_1|n\rangle + \sum_{m} \frac{\left|\langle n|\lambda\hat{H}_1|m\rangle\right|^2}{E_n^0 - E_m^0}.$$

5.
$$E_n = E_n^0 - \langle n|\lambda\hat{H}_1|n\rangle + \sum_{m} \frac{\left|\langle n|\lambda\hat{H}_1|m\rangle\right|^2}{E_n^0 - E_m^0}.$$

Solution

$$E_n = E_n^0 + < n|\lambda\hat{H}_1|n > + \sum_{m(\neq n)} \frac{\left|< n|\lambda\hat{H}_1|m >\right|^2}{E_n^0 - E_m^0}.$$

Problem 17. Spin Precession

We consider the interaction between a magnetic moment of a spin 1/2 particle and an external magnetic field along the z axis. The Hamilton operator is

$$\hat{H} = -\omega \hat{S}_z,$$

where $\omega = gqB/(2m)$ is determined by the particle mass m, the electric charge q, Lande's g-factor and the external magnetic field. We assume that we measure at time $t = 0$ the spin component along the x axis and find the value $+\hbar/2$. What is the probability to find the spin directed along the x axis ($\hat{S}_x$) at a later time, $p(t)$?

1. $$p(t) = \sin(\omega t).$$

2. $$p(t) = \exp(-i\omega t).$$

3. $$p(t) = \sin(\frac{\omega}{2} t).$$

4. $$p(t) = \exp(+i\omega t).$$

5. $$\boxed{p(t) = \cos^2(\frac{\omega}{2} t).}$$

Solution

$$p(t) = \cos^2 \frac{\omega}{2} t.$$

Problem 18. The Stark Effect in Hydrogen

We consider the induced changes in the energy levels of the hydrogen atom caused by an external electric field. We assume that the electric field is directed along the z direction. The Hamiltonian therefore has an additional term due to the external electric field:

$$\lambda \hat{H}_1 = -e\mathcal{E}\hat{z},$$

where $\mathcal{E}$ is the electric field strength and $\hat{z}$ is the position operator along the z axis. We assume that the perturbation is weak. We disregard the electron spin. The electron states in the hydrogen atom are given by the quantum numbers n, l og m ($n = 0, 1, 2, ...$; $l = 0, 1, ..., n-1$; $m = -l, -l+1, ..., l-1, l$). The energy eigenvalues are $E_n = -E_0/n^2$, where E_0 is a constant. What is the induced change in the ground state energy and in the energy of the first excited state?

1. The lowest order correction to the ground state energy is quadratic in the electric field strength $\mathcal{E}$ and the lowest order correction to the first excited state is linear in the electric field strength $\mathcal{E}$.

2. The lowest order correction to the ground state energy is linear in the electric field strength $\mathcal{E}$ and the lowest order correction to the first excited state is linear in the electric field strength $\mathcal{E}$.

3. The lowest order correction to the ground state energy is quadratic in the electric field strength $\mathcal{E}$ and the lowest order correction to the first excited state is quadratic in the electric field strength $\mathcal{E}$.

4. The lowest order correction to the ground state energy is linear in the electric field strength $\mathcal{E}$ and the lowest order correction to the first excited state is quadratic in the electric field strength $\mathcal{E}$.

5. The lowest order correction to the ground state energy is cubic in the electric field strength $\mathcal{E}$ and the lowest order correction to the first excited state is linear in the electric field strength $\mathcal{E}$.

Solution

The lowest order correction to the ground state energy is quadratic in the electric field strength $\mathcal{E}$ and the lowest order correction to the first excited state is linear in the electric field strength $\mathcal{E}$.

Problem 19. State Vectors

We assume that the state vector $|a\rangle$ has norm equal to 1, *e.g.* $\langle a|a\rangle = 1$. Which of the following statements is true when $|b\rangle = (2+i)|a\rangle$?

1.
$$\langle a|b\rangle = 2 - i.$$

2.
$$\boxed{\langle b|a\rangle = 2 - i.}$$

3.
$$\langle b| = 2 + i.$$

4.
$$(\langle a|b\rangle)^2 = -1 - 2i.$$

5.
$$(\langle a|b\rangle)^2 = -1 + 2i.$$

Solution

$$\langle b|a\rangle = 2 - i.$$

Problem 20. The Ladder Operator

We assume that $\hat{a}$ is an annihilitation operator and $\hat{a}^\dagger$ is a creation operator so that

$$\left[\hat{a}, \hat{a}^\dagger\right] = 1$$

and

$$[H, \hat{a}] = -\hbar\omega\hat{a}.$$

Which of the following statements is true?

1. $\boxed{\text{An eigenstate to the operator } \hat{a}^\dagger \text{ does not exist.}}$
2. $\hat{a}$ and $\hat{a}^\dagger$ are hermitian operators.
3. The energy difference between adjacent states is $\hbar\omega/2$.
4. $$\hat{a} = \hat{a}^\dagger.$$
5. $$\left[H, \hat{a}^\dagger\right] = -\hbar\omega\hat{a}^\dagger.$$

Solution

An eigenstate to the operator $\hat{a}^\dagger$ does not exist.