NTNU
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Department of Physics

Examination paper for
TFY4210 Quantum field theory of many-particle systems

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Examination date: May 12 2020

Examination time (from–to): 09:00–13:00

Permitted examination support material: B: All printed and handwritten material. Approved calculator.

Other information:
If a question is unclear/vague – make your own assumptions and specify in your answer the premises you have made. Only contact academic contact in case of errors or insufficiencies in the question set.
Cheating/Plagiarism: The exam is an individual, independent work. Examination aids are permitted. All submitted answers will be subject to plagiarism control.
All files must be uploaded before the examination time expires.

Technical support during examination: Orakel support service. Phone: 73591600

Language: English

Number of pages: 2

Number of pages enclosed: 0

Checked by:

Informasjon om trykking av eksamensoppgave	
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Date Signature

Problem 1 Consider a tight-binding Hamiltonian with nearest-neighbor hopping t and next-nearest neighbor hopping t' on a general two dimensional Bravais lattice with N lattice points, in contact with a particle reservoir. All other hopping matrix elements may be ignored. The chemical potential of the system is denoted μ. The Hamiltonian is given by

$$H = -\sum_{i,j,\sigma} t_{ij}\; c^\dagger_{i,\sigma} c_{j,\sigma} - \mu \sum_{i,\sigma} c^\dagger_{i,\sigma} c_{i,\sigma}$$

where t_{ij} are the hopping matrix elements between lattice sites i and and j. Here, $(c^\dagger_{i,\sigma}, c_{i,\sigma})$ create and destroy particles in spin-state σ on site i. Introduce Fourier-transformed operators

$$\begin{aligned} c^\dagger_{\mathbf{k},\sigma} &= \frac{1}{\sqrt{N}} \sum_i c^\dagger_{i,\sigma}\; e^{-i\mathbf{k}\cdot\mathbf{r}_i} \\ c_{\mathbf{k},\sigma} &= \frac{1}{\sqrt{N}} \sum_i c_{i,\sigma}\; e^{i\mathbf{k}\cdot\mathbf{r}_i} \end{aligned}$$

where $\mathbf{r}_i$ is the position at lattice site i. The lattice constant may be set to unity.

a) Show that Hamiltonian may be written on form

$$H = \sum_{\mathbf{k},\sigma} E_{\mathbf{k}}\; c^\dagger_{\mathbf{k},\sigma} c_{\mathbf{k},\sigma}$$

and give an expression for $E_{\mathbf{k}}$ for a general two-dimensional Bravais lattice.

b) Specialize to the case of a two-dimensional square lattice, and give the expression for $E_{\mathbf{k}}$ in this case.

Hint: To simplify the expression in **b)**, you may find the following identity useful:

$$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$$

c) Set $\mu = 0$ and sketch the Fermi-surface qualitatively for $t' = 0$ and $t' > 0$ in the $2D$ Brillouin-zone.

d Derive an equation determining the average number of particles per lattice site as a function of μ.

Problem 2 A model for an antiferromagnetic insulator is a Heisenberg Hamiltonian with nearest neighbor exchange interaction and easy-axis anisotropy defined on a two-dimensional square lattice

$$H_{\text{AFMI}} = -J \sum_{\langle \boldsymbol{i},\boldsymbol{j} \rangle} \boldsymbol{S_i} \cdot \boldsymbol{S_j} - K \sum_{\boldsymbol{i}} S_{\boldsymbol{i}z}^2$$

Here, $J < 0$ and $K > 0$.

a) Use the Holstein-Primakoff transformation to find the magnon-spectrum for the model given above, calculating to quadratic order in magnon-operators.

Hint: You may find it helpful to use relevant results from the lectures ($K = 0$). You do not need to present a derivation of the latter.

b) Consider low temperatures, and compute the temperature-corrections to the magnetization.

c) Give a brief physical explanation for the result you find in problem **b**.