NTNU

Norwegian University of
Science and Technology

Department of Physics

Examination paper for
TFY4210/FY8302 Quantum theory of solids

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Examination date: May 26 2021

Examination time (from–to): 09:00–13:00

Permitted examination support material: B: All printed and handwritten material. Approved calculator.

Other information:
If a question is unclear/vague – make your own assumptions and specify in your answer the premises you have made. Only contact academic contact in case of errors or insufficiencies in the question set.
Cheating/Plagiarism: The exam is an individual, independent work. All submitted answers will be subject to plagiarism control.
The number of points that can be scored on each question is given in the problem set. Maximum number of points is 80.
If you use results from lecture notes or other printed material, cite your sources.
If you wish to submit a blank test/withdraw from the exam, go to the menu and click "Submit blank". This cannot be undone, even if the test is still open.

Technical support during examination: Orakel support service. Phone: 73591600

Language: English

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Checked by:

Informasjon om trykking av eksamensoppgave	
Originalen er:	
1-sidig ☐	**2-sidig** ☒
sort/hvit ☒	**farger** ☐
skal ha flervalgskjema ☒	

Date Signature

Problem 1 (Points: 10+15+10=35)

A two-dimensional antiferromagnetic insulator may be modelled by a Hamiltonian with nearest neighbor exchange interaction defined on a square lattice

$$H_{\mathrm{AFMI}} = -\sum_{\alpha,\langle i,j\rangle} J_\alpha \; S_{\alpha i} S_{\alpha j}; \alpha \in (x, y, z)$$

Here, $J_\alpha < 0$, $J_z = J$, and $J_x/J = J_y/J = \Delta$, with $\Delta > 0$.

a) Use the Holstein-Primakoff transformation to find the excitation-spectrum for the magnons of the model given above, calculating to quadratic order in magnon-operators. Sketch the spectrum for small momenta for $\Delta = 1$ and $\Delta < 1$.

Hint: You may find it helpful to use relevant results from the lectures for the case $\Delta = 1$. You do not need to present a derivation of the latter.

b) Consider low temperatures $\beta|J| \gg 1$, and compute the leading thermal corrections to the magnetization for $\Delta < 1$. Here, $\beta = 1/k_B T$, k_B is Boltzmann's constant, and T is the temperature.

Hint: Keep the magnon-spectrum on the form $\sim \sqrt{a + bq^2}$ for small q.

c) Give a brief physical explanation for the difference in the magnon-spectrum for $\Delta < 1$ and $\Delta > 1$.

$$\frac{1}{e^x - 1} = e^{-x} \sum_{n=0}^{\infty} e^{-nx}; x > 0$$

$$\int_0^\infty \frac{dx}{\sqrt{a + bx}} \; e^{-c\sqrt{a+bx}} = \frac{2}{bc} \; e^{-c\sqrt{a}}$$

Problem 2 (Points: 5+10+10+10+10=45)

The BCS-model of superconductivity in a system with a single energy band crossing the Fermi-level, is defined by an effective Hamiltonian of the form

$$H = \sum_{\mathbf{k},\sigma} (\varepsilon_{\mathbf{k}} - \mu)\ c^{\dagger}_{\mathbf{k},\sigma}\ c_{\mathbf{k},\sigma} + \sum_{\mathbf{k},\mathbf{k}'} V_{\mathbf{k},\mathbf{k}'} P^{\dagger}_{\mathbf{k}}\ P_{\mathbf{k}'}$$

where $P_{\mathbf{k}} = c_{-\mathbf{k},\downarrow} c_{\mathbf{k},\uparrow}$ and $P^{\dagger}_{\mathbf{k}}$ is the corresponding adjoint operator. $V_{\mathbf{k},\mathbf{k}'}$ is an effective interaction, the c-operators are fermionic creation- and destruction operators, $\varepsilon_{\mathbf{k}}$ is the single-particle excitation energy of the non-interacting system, and μ is the chemical potential.

a) Describe under what restrictions this effective model is derived.

b) We now assume that the effective interaction scattering pairs of electrons is given on the form

$$V_{\mathbf{k},\mathbf{k}'} = -V\ g(\mathbf{k})\ g(\mathbf{k}');\quad V > 0$$

where $g(\mathbf{k})$ is a given function which we assume is known. Define the superconducting gap-function $\Delta(\mathbf{k})$ in the standard way and write down an equation determining $\Delta(\mathbf{k})$. You do not need to derive this equation.

c) Show that $\Delta(\mathbf{k})$ may be written on the form

$$\Delta(\mathbf{k}) = \Delta_0(T)\ F(\mathbf{k})$$

and give an expression for $F(\mathbf{k})$ and an equation determining $\Delta_0(T)$.

d) Consider now the model defined on a two-dimensional square lattice with lattice constant $a = 1$. Which of the following forms of $g(\mathbf{k})$ are permissible expressions in the Hamiltonian given above, if we consider only spin-singlet pairing? Give reasons for your answer. An answer without reasoning will not be given any points.

$$\begin{aligned}
g(\mathbf{k}) &= \sin(k_x) \\
g(\mathbf{k}) &= \sin\left(k_y\right) \\
g(\mathbf{k}) &= \sin\left(k_y\right)\sin(k_x) \\
g(\mathbf{k}) &= \cos(k_x) + \cos\left(k_y\right) \\
g(\mathbf{k}) &= \cos(k_x) - \cos\left(k_y\right)
\end{aligned}$$

e) Use a permissible function $g(\mathbf{k})$ form the set given in **d)** and sketch $\Delta(\mathbf{k})$ on the Fermi-surface for a situation where $\varepsilon_{\mathbf{k}} = -2t\left(\cos(k_x) + \cos\left(k_y\right)\right)$ and the system has an average number of particles per site less than unity. Here, t is a nearest neighbor hopping matrix element.