Front page

Department of Physics

Examination paper for TFY4220 Solid State Physics

Examination date: 28 May 2022

Examination time (from-to): 09:00 - 13:00

Permitted examination support material: All support material is allowed.

Academic contact during examination: Prof. Dag W. Breiby
Phone: 9845 4213

Technical support during examination: Orakel support services
Phone: 73 59 16 00

OTHER INFORMATION

Saving: Answers written in Inspera are automatically saved every 15 seconds.

Cheating/Collaboration: The exam is an individual, independent work.

Notifications: If there is a need to send a message to the candidates during the exam (e.g. if there is an error in the question set), this will be done by sending a notification in Inspera. A dialogue box will appear. You can re-read the notification by clicking the bell icon in the top right-hand corner of the screen. All candidates will also receive an SMS to ensure that nobody misses out on important information. Please keep your phone available during the exam.

Weighting: The multiple choice questions will be equally weighted, with 1 point for a correct answer and 0 points for a wrong answer. No points will be given for uploaded explanations etc.

Submission: Your answer will be submitted automatically when the examination time expires and the test closes, if you have answered at least one question. This will happen even if you do not click “Submit and return to dashboard” on the last page of the question set. You can reopen and edit your answer as long as the test is open. If no questions are answered by the time the examination time expires, your answer will not be submitted.

Withdrawing from the exam: If you wish to submit a blank test/withdraw from the exam, go to the menu in the top right-hand corner and click “Submit blank”. This can not be undone, even if the test is still open.

Accessing your answer post-submission: You will find your answer in Archive when the examination time has expired.

1 Task

In a powder diffraction experiment, X-ray radiation with photon energy E = 8040 eV is used. The unit cell of the material is known to be cubic. The *311* Bragg diffraction peak is observed at a scattering angle 2θ = 60.7°.

What is the corresponding unit cell dimension a?

Select one alternative:

- 5.1 Å ✔
- 9.7 Å
- Cannot be answered because the sample orientation is not specified.
- 3.1 Å
- 13.1 Å

Maximum marks: 1

2 Task

A material with a fcc unit cell and one atom per Bravais point has a lattice constant of 0.288 nm.

What is the number density of atoms in the (111) plane?

Select one alternative:

- $2.4 \cdot 10^{13}$ mm^{-2}
- $2.9 \cdot 10^{13}$ mm^{-2}
- $3.8 \cdot 10^{13}$ mm^{-2}
- $1.5 \cdot 10^{13}$ mm^{-2}
- $2.8 \cdot 10^{13}$ mm^{-2} ✔

Maximum marks: 1

3 Task

According to the free electron Fermi gas model in 3D, the density of states is proportional to the square root of the energy, $D(E) \propto \sqrt{E}$. How many electrons fill the states from an energy of two thirds the Fermi energy, i.e. $2E_F/3$, up to the Fermi level E_F, at $T = 0$ K?

Select one alternative:

- 0.60 *N*
- 0.46 *N* ✔
- 0.78 *N*
- 0
- 0.35 *N*

Maximum marks: 1

4 Task

A face-centered orthorhombic unit cell has conventional unit cell vectors that can be described by $\mathbf{a} = a\hat{\mathbf{x}}; \mathbf{b} = b\hat{\mathbf{x}}; \mathbf{c} = c\hat{\mathbf{z}}$, where $\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$ form an orthogonal coordinate system.

What is the volume of the reciprocal space unit cell corresponding to the *primitive* cell?

Select one alternative:

- $V^* = 16\pi^3/(abc)$
- $V^* = abc$
- $V^* = 8\pi^3/(abc)$
- $V^* = 32\pi^3/(abc)$ ✔
- $V^* = 4\pi^3/(abc)$

Maximum marks: 1

5 Task

What is the approximate de Broglie wavelength of an electron having energy E equal to half the Fermi energy, $E = E_F/2$? Here, n denotes the electron number density.

Select one alternative:

- 3.52 $n^{-1/3}$
- 1.43 $n^{-1/3}$
- 2.87 $n^{-1/3}$ ✔
- 2.03 $n^{-1/3}$
- 4.06 $n^{-1/3}$

Maximum marks: 1

6 Task

A trivalent metal has a face centred cubic unit cell with a = 4.15 Å. What is the Fermi temperature of the material?

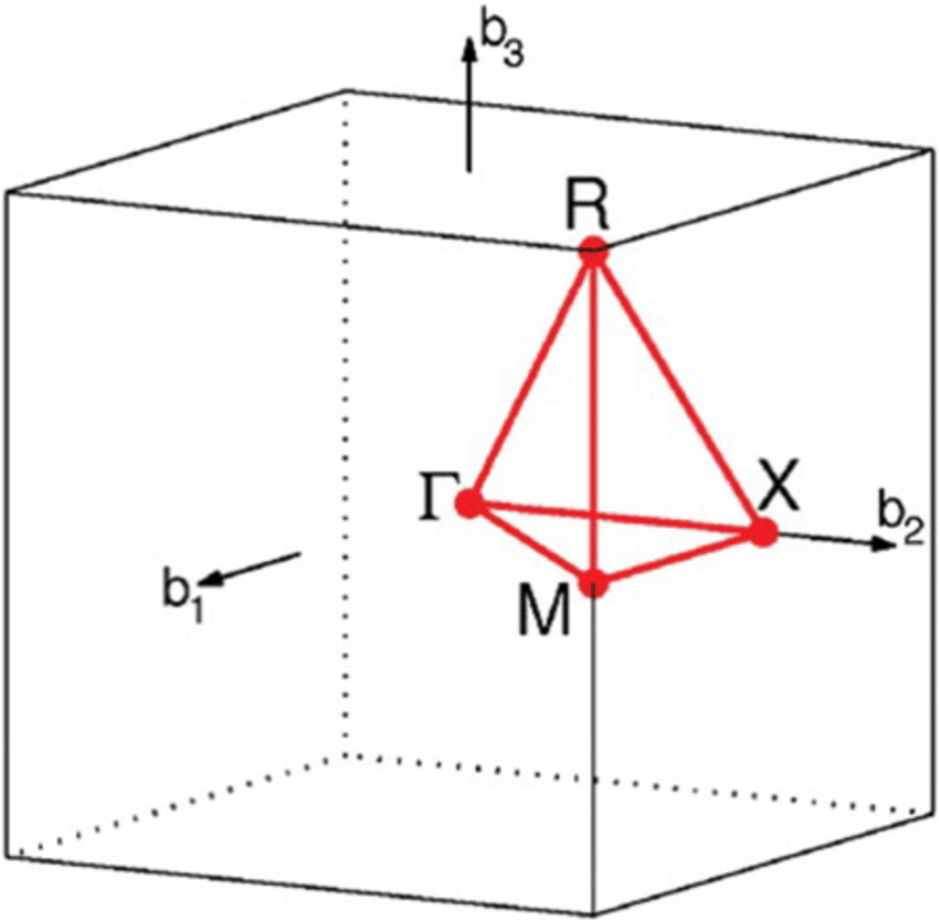
Select one alternative:

- $1.62 \cdot 10^5$ K
- $1.29 \cdot 10^5$ K ✔
- $0.92 \cdot 10^5$ K
- Approx. 293 K (at room temperature)
- $1.03 \cdot 10^5$ K

Maximum marks: 1

7 Task

The figure shows the first Brillouin zone of a simple cubic lattice.



According to the free electron Fermi model, what is the ratio of the electron energies at the *M* point to the *R* point in reciprocal space (see sketch)?

(Take the energy to be zero at the Brillouin zone center).

Select one alternative:

- 2/3 ✔
- 5/3
- 3/4
- 4/9
- 3

Maximum marks: 1

8 Task

A two-dimensional solid is in the form of a square with side length L = 1.00 μm. The number of phonon states n within a circle of radius K is given by $n = \pi K^2 \left(\frac{L}{2\pi}\right)^2$. What is the number of phonon states in a ring of radius K = 1.00 Å^{-1} and thickness ΔK = 0.01 Å^{-1}?

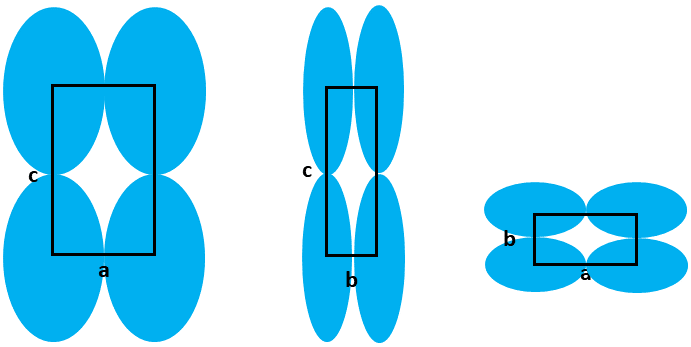
Select one alternative:

- $1.6 \cdot 10^5$ ✔
- $1.6 \cdot 10^7$
- $1.6 \cdot 10^9$
- $1.6 \cdot 10^{11}$
- $6.0 \cdot 10^{23}$

Maximum marks: 1

9 Task

A molecular crystal consists of densely packed compact spheroide-shaped molecules with radii r_a, r_b and r_c. There is one molecule per unit cell. The crystal structure is orthogonal, the molecules are oriented with their principal axes along the *a*, *b* and *c* axes, with $a = 2r_a$; $b = 2r_b$; and $c = 2r_c$, as shown in the figure. In-between the molecules there is a *pore* (void region).



View along **b** View along **a** View along **c**

What is the approximate *porosity* (the volume fraction of empty space)?

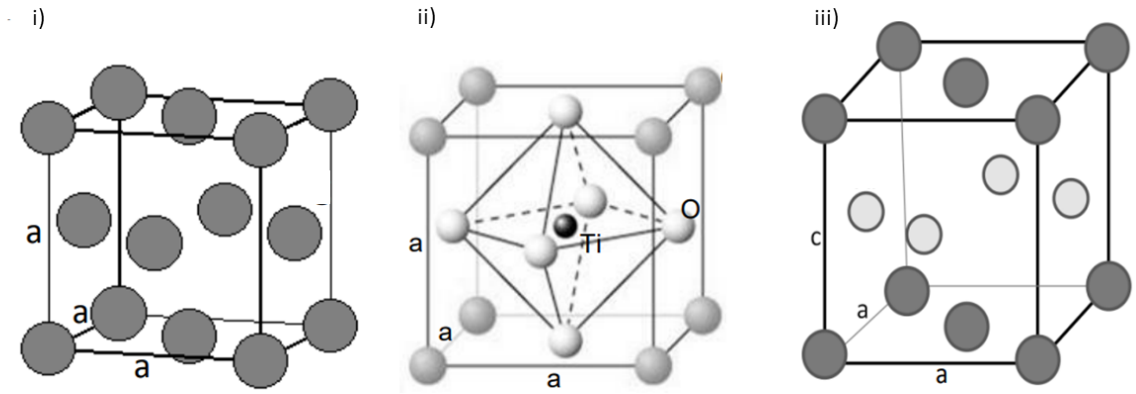
Select one alternative:

- 0.38
- 0.48 ✔
- 0.60
- 0.58
- 0.54

Maximum marks: 1

10 Task

Three conventional unit cells are shown in the figure. All the angles $\alpha, \beta, \gamma = 90^o$.



Which statement is correct?

The figure shows ...
Select one alternative:

- ... i) an fcc structure with one atom in the basis, ii) an fcc structure with 3 atoms in the basis, and iii) a hexagonal cell with four atoms in the basis
- ... three unit cells that are all face-centered.
- ... i) an fcc structure with four atoms in the basis, ii) a simple cubic structure, and iii) a tetragonal cell with six atoms in the basis
- ... i) an fcc structure with one atom in the basis, ii) a simple cubic structure with five atoms in the basis, and iii) a side-centered tetragonal cell with two atoms in the basis ✔
- ... i) an fcc structure with one atom in the basis, ii) a simple cubic structure, and iii) an fcc structure with four atoms in the basis

Maximum marks: 1

11 Task

X-ray diffraction is used to study gallium arsenide (GaAs) with indium (In) substitutions. The unit cell structure factor F_{hkl} for GaAs (without In) can be expressed by

$$F_{hkl} = 4(f_{Ga} + f_{As}), \text{if } h+k+l = 4N$$
$$F_{hkl} = 4(f_{Ga} \pm f_{As}), \text{if } h+k+l = 2N+1$$
$$F_{hkl} = 4(f_{Ga} - f_{As}), \text{if } h+k+l = 4N+2$$

In the compound semiconductor $Ga_{1-x}\ In_xAs$, indium (In) substitutes into random Ga sites with a fraction x.

Find the value of x that gives $F_{200} = 0$.
Hint: Use the small angle scattering approximation of $f \sim Z$. The atomic numbers are $Z_{Ga} = 31$, $Z_{As} = 33$ and $Z_{In} = 49$.

Select one alternative:

- 0.14
- 0.17
- 0.19
- 0.11 ✔
- 0.21

Maximum marks: 1

12 Task

A specially prepared sample has total phonon energy given by
$E = 4Nk_B\frac{T^3}{\Theta_D^2}\int_0^{\Theta_D/T}\frac{x^2}{e^x-1}dx$, where $x = \frac{\hbar\omega}{k_BT}$ and Θ_D is the Debye temperature.
What is the corresponding heat capacity for a sample with $N = N_A$ (Avogadro's number) atoms in the high-T limit?
(R is the universal gas constant.)

Select one alternative:

- 1.5 R
- 2.5R
- 3.0 R
- 2.0 R ✔
- 1.0 R

Maximum marks: 1

13 Task

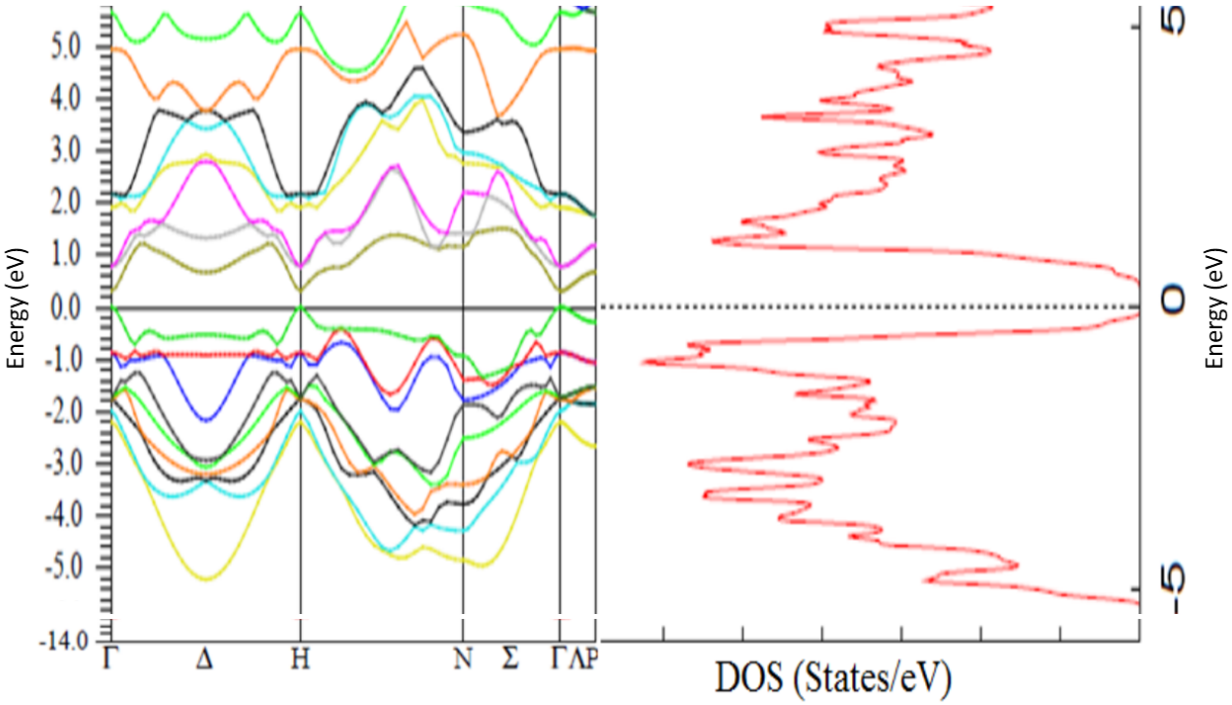
Which statement is correct for a semiconductor with a donor level ("*n*-type")?

Select one alternative:

- No statements about the position of μ can be made without specifying more details of the band structure, in particular the effective masses.
- At low T, most of the donor electrons are at the donor level; μ is then between the midpoint of the bandgap and the donor level. At higher T, μ moves towards the conduction band.
- At low T, most of the donor electrons are at the donor level; μ is then between the donor level and the conduction band. At higher T, μ moves towards the midpoint of the bandgap.
- At all T, μ is near the midpoint of the bandgap.
- At low T, most of the donor electrons are at the donor level; μ is then between the donor level and the conduction band. At higher T, μ moves towards the conduction band.

Maximum marks: 1

14 Task



The figure shows the 3D band structure and DOS of a material. The energy of the highest filled electronic state at $T = 0$ defines $E = 0$.

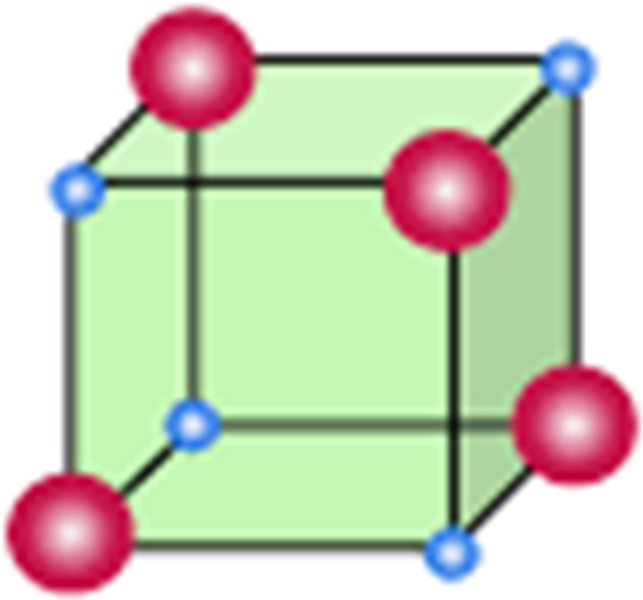
Which statement is correct?

Select one alternative:

- The material is an insulator.
- The material is a metal.
- The material is a semiconductor with a direct bandgap of < 1 eV. ✔
- The material is a semiconductor with an indirect bandgap of < 1 eV.
- The material is a semimetal

Maximum marks: 1

15 Task



A structural arrangement is illustrated in the figure.
(The red and blue spheres indicate atoms, the green shading and black lines are just for visualization purposes. The side lengths are equal and the angles of the corners are all 90^{o}).

Which statement is correct?

Select one alternative:

- The figure illustrates a triangular unit cell
- The figure illustrates a simple cubic unit cell with two atoms in the basis
- This structure can be a subunit ("building block") of any crystal system. ✔
- The figure illustrates a simple cubic unit cell containing one atom in average per unit cell.
- The illustrated structure cannot be a subunit of a cubic crystal structure

Maximum marks: 1

16 Task

Estimate the chemical potential μ at T = 320 K for an intrinsic semiconductor with bandgap E_g = 0.66 eV having effective masses m_h = 0.50 m and m_e = 0.10 m.

Select one alternative:

- 1.32 eV
- 0.36 eV ✔
- 0.30 eV
- 0.33 eV
- 0.66 eV

Maximum marks: 1

17 Task

It can be shown that the dispersion relation for phonons on a 1D string with next-nearest neighbors included is given by

$\omega^2 = \frac{4}{M}(C_1 \sin^2 \frac{ka}{2} + C_2 \sin^2 ka)$.

Here, *M* is the mass of the atoms, C_1 and C_2 are force constants, *a* denotes the interatomic spacing and *k* is the wave vector.
Now assume that for a specifically engineered material $C_2 = -C_1/4$.

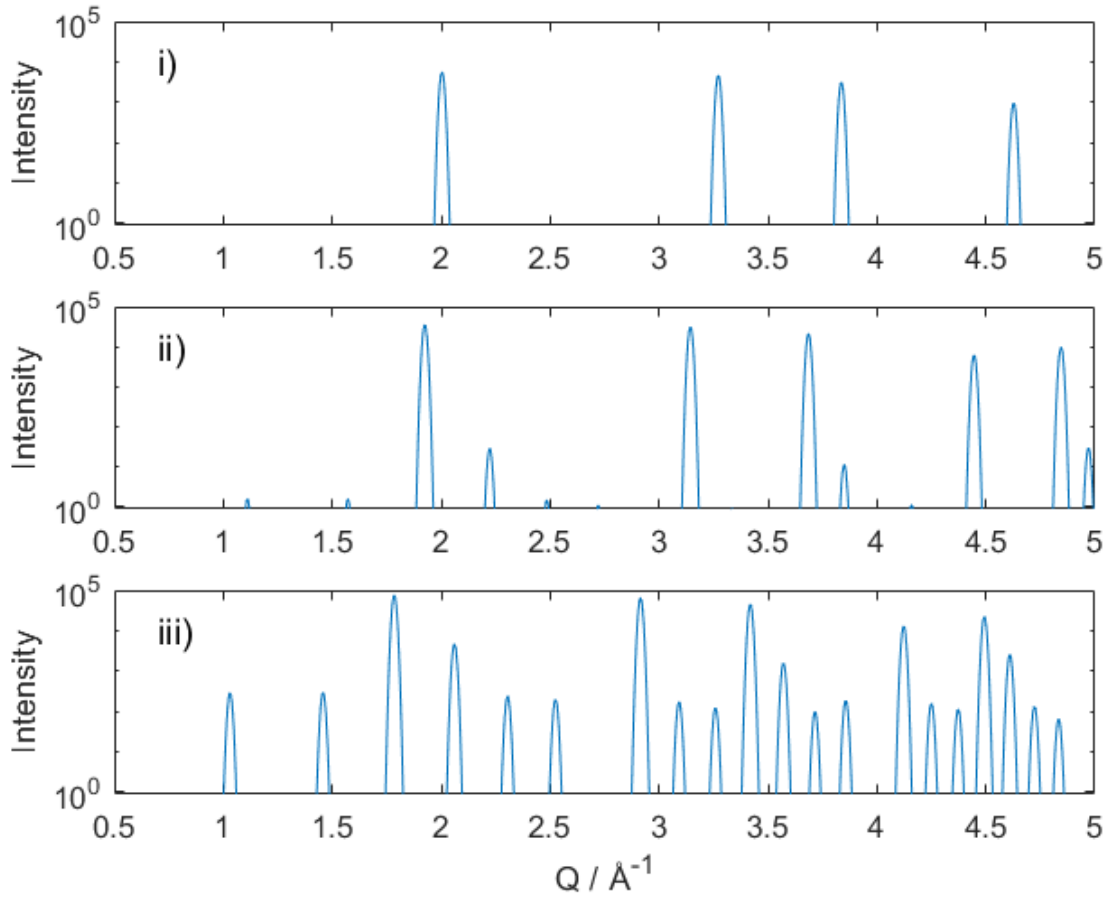
What is the limiting value of the group velocity for large wavelengths?

Select one alternative:

- $a\sqrt{\frac{3C_1}{M}}$
- $3a\sqrt{\frac{C_2}{M}}$
- 0 ✔
- $\frac{2\pi a^2}{L}\sqrt{\frac{3C_1}{M}}$, where *L* is the length of the object
- $a\sqrt{\frac{C_1}{3M}}$

Maximum marks: 1

18 Task



The figure presents powder diffraction data for GaAs, GaSb and Si, all having variations of the diamond or zinc blende structure.

Which assignment is correct?

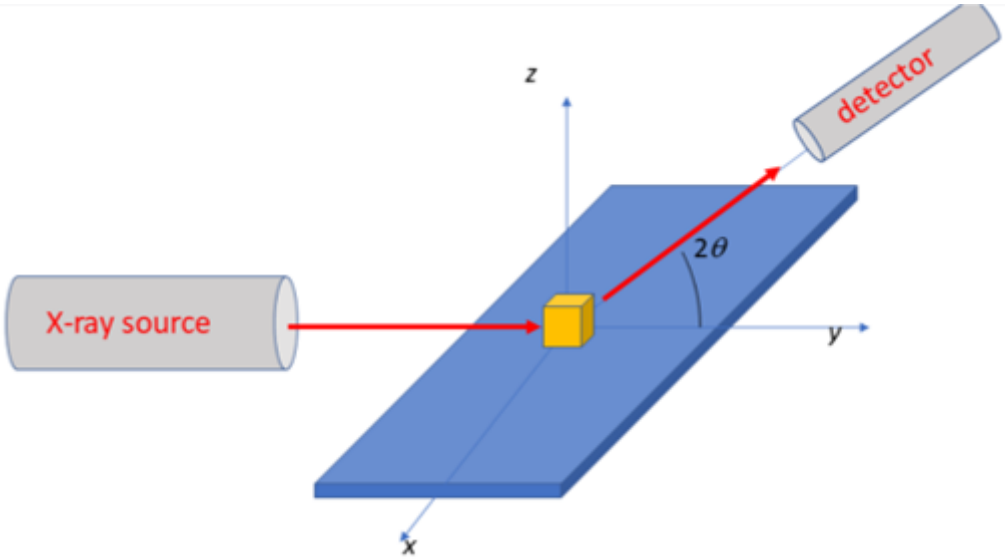
Select one alternative:

- i) Si; ii) GaAs; iii) GaSb ✔
- i) GaAs; Si; GaSb
- i) Si; ii) GaSb; iii) GaAs
- i) GaAs; ii) GaSb; iii) Si
- i) GaSb; ii) GaAs; iii) Si

Maximum marks: 1

19 Task

A diffractometer is endowed with an orthogonal laboratory coordinate system (x, y, z), where $\mathbf{z}$ is the vertical direction. A powder sample of a crystalline material is placed in a small container at the origin. The crystal structure is tetragonal, and it is known that the unique axis *c* is three times as long as the a axis. The incoming X-ray beam is directed along $\hat{\mathbf{y}}$. The point detector (sensor) is placed in the (*y*,*z*) plane, and is at an angle 2θ with respect to the horizontal, as shown in the figure. With the wavelength λ = 1.542 Å and the chosen angle 2θ, the diffraction condition for the *111* reflection is fulfilled.



Which expression gives a correct representation for the scattered wave vector measured in this geometry?

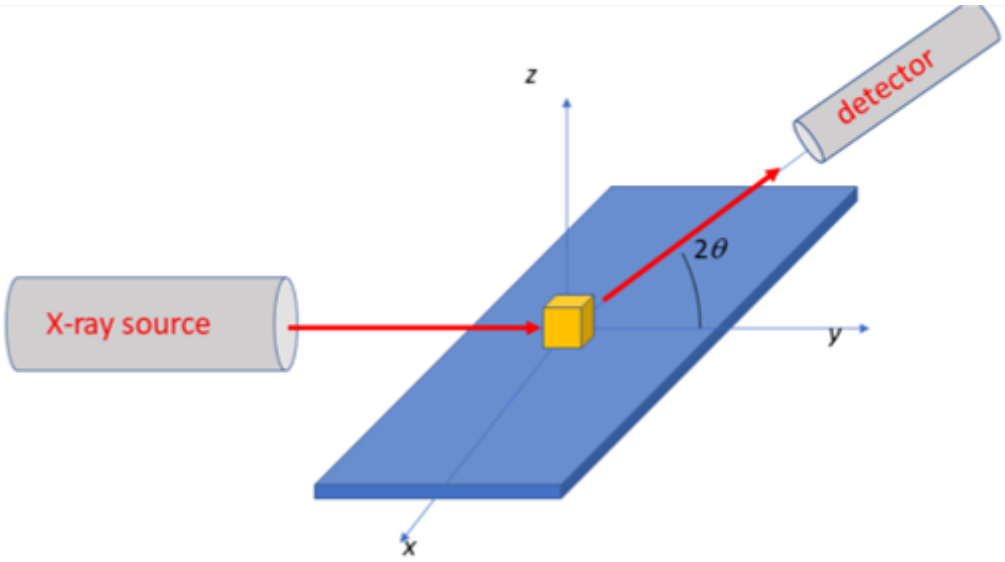
Select one alternative:

- $\mathbf{k_f} = 2\pi < \frac{1}{a}\sin 2\theta, 0, \frac{1}{c}\cos 2\theta >$
- The question cannot be answered without further information.
- $\mathbf{k_f} = \frac{2\pi a}{\lambda^2} < 1, 0, -1 >$
- $\mathbf{k_f} = \frac{2\pi}{\lambda} < 0, \cos 2\theta - 1, \sin 2\theta >$
- $\mathbf{k_f} = \frac{2\pi}{\lambda} < 0, \cos(2\theta), \sin(2\theta) >$ ✔

Maximum marks: 1

20 Task

A diffractometer is endowed with an orthogonal laboratory coordinate system (x, y, z), where $\mathbf{z}$ is the vertical direction. A single crystal is placed at the origin. The crystal structure is tetragonal, and its unique axis *c* is three times as long as the *a* axis. The **c** axis is oriented along $\hat{\mathbf{x}}$, and the **a** and **b** axes are in the (*y*,*z*)-plane. The incoming X-ray beam is monochromatic and directed along $\hat{\mathbf{y}}$. The point detector (sensor) is placed in the (*y*,*z*) plane, and is scanned by the angle 2θ with respect to the horizontal, as shown in the figure.



Complete the following statement with the correct alternative:

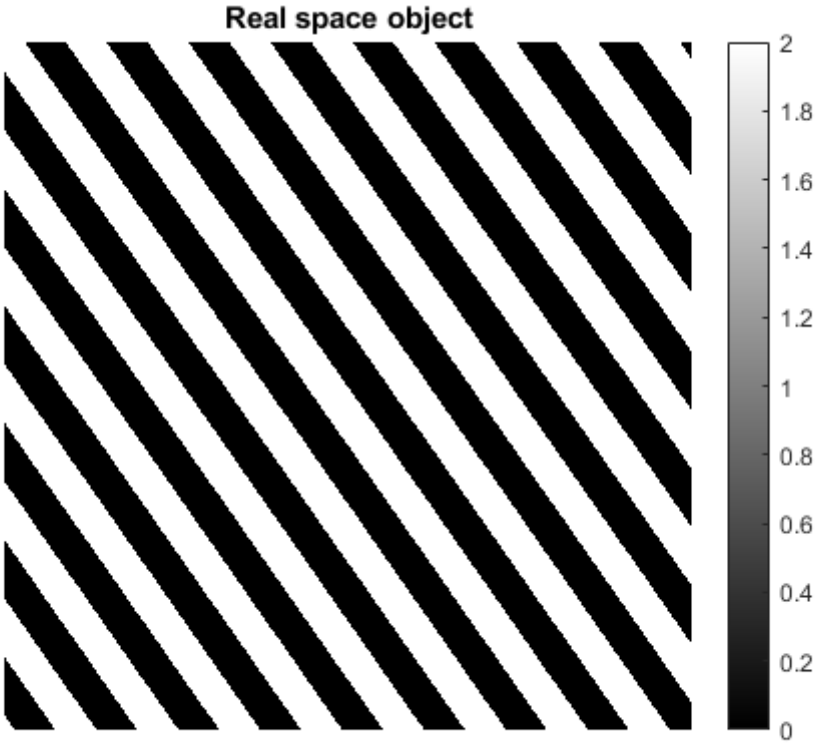
If the detector angle 2θ is scanned from 0^o to 180^o , while the sample is kept stationary ...
Select one alternative:

- ... only *hk0* Bragg peaks can potentially be observed ✔
- ... it is certain that many Bragg reflections will be observed.
- ... only Bragg reflections giving $|\mathbf{k_f}| = 3|\mathbf{k_i}|$ can potentially be observed.
- ... only Bragg reflections with $|\mathbf{G_{hkl}}| = \frac{2\pi}{\lambda}\sin(\theta)$ can potentially be observed
- ... any *hkl* Bragg peaks can potentially be observed

Maximum marks: 1

21 Task

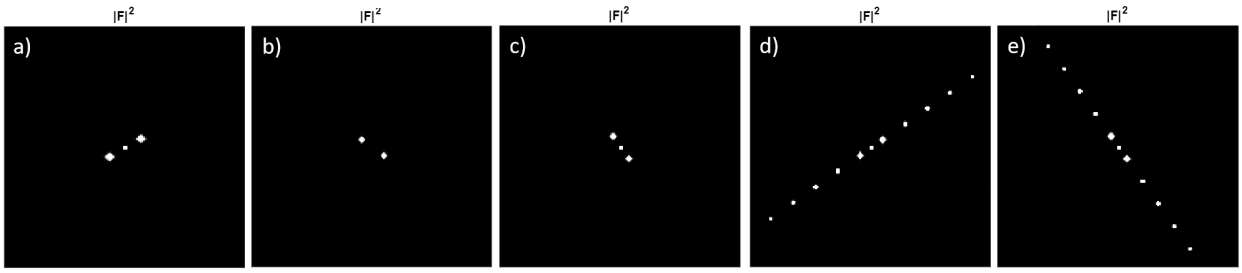
A real space density distribution (extending to infinity in all directions) is shown as the "Real space object":



The density variation $\rho(x, y)$ is given by a planar square wave on a flat constant background, ensuring that the density is everywhere ≥ 0.

Which of the suggested patterns depicts the corresponding absolute square ("power spectrum") of the Fourier transform, i.e., $|FT(\rho)|^2$?

(The origin of Fourier space is in the middle of these patterns).



Select one alternative:

- pattern a)
- pattern b)
- pattern c)
- pattern d) ✔
- pattern e)

Maximum marks: 1

22 Task

For an intrinsic 1D semiconductor, the valence band is modeled by
$\epsilon_1 = -A\sin^2(\frac{ka}{2})$
and the conduction band by
$\epsilon_2 = A(3-\cos(ka))$,
where $A > 0$ is a constant. a is the lattice constant.
At $T = 0$, the valence band is completely filled and the conduction band is empty.

Which statement is *incorrect*?
Select one alternative:

- The effective mass of both holes and electrons change sign at $k = \pm\frac{\pi}{2a}$
- Here, $|m_h| = 2|m_e|$ for all k. m_h is the effective mass of the holes (in the valence band), and m_e is the effective mass of the electrons (in the conduction band).
- There is a direct bandgap at the Γ-point
- The 1st Brillouin zone extends from 0 to $\frac{2\pi}{a}$ ✔
- The bandgap is 2A. At $T = 0$, the chemical potential is given by $\mu = A$.

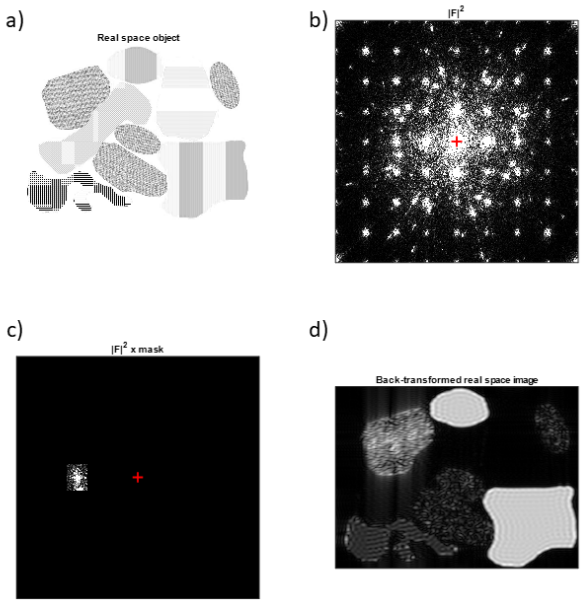
Maximum marks: 1

23 Task

The figure illustrates Fourier filtering. a) Artificially generated object. b) Fourier transform squared ("power spectrum") of the object shown in (a). c) Illustration of a mask applied to the Fourier transform, visualized as the masked power spectrum.

d) Resulting image after inverse Fourier transformation of the masked signal.

The red crosses mark the origin of Fourier space ($k_x = k_y = 0$).



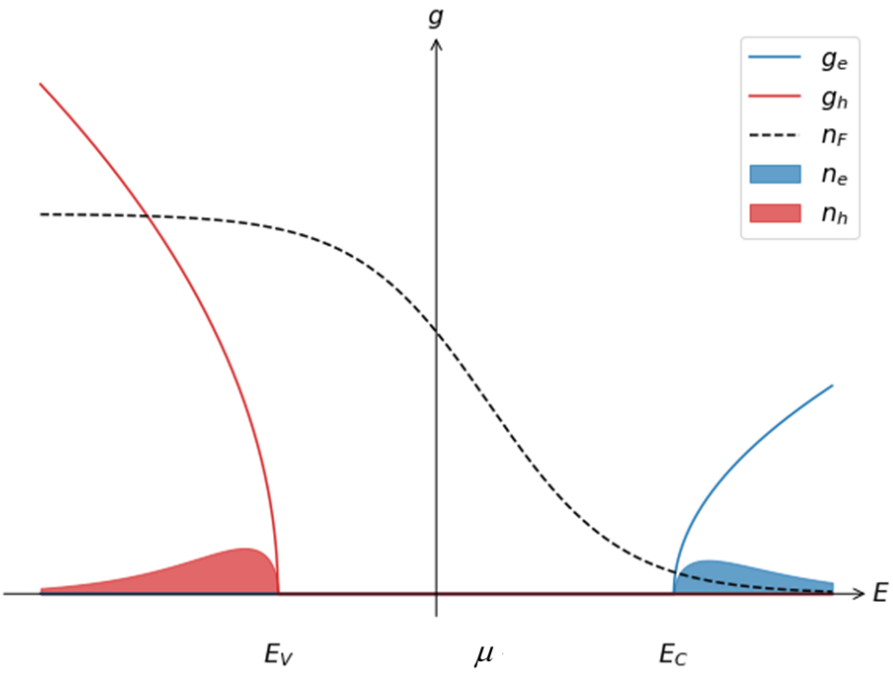
Which of the following statements is *incorrect*?

Select one alternative:

- The mask transmits spatial frequencies that are found (almost) exclusively in the part of the object lighting up in fig. d).
- The process in the figure corresponds to dark-field imaging.
- In general, a reciprocal lattice is the Fourier transform of a real space lattice.
- Points far away from the origin of Fourier space (cf. fig. b), correspond to plane waves with high spatial frequency.
- The process in the figure corresponds to bright-field imaging. ✔

Maximum marks: 1

24 Task



Which statement is *incorrect*?

Select one alternative:

- The chemical potential for an intrinsic semiconductor is at the top of the valence band for $T = 0$. ✔
- For an *n*-type extrinsic semiconductor, the number of electrons in the conduction band is higher than the number of holes in the valence band.
- For an intrinsic semiconductor, the number of electrons in the conduction band must equal the number of holes in the valence band.
- The figure illustrates the position of the chemical potential in a semiconductor, the density of states and the Fermi-Dirac distribution for $T > 0$.
- For *p*-type semiconductors, there are no electrons in the conduction band at low temperature.

Maximum marks: 1

25 Task

A germanium sample is uniformly doped with $2 \cdot 10^{18}$ phosphorus atoms/cm^3 and $2 \cdot 10^{16}$ boron atoms/cm^3. If all the dopants are fully ionized, the material is:

Select one alternative:

- *n*-type with carrier concentration of $2 \cdot 10^{18}$ cm^{-3} ✔
- *p*-type with carrier concentration of $1 \cdot 10^{18}$ cm^{-3}
- *p*-type with carrier concentration of $4 \cdot 10^{16}$ cm^{-3}
- Intrinsic
- *n*-type with carrier concentration of $2 \cdot 10^{16}$ cm^{-3}

Maximum marks: 1

26 Task

A monatomic linear chain has acoustic lattice vibrations with phase velocity v_p = 2200 m/s, and the maximum frequency is $\omega_0 = 2 \cdot 10^{13}$ rad s^{-1}.

What is the lattice constant *a*?

Select one alternative:

- 3.2 Å
- 2.1 Å
- 4.2 Å
- 2.2 Å ✔
- 2.3 Å

Maximum marks: 1

27 Task

For electrons in a material, the probability of occupation of an energy level E, when $E - E_F = -2k_BT$, is:

Select one alternative:

- 0.12
- 0.88 ✔
- 0.50
- 0.27
- Cannot be determined without more information.

Maximum marks: 1

28 Task

$$\begin{array}{ccccccccc}
U_2 & U_1 & \lambda_{k-2g}+U_0-\varepsilon & U_1 & U_2 & 0 & 0 & 0 & 0 \\
0 & U_2 & U_1 & \lambda_{k-g}+U_0-\varepsilon & U_1 & U_2 & 0 & 0 & 0 \\
0 & 0 & U_2 & U_1 & \lambda_k+U_0-\varepsilon & U_1 & U_2 & 0 & 0 \\
0 & 0 & 0 & U_2 & U_1 & \lambda_{k+g}+U_0-\varepsilon & U_1 & U_2 & 0 \\
0 & 0 & 0 & 0 & U_2 & U_1 & \lambda_{k+2g}+U_0-\varepsilon & U_1 & U_2
\end{array}$$

Here, U_i are Fourier coefficients of the potential, and $\lambda_k = \hbar k/2m$.
Which statement relating to the central equation is *incorrect*?

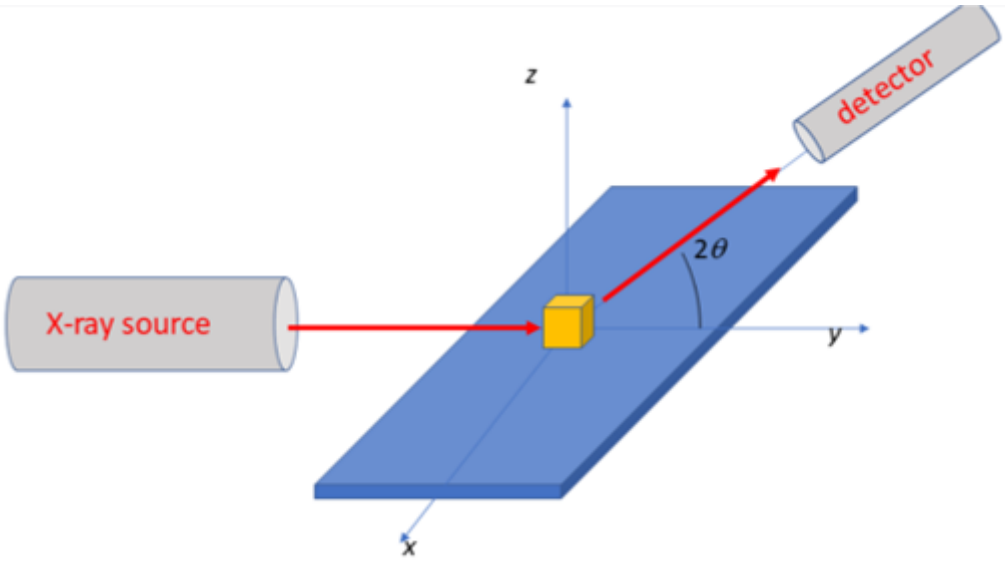
Select one alternative:

- U_2 and higher order terms of the potential can be omitted for sufficiently weak and smoothly varying electron-lattice potentials.
- U_0 is half the spatially averaged value of the potential ✔
- It is always sufficient to solve for the k values within the 1st Brillouin zone.
- The given block is a determinant whose solutions give the energy eigenvalues for the chosen value of k.
- The central equation is an algebraic equation containing exactly the same information as the Schrödinger equation for a periodic potential.

Maximum marks: 1

29 Task

A powder sample is placed in a small container at the origin of the orthogonal laboratory coordinate system $(\mathbf{x}, \mathbf{y}, \mathbf{z})$, where $\mathbf{z}$ is the vertical direction. The crystal belongs to the body-centered cubic (b.c.c.) crystal system, with *a* = 3.16 Å. The incoming X-ray beam with wavelength $\boldsymbol{\lambda}$=1.64 Å is directed along $\hat{\mathbf{y}}$. The point detector (sensor) is placed in the (*y*,*z*) plane, and is scanned by angle $2\boldsymbol{\theta}$ with respect to the horizontal (about the *x* axis), as shown in the figure.



Which statement is correct?

Select one alternative:

- The *400* peak cannot be observed with the chosen X-ray photon energy. ✔
- The *100* reflection is expectedly missing because the structure factor F_{100} = 0. The *200* diffraction peak is seen at $2\boldsymbol{\theta}$=58.3°.
- Any reciprocal lattice vector $\mathbf{G_{h00}}$ fulfilling the diffraction condition will have a positive *y*-component
- $\mathbf{G_{100}} = \mathbf{0}$ because of extinction
- Because of the powder geometry, all the diffraction peaks will be smeared out in reciprocal space

Maximum marks: 1

30 Task

The product of the charge carrier concentrations in the valence and conduction bands is given by *np*,

$$np = 4\left(\frac{k_B T}{2\pi\hbar^2}\right)^3 (m_e m_h)^{3/2} \exp\left(-E_g/k_B T\right).$$

If we assume an intrinsic semiconductor with bandgap E_g = 1.26 eV, effective masses $m_e = m_h$ = 0.20 m, and with mobilities μ_e= 0.42 $m^2V^{-1}s^{-1}$ and μ_h= 0.22 $m^2V^{-1}s^{-1}$, what is the conductivity at T = 280 K?

Select one alternative:

- 0.22 S/m
- 2280 S/m
- $9.70 \cdot 10^{-7}$ S/m ✔
- 280 S/m
- $9.70 \cdot 10^{-4}$ S/m

Maximum marks: 1