Norges teknisk–naturvitenskapelige universitet
NTNU

Institutt for fysikk
Fakultet for fysikk, informatikk og matematikk

Contact during the exam:
Jan Myrheim
Telephone: 93653

Exam in SIF4052 SOLID STATE PHYSICS 1

Friday, December 7, 2001
09:00–14:00

Allowed help: Alternativ C

Sensur: Se fagets webside: http://www.phys.ntnu.no/˜alexh/Underv/SIF4052/

This problem set consists of 3 pages.

Problem 1

A one-dimensional Bravais lattice consists of identical atoms with mass m that interact through through harmonic forces with spring constant K. The lattice constant is a.

a) Show that the equations of motion are

$$m\,\frac{d^2u_n}{dt^2} = K(u_{n-1} + u_{n+1} - 2u_n)\;, \tag{1.1}$$

for all n. Determine the dispersion relation and group velocity for vibrations in the lattice.

b) Demonstrate the connection between Eq. (1.1) and the macroscopic wave equation

$$\frac{\partial^2 u}{\partial^2 t} = c\frac{\partial^2 u}{\partial^2 x}\;. \tag{1.2}$$

What is the connection between c, a, m, and K?

c) Determine the density of states $g(\omega)$ for the one-dimensional lattice in a).

Problem 2

a) Define the reciprocal lattice of a Bravais lattice. Show that the reciprocal lattice may be generated by the primitive vectors

$$\vec{a}^* = 2\pi \, \frac{\vec{b} \times \vec{c}}{\vec{a} \cdot \vec{b} \times \vec{c}} \, , \tag{2.1}$$

$$\vec{b}^* = 2\pi \, \frac{\vec{c} \times \vec{a}}{\vec{a} \cdot \vec{b} \times \vec{c}} \, , \tag{2.2}$$

and

$$\vec{c}^* = 2\pi \, \frac{\vec{a} \times \vec{b}}{\vec{a} \cdot \vec{b} \times \vec{c}} \, , \tag{2.3}$$

where $\vec{a}$, $\vec{b}$ and $\vec{c}$ are the primitive vectors of the Bravais lattice.

b) What is the reciprocal of a reciprocal lattice?

c) Define primitive cell, the Wigner-Seitz cell and the 1st Brillouin cell.

Problem 3

In a semiconductor in thermal equillibrium at a temperature T, the bottom of the conduction band has the form

$$E(\vec{k}) = E_c + \frac{\hbar^2 k^2}{2{m_e}^*} \, . \tag{3.1}$$

The density of states in the conduction band is given by

$$g(E) = \frac{1}{2\pi^2} \left(\frac{2{m_e}^*}{\hbar^2} \right)^{3/2} (E - E_c)^{1/2} \tag{3.2}$$

and the Fermi-Dirac distribution is given by

$$f(E) = \frac{1}{e^{(E-\mu)/k_B T} + 1} \, . \tag{3.3}$$

Hint: You may find the integral $\int_0^\infty x^{1/2} e^{-x} dx = \sqrt{\pi}/2$ useful in the following.

a) Find an expression for the electron density n of electrons in the conduction band. Show that when the chemical potential μ is sufficiently far below the bottom of the conduction band, $E_c - \mu \gg k_B T$, we have

$$n = N_c e^{(\mu - E_c)/k_B T} \, , \tag{3.4}$$

where

$$N_c = 2 \left(\frac{2\pi {m_e}^* k_B T}{h^2} \right)^{3/2} \, . \tag{3.5}$$

b) The semiconductor is n-type with N_d dopant atoms per unit volume. The density n_d of excited donor electrons depends on temperature as

$$n_d = \frac{N_d}{1 + 2e^{(\mu - E_d)/k_B T}} . \tag{3.6}$$

Here E_d is the donor level. Derive this result keeping in mind that the electron can either have spin up or spin down.

Between what bounds must μ lie when $T = 0$? Construct an equation that determines the connection between the chemical potential and the temperature when the valence band is assumed to be fully occupied. Make the assumption that there are few electrons in the conduction band.

c) Show that with the assumptions made above, the chemical potential is given by

$$\mu = E_d + k_B T \log \left[\frac{1}{4} \sqrt{1 + \frac{8N_d}{N_c} e^{(E_c - E_d)/k_B T}} - \frac{1}{4} \right] . \tag{3.7}$$

What is μ at $T = 0$?

d) Show that n is given by

$$n = \frac{2N_d}{1 + \sqrt{1 + 8N_d N_c^{-1} e^{(E_c - E_d)/k_B T}}} . \tag{3.8}$$

e) When the temperature is high, at least one of the assumptions made so far breaks down and the semiconductor changes from extrinsic to intrinsic behavior. What does this mean? Sketch the development of μ and n as a function of T, also outside the range of validity of Eqs. (3.7) and (3.8).