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E6 (6)



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Engelsk

NORGES TEKNISK-
NATURVITENSKAPELIGE UNIVERSITET
INSTITUTT FOR FYSIKK

Contact during exam:
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**Exam in course TFY4220 Solid state physics**

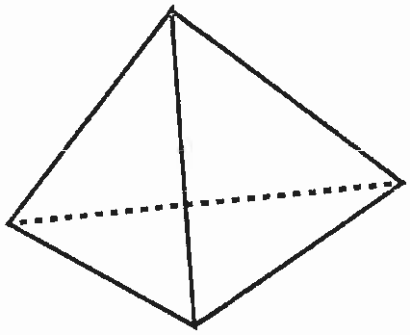
Thursday May 28, 2009
Time 09.00-13.00

Allowed at exam: Alternative C
Approved pocket calculator
K. Rottman: Matematical formulas
English dictionary

**Appendix:**
- constants and formulas

**Problem 1**

(a) Derive the primitive lattice vectors for the BCC unit cell in terms of the lattice parameter $a$.
Find the angles between the primitive lattice vectors.
What is the volume of the primitive cell?
Find the reciprocal lattice vectors. What is the lattice structure in reciprocal space?



(b) The figure above shows a tetrahedron.
How many GEP (general equivalent points) are contained in the tetrahedron?
Write down the symmetry elements in the point group of the tetrahedron.

(c) Formulate the Laue condition for x-ray diffraction and define the involved quantities.
Derive the extinction rules (norsk: utslokkingregler) for the FCC structure.

(d) The total lattice energy for an ionic solid may be described by the Born-Landé equation:

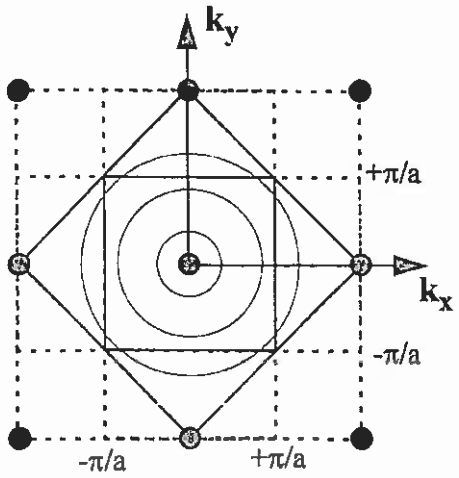
$$U = N\left(\frac{\beta b}{R^n} - \frac{Aq^2}{4\pi\varepsilon_0 R}\right)$$

Define the quantities in the above equation.
The bulk modulus B is a measure of the stiffness of the ionic solid, and is given by:

$$B = V\left(\frac{\partial^2 U}{\partial V^2}\right)_0$$

where the subscript 0 refers to the equilibrium distance $R_0$ between atoms in the solid.
Find an expression for the bulk modulus B expressed by $R_0$, $A$, $q$ and $n$.

**Problem 2**




The figure shows a two dimensional reciprocal lattice. The circles represent constant energy contours in the free electron model.

(a) Derive the 3D density of states $g(E)$, find the Fermi energy in terms of the electron density $n$, and find the average electron energy at $T = 0\ K$ in the free electron model.

(b) Find the Fermi wave vector $k_F$ for the 2D system that corresponds to the figure above, in the case of 2 electrons per unit cell. Use this to sketch qualitatively the Fermi surface in the 1st and 2nd Brillouin zones in the repeated zone scheme. Does the Fermi surface exist in the 3rd Brillouin zone? Explain.

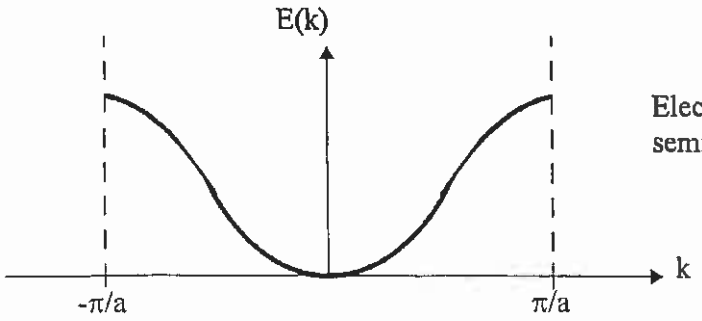
(c) Compute the three lowest energy bands in the empty-lattice approximation in one dimension. Plot these bands in an E(k) versus k diagram, using the reduced-zone scheme. Show qualitatively how these bands are modified in the NFE (nearly free electron) model.

(d) By Fourier expansion of the wavefunction and the interaction potential, the algebraic form of the Schrödinger equation may be found to be:

$$\left(\frac{\hbar^2 k^2}{2m} - E\right) C_k + \sum_G V_G C_{k-G} = 0$$

In the nearly-free-electron (NFE) model it is assumed that a deviation from the free-electron model is only observed near the zone boundaries $k = +\pi/a = +G/2$ and $k = -\pi/a = -G/2$. Assume that $V_G = V_{-G} = |V_G| = 0.1\ eV$ and estimate the energy bandgap at the zone boundaries.

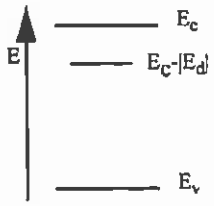
**Problem 3**




Electron energy band an a semiconductor crystal.

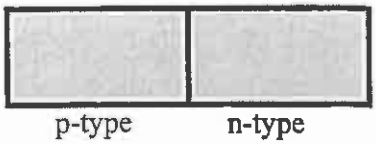
(a) Sketch qualitatively the effective electron mass corresponding to the electron band shown in the figure above. Please explain.

(b) Find the concentration $n$ of electrons in the conduction band for an intrinsic semiconductor when $T = 300\ K$ and the value of the energy gap is $E_g = 1.2\ eV$.
Assume that the effective mass of the electrons is 50% of the mass of a free electron.
Find the concentration $p$ of holes in the valence band.




The figure shows the donor level for a hydrogenic dopant in an extrinsic semiconductor.

(c) Find the position of the donor level relative to the bottom of the conduction band for the semiconductor as shown in the figure above, when the effective mass of the electron $m_e^* = 0.1\ m_e$ and the dielectric constant of the semiconductor $\varepsilon = 10\ \varepsilon_0$.
Also estimate the radius $r_d$ of the electron orbit corresponding to the donor state.




The figure shows a semiconductor pn-junction.

(d) Draw the energy diagram for p-type and n-type semiconductor before they are contacted.
Also draw the energy diagram after they are contacted. Show the Fermi level.
What is the origin of the potential difference across the junction?
Give a brief qualitatively discription of the resistance and number of charge carriers in the depletion region.

**Vedlegg (Appendix)**: Some constants and expressions that may or may not be of use.

| | | |
|---|---|---|
| $k_B = 1.38 \cdot 10^{-23}$ J/K | $e = 1.6 \cdot 10^{-19}$ C | $\varepsilon_0 = 8.85 \cdot 10^{-12}$ $s^4A^2kg^{-1}m^{-3}$ |
| $h = 6.63 \cdot 10^{-34}$ Js | $m_e = 9.1 \cdot 10^{-31}$ kg | $c = 3.0 \cdot 10^8$ m/s |
| $\hbar = h/2\pi = 1.05 \cdot 10^{-34}$ Js | $R_0 = 13.6$ eV | $a_0 = 0.53$ Å |

$$\left.\begin{aligned} \frac{1}{1-x} &\approx 1 + x + x^2 \ldots \\ \sin x &\approx x - \frac{1}{3!}x^3 \ldots \\ \sqrt{1+x} &\approx 1 + \frac{x}{2} + \ldots \end{aligned}\right\} \quad \textit{for } x << 1$$

$$n(\omega) = \frac{1}{e^{\hbar\omega/k_BT} - 1} \qquad \textit{Planck distribution law for phonons (average phonon occupation number)}$$

$$\omega(k) = 2\sqrt{\frac{K}{M}}\left|\sin\frac{ka}{2}\right| \qquad \textit{phonon dispersion relation}$$

$$g(E) = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}} E^{1/2} \qquad \textit{3D free-electron density of states}$$

$$g(E) = \frac{mL^2}{\hbar^2\pi} \qquad \textit{2D free-electron density of states}$$

$$E_n = -R_0/n^2 \qquad \textit{energy levels of the hydrogen atom}$$

$$R_0 = \frac{e^4 m_e}{32\pi^2\varepsilon_0^2\hbar^2} \qquad \textit{the Rydberg constant}$$

$$a_0 = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2} \qquad \textit{the Bohr radius}$$

$$f(E) = \frac{1}{e^{(E-\mu)/k_BT} + 1} \qquad \textit{Fermi-Dirac distribution function (Fermi-function)}$$

$$n = 2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{\frac{3}{2}} e^{-(E_c-\mu)/k_BT} = N_c e^{-(E_c-\mu)/k_BT}$$

$$p = 2\left(\frac{2\pi m_h^* k_B T}{h^2}\right)^{\frac{3}{2}} e^{(E_v-\mu)/k_BT} = N_v e^{(E_v-\mu)/k_BT}$$