Engelsk

NORGES TEKNISK-
NATURVITENSKAPELIGE UNIVERSITET
INSTITUTT FOR FYSIKK

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Exam TFY4220 Solid state physics

Thursday June 10, 2010
Time 09.00-13.00

Allowed at exam: Alternative C
Approved pocket calculator
K. Rottman: Mathematical formulas
English dictionary

Vedlegg / Appendix:
- constants and formulas

Problem 1

(a) Define primitive lattice vectors for the simple hexagonal structure in terms of the lattice parameters *a* and *c*. What are the angles between the primitive lattice vectors.
What is the volume of the primitive cell?
Find the reciprocal lattice vectors.

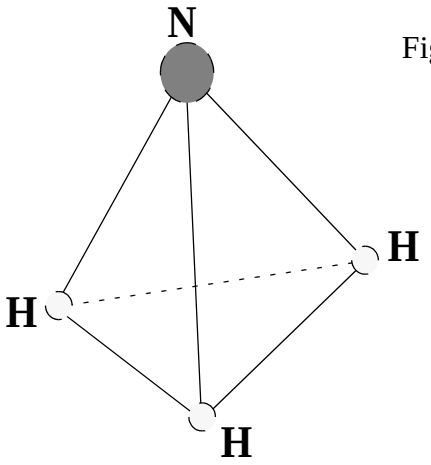


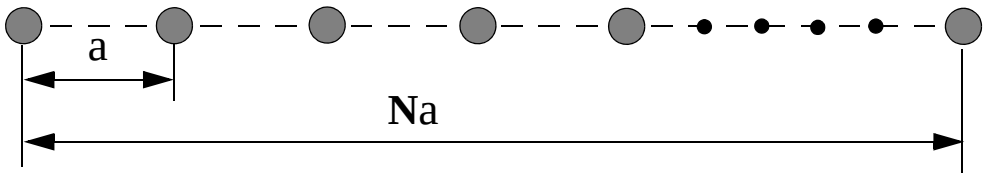
Figure showing the NH_3 molecule.

(b) The figure above shows the NH_3 molecule.
How many GEP (general equivalent points) may be found in the molecule?
Write down the symmetry elements in the point group of the molecule.

(c) Formulate the Laue condition for x-ray diffraction and define the involved quantities.
Derive the extinction rules (norsk: utslokkingregler) for the BCC structure.

(d) List the most common types of bonding in solids, and give a brief description of each.

Problem 2



The figure shows a 1D chain of N atoms. The lattice parameter is a.

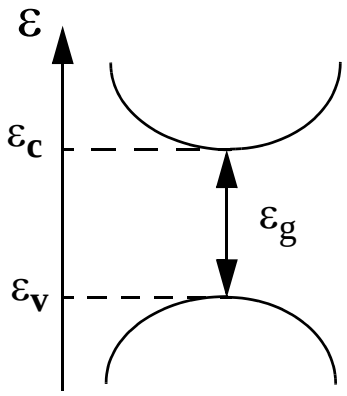
(a) Consider the 1D chain of atoms in the figure. Use the Bloch wave function and periodic boundary conditions to find the spacing of states in k-space.
Show how many distinct states are found within the first Brillouin zone.

(b) Derive the 3D density of states *g(E)*, find the Fermi energy in terms of the electron density *n*, and find the average electron energy at $T = 0\ K$ in the free electron model.

(c) Compute the three lowest energy bands in the empty-lattice approximation in one dimension. Plot these bands in an E(k) versus k diagram, using the reduced-zone scheme. Show qualitatively how these bands are modified in the NFE (nearly free electron) model.

(d) Consider a 2D free electron solid and derive the value for the fermi wave number k_F in the case of two conduction electrons per atom.

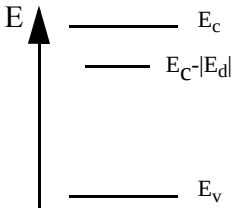
Problem 3



The figure shows a schematic representation of the direct band gap semiconductor InP. The semiconductor is undoped. The band gap is 1.42 eV.
The effective masses are: $m_e^* = 0.07m_e$, $m_h^* = 0.05m_e$
The temperature T = 300 K.

(a) Consider the figure above. Find the concentration of electrons in the conduction band and the concentration of holes in the valence band. Find an expression for the variation of the chemical potential μ as a function of temperature T.

(b) The mobilities of electrons and holes in InP are: $\mu_e = 0.45$ and $\mu_h = 0.01$ m^2/Vs.
Find the electrical conductivity at T = 300 K (mobility $v = \mu E$).
How does the conductivity vary with temperature?



The figure shows the donor level for a hydrogenic dopant in an extrinsic semiconductor.

(c) Find the position of the donor level relative to the bottom of the conduction band for the semiconductor as shown in the figure above, when the effective mass of the electron $m_e^* = 0.1\ m_e$ and the dielectric constant of the semiconductor $\varepsilon = 10\ \varepsilon_0$.
Also estimate the radius r_d of the electron orbit corresponding to the donor state.

(d) Give a physical explanation for the appearance of negative effective electron masses in solids.

Vedlegg (Appendix): Some constants and expressions that may or may not be of use.

$k_B = 1.38 \cdot 10^{-23}$ J/K $e = 1.6 \cdot 10^{-19}$ C $\varepsilon_0 = 8.85 \cdot 10^{-12}$ $s^4A^2kg^{-1}m^{-3}$

$h = 6.63 \cdot 10^{-34}$ Js $m_e = 9.1 \cdot 10^{-31}$ kg $c = 3.0 \cdot 10^8$ m/s

$\hbar = h/2\pi = 1.05 \cdot 10^{-34}$ Js $R_0 = 13.6$ eV $a_0 = 0.53$ Å

$$\left.\begin{aligned} \frac{1}{1-x} &\approx 1 + x + x^2 \ldots \\ \sin x &\approx x - \frac{1}{3!}x^3 \ldots \\ \sqrt{1+x} &\approx 1 + \frac{x}{2} + \ldots \end{aligned}\right\} \quad \textit{for } x << 1$$

$$n(\omega) = \frac{1}{e^{\hbar\omega/k_BT} - 1}$$ *Planck distribution law for phonons (average phonon occupation number)*

$$\omega(k) = 2\sqrt{\frac{K}{M}}\left|\sin\frac{ka}{2}\right|$$ *phonon dispersion relation*

$$g(E) = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}}E^{1/2}$$ *3D free-electron density of states*

$$g(E) = \frac{mL^2}{\hbar^2\pi}$$ *2D free-electron density of states*

$$E_n = -R_0/n^2$$ *energy levels of the hydrogen atom*

$$R_0 = \frac{e^4 m_e}{32\pi^2\varepsilon_0^2\hbar^2}$$ *the Rydberg constant*

$$a_0 = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2}$$ *the Bohr radius*

$$f(E) = \frac{1}{e^{(E-\mu)/k_BT} + 1}$$ *Fermi-Dirac distribution function (Fermi-function)*

$$n = 2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{\frac{3}{2}} e^{-(E_c-\mu)/k_BT} = N_c e^{-(E_c-\mu)/k_BT}$$

$$p = 2\left(\frac{2\pi m_h^* k_B T}{h^2}\right)^{\frac{3}{2}} e^{(E_v-\mu)/k_BT} = N_v e^{(E_v-\mu)/k_BT}$$