Department of physics

Examination paper for TFY4220 Solid State Physics

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Examination time (from - to):	**09:00 – 13:00**
Permitted examination support material:	**C**

Other information

All the subtasks a), b), … are given equal weight.

Language:	**English**
Number of pages (front page excluded):	**3**
Number of pages enclosed (“vedlegg”):	**1**

Informasjon om trykking av eksamensoppgave
Originalen er:

1-sidig ☐ **2-sidig** ☐

sort/kvit ☐ **farger** ☐

Skal ha flervalgsskjema ☐

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Date Signature

NB! Attachment.

Problem 1

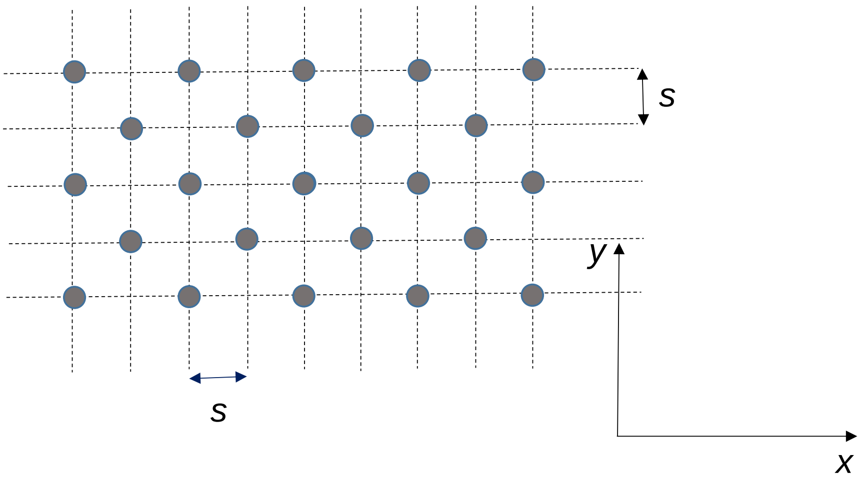

Figure 1.

Figure 1 shows a hypothetical crystal consisting of atoms arranged on a square grid in 2D. *s* denotes the lattice plane spacing in the *x* and *y* directions.

a) What is a primitive cell? Indicate an example of a primitive unit cell in the arrangement shown in Fig. 1. Write down the basis vectors $\mathbf{a}_1$, $\mathbf{a}_2$ of the primitive cell, expressed by *s*, **i** and **j**, where **i** and **j** denote unit vectors in the *x* and *y* directions.

b) Define mathematically the 2-dimensional ”reciprocal lattice” for the lattice of Fig. 1.
Hint: A plane normal of length unity can be employed for convenience.

Explain the connection of the reciprocal lattice to Bragg diffraction (Laue conditions).

c) Sketch the 2-dimensional reciprocal lattice and the first Brillouin zone for the lattice in Fig. 1. How is this zone related to Bragg diffraction?

d) State and explain Bloch’s theorem. Comment shortly on the choice of boundary conditions.

Problem 2

A beam of electrons with kinetic energy 1.0 keV is diffracted by a polycrystalline metal foil. The metal has a simple cubic crystal structure with a lattice constant of 1.0 Å. Given *m*, *q*, *h*, *c*.

a) Calculate the wavelength of the electron beam.
Calculate the Bragg angle for the first order diffraction maximum.

b) Explain briefly the difference between dark field and bright field imaging in transmission electron microscopy (TEM). What is *diffraction contrast*?

Problem 3

Harmonic lattice vibrations for a 1-dimensional chain with a 2-atom basis has the following dispersion relation:

$$\omega^2 = \gamma\left(\frac{1}{M_1}+\frac{1}{M_2}\right) \pm \gamma\sqrt{\left(\frac{1}{M_1}+\frac{1}{M_2}\right)^2 - \frac{4\sin^2\left(\frac{kb}{2}\right)}{M_1 M_2}}$$

Here γ is the effective spring constant, M_1 and M_2 the two masses, and k is the wave number. The distance between the masses is $b/2$ such that b is the repetition distance.

a) Sketch the dispersion relation. Indicate acoustic and/or optical branches. Are all k-values of equal interest?

b) How is the dispersion relation changed under the assumption that the crystal has a finite number of unit cells N? Assume periodic boundary conditions.

c) Sketch qualitatively the density of states D as function of ω. Sketch also the group velocity v_g as function of k.

d) Let $M_1 = M_2$. What is the dispersion relation in this case, and what is the relation between this expression and the dispersion relation for a 1-atom basis?
Hint: Sketch both dispersion relations in the same figure.

e) Derive Dulong-Petit's classical expression for the heat capacity of a crystal. Then write down an integral expression for the internal energy $U(T)$ for the phonon modes in a crystal with density of states $D(\omega)$.

Problem 4

a) Free-electron-model for a three-dimensional system: Show that the highest occupied energy at $T = 0$ level for N free electrons in a volume V can be written

$$E(N) = A\left(3\pi^2 \frac{N}{V}\right)^{2/3},$$

where A is a constant. Find an expression for A.

b) Use the expression from a) to argue for the following expression for the "density of states» for conduction electrons in metals:

$$D(E) = \frac{V}{2\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{\frac{3}{2}}(E - E_0)^{\frac{1}{2}}.$$

Then show by integrating over an energy interval ΔE that the effective mass m^* is given by the expression

$$m^* = \frac{\hbar^2}{2\Delta E}(3\pi^2 N / V)^{2/3}.$$

Sodium forms a body-centered cubic lattice with lattice constant a = 4.25 Å. Sodium is an alkali metal (1^{st} column of the periodic table).

c) Why is it a better assumption for sodium than for most other metals that the conduction electrons can be described by a free-electron-model? Experimentally one can measure the energy range ΔE of the conduction electrons, and one finds ΔE = 3.0 eV. Calculate the effective mass m^* for conduction electrons in sodium!

Attachment

Some expressions and constants that may prove useful:

Physical constants

One mole: $M(^{12}C) = 12$ g	1u = $1.6605 \cdot 10^{-27}$ kg	$N_A = 6.0221 \cdot 10^{23}$ mol^{-1}
$k_B = 1.3807 \cdot 10^{-23}$ J/K	$R = N_A\, k_B = 8.3145$ J mol^{-1} K^{-1}	0°C = 273.15 K
$\varepsilon_0 = 8.8542 \cdot 10^{-12}$ C^2/Nm^2	$\mu_0 = 4\pi \cdot 10^{-7}$ N/A^2	
$e = 1.6022 \cdot 10^{-19}$ C	$m_e = 9.1094 \cdot 10^{-31}$ kg	
$c = 2.998 \cdot 10^8$ m/s	$h = 6.6261 \cdot 10^{-34}$ Js	$g = 9.81$ m/s^2

Mean occupation number for fermions

$$f(E) = \frac{1}{\mathrm{e}^{(E-\mu)/k_B T} + 1}$$

Mean occupation number for fermions

$$f(\omega) = \frac{1}{\mathrm{e}^{\hbar\omega/k_B T} - 1}$$

Density of states for free electrons ($E(k) = E_0 + \hbar^2 k^2 / 2m$):

$$D(E) = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}} (E - E_0)^{\frac{1}{2}}$$

Phonon dispersion relation for a one-atomic basis:

$$\omega^2 = \frac{4\alpha}{M}\sin^2\frac{ka}{2}$$