Forside

Department of Physics

Examination paper for TFY4220 Solid State Physics

Examination date: 3 June 2021

Examination time (from-to): 09:00 - 13:00

Permitted examination support material: All support material is allowed.

Academic contact during examination: Prof. Dag W. Breiby
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OTHER INFORMATION

Saving: Answers written in Inspera are automatically saved every 15 seconds.

Cheating/Collaboration: The exam is an individual, independent work.

Notifications: If there is a need to send a message to the candidates during the exam (e.g. if there is an error in the question set), this will be done by sending a notification in Inspera. A dialogue box will appear. You can re-read the notification by clicking the bell icon in the top right-hand corner of the screen. All candidates will also receive an SMS to ensure that nobody misses out on important information. Please keep your phone available during the exam.

Weighting: The multiple choice questions will be equally weighted, with 1 point for a correct answer and 0 points for a wrong answer.

Submission: Your answer will be submitted automatically when the examination time expires and the test closes, if you have answered at least one question. This will happen even if you do not click "Submit and return to dashboard" on the last page of the question set. You can reopen and edit your answer as long as the test is open. If no questions are answered by the time the examination time expires, your answer will not be submitted.

Withdrawing from the exam: If you wish to submit a blank test/withdraw from the exam, go to the menu in the top right-hand corner and click "Submit blank". This can not be undone, even if the test is still open.

Accessing your answer post-submission: You will find your answer in Archive when the examination time has expired.

1 Task

What is the crystal structure ofthe CsCl structure?

Select one alternative:

- Simple cubic (sc)
- 4 Cl^- ions and 1 Cs^+ ion
- $0.5(f_{Cs^+} + f_{Cl^-})$
- 1 Na^+ ion and 1 Cl^- ion
- Body centered cubic (bcc)

Maximum marks: 1

2 Task

A diffraction peaks is observed at a scattering angle $2\theta = 43.2°$ using X-ray radiation with a wavelength of 1.542 Å. What is the corresponding lattice spacing d?

Select one alternative:

- 2.26 Å
- 20.9 Å
- 2.09 Å
- 21.2 Å
- 24.2 Å

Maximum marks: 1

3 Task

Consider the 1D monatomic chain in the harmonic lattice model as described in the lectures. Given that the mass M = 42 u and the force constant γ = 1.0 N/m. What is the maximum frequency ω_{max} that can be sustained in the lattice?

Select one alternative:

- 4.3 THz
- 7.6 THz
- 3.8 THz
- ✓ 8.7 THz
- 10.7 THz

Maximum marks: 1

4 Task

A semiconductor piece measures 10 x 100 x 100 nm^3. The effective electron mass is 1.12 m_e. What is the number of states per unit energy 0.2 eV above the conduction band edge?

Select one alternative:

- $2.28 \cdot 10^6$ eV^{-1}
- $2.54 \cdot 10^5$ eV^{-1}
- $2.28 \cdot 10^4$ eV^{-1}
- ✓ $3.60 \cdot 10^5$ eV^{-1}
- $2.54 \cdot 10^6$ eV^{-1}

Maximum marks: 1

5 Task

Consider a BCC material with one conduction electron per primitive cell, in the free electron Fermi gas model. The lattice parameter a = 4.2 Å.

What is the magnitude of the Fermi wave vector?

Select one alternative:

- 0.22 1/Å
- 0.75 1/Å
- 2.18 1/Å
- ✓ 0.93 1/Å
- 1.17 1/Å

Maximum marks: 1

6 Task

A material with a bcc unit cell and one atom per Bravais point has a lattice constant of 0.228 nm.

What is the number density of atoms in the (110) plane?

Select one alternative:

- $38 \cdot 10^{12}$ mm^{-2}
- ✓ $27 \cdot 10^{12}$ mm^{-2}
- $24 \cdot 10^{12}$ mm^{-2}
- $17 \cdot 10^{12}$ mm^{-2}
- $12 \cdot 10^{12}$ mm^{-2}

Maximum marks: 1

7 Task

Assuming that the primitive translation vectors of a lattice are given by $\mathbf{a} = \sqrt{2}(\hat{\mathbf{x}} - \hat{\mathbf{y}}); \mathbf{b} = \sqrt{2}(\hat{\mathbf{x}} + \hat{\mathbf{y}}); \mathbf{c} = 3\hat{\mathbf{z}}$.

What is the corresponding volume of the reciprocal space unit cell?

Select one alternative:

- $\frac{\pi}{6}$
- $\frac{4}{9}\pi^3$
- $\frac{2}{3}\pi^3$
- $\frac{2\pi}{27}$
- $\frac{8}{27}\pi^3$

Maximum marks: 1

8 Task

According to the free electron Fermi gas model in 3D, the density of states is proportional to the square root of the energy, $D(E) \propto \sqrt{E}$. The kinetic energy of a 3D gas of N free electrons at T = 0 K is $U_0 = \frac{3}{5} N E_F$. Consider the electrons filling the states up to an energy of 20% of the Fermi energy, i.e. $E_F/5$, what is their average energy at T = 0 K?

Select one alternative:

- $\frac{3}{5} E_F$
- $\frac{3}{125} E_F$
- ✓ $\frac{3}{25} E_F$
- $\frac{2}{7} E_F$
- E_F

Maximum marks: 1

9 Task

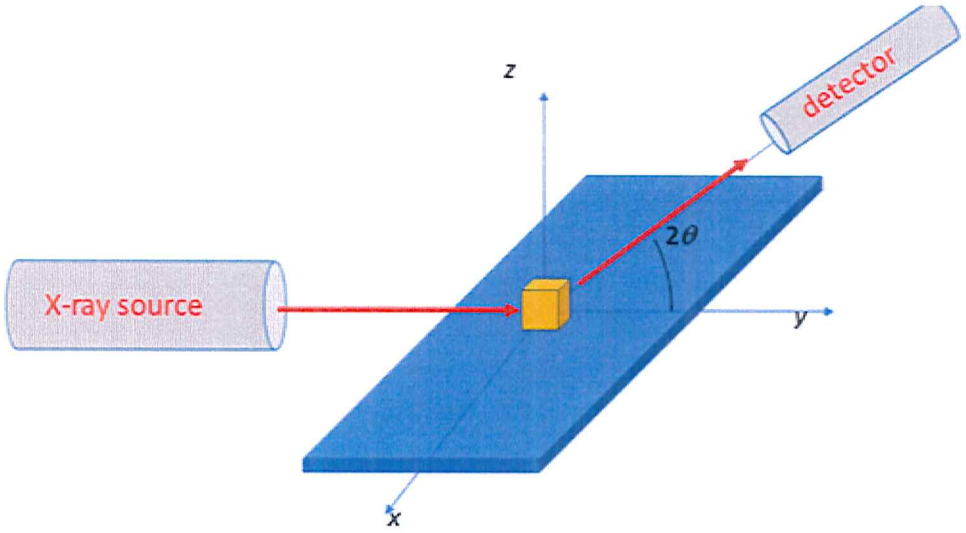
Thermodynamically, the pressure is related to the energy by $p = -\frac{\partial E}{\partial V}|_{N\text{const}}$. One can thus derive that because of the volume dependence of the Fermi energy, $E_F \propto (\frac{N}{V})^{2/3}$, the electron gas exerts a pressure.
The volume dependence of the pressure of the free electron Fermi gas is given by:
Select one alternative:

- $p \propto V^{-5/3}$
- $p = 0$
- $p \propto V^{-1}$
- $p \propto V$
- $p \propto V^{-7/5}$

Maximum marks: 1

10 Task

A single crystal is oriented in the orthogonal laboratory coordinate system $(\mathbf{x}, \mathbf{y}, \mathbf{z})$, where $\mathbf{z}$ is the vertical direction. The crystal belongs to the monoclinic crystal system with primitive translation vectors oriented with $\mathbf{a} = a\hat{\mathbf{x}}$, $\mathbf{b} = b\hat{\mathbf{y}}$ and $\mathbf{c} = c\cos(\beta)\hat{\mathbf{x}} + c\sin(\beta)\hat{\mathbf{z}}$. The incoming X-ray beam is directed along $\hat{\mathbf{y}}$. The point detector (sensor) is placed in the (*y*,*z*) plane, and is at an angle 2θ with respect to the horizontal, as shown in the figure.



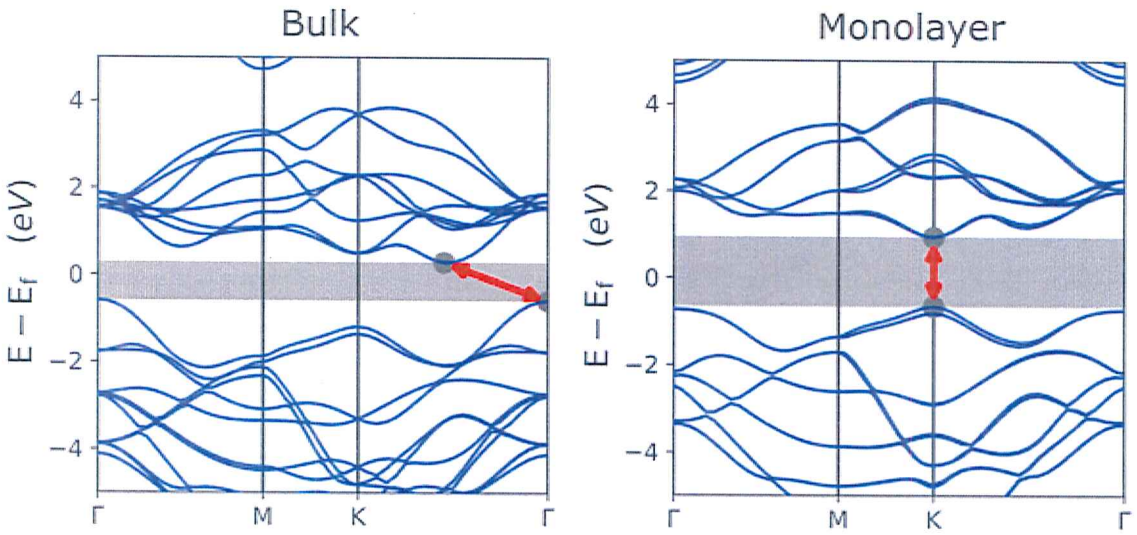
Define a reciprocal coordinate system (*X*, *Y*, *Z*) with $\hat{\mathbf{X}}||\hat{\mathbf{x}}; \hat{\mathbf{Y}}||\hat{\mathbf{y}}; \text{and } \hat{\mathbf{Z}}||\hat{\mathbf{z}}$.
Which expression gives a correct representation of the scattering vector **Q** that can be measured with the given geometry?
Select one alternative:

- The question cannot be answered without further information.
- $\mathbf{Q} = \frac{2\pi}{\lambda} < 0, \cos(2\theta), \sin(2\theta) >$
- ✓ $\mathbf{Q} = \frac{2\pi}{\lambda} < 0, \cos 2\theta - 1, \sin 2\theta >$
- $\mathbf{Q} = \frac{2\pi}{\lambda}\hat{\mathbf{Y}}$
- $\mathbf{Q} = \mathbf{G}_{100}$

Maximum marks: 1

11 Task

A bulk material that can be exfoliated (turned into a single molecular layer, "2D" monolayer) has electronic band structure as shown in the figure. Which statement is correct?



Select one alternative:

- The bandgaps are equally large.
- The monolayer bandgap is direct. The monolayer bandgap is smaller than the bulk bandgap.
- The monolayer bandgap is indirect. The monolayer bandgap is smaller than the bulk bandgap.
- The monolayer bandgap is indirect. The monolayer bandgap is larger than the bulk bandgap.
- ✓ The monolayer bandgap is direct. The monolayer bandgap is larger than the bulk bandgap.

Maximum marks: 1

12 Task

In a 1D phonon collision process two transverse acoustic phonons combine to give one longitudinal acoustic phonon. The dispersion relations for transverse and longitudinal phonons are

$\omega_{TA} = \omega_0 |\sin(ka/2)|$

and

$\omega_{LA} = 2\omega_0 |\sin(ka/2)|$.

For which of the following acoustic phonon wave vectors k_1 and k_2 is the process allowed?

Select one alternative:

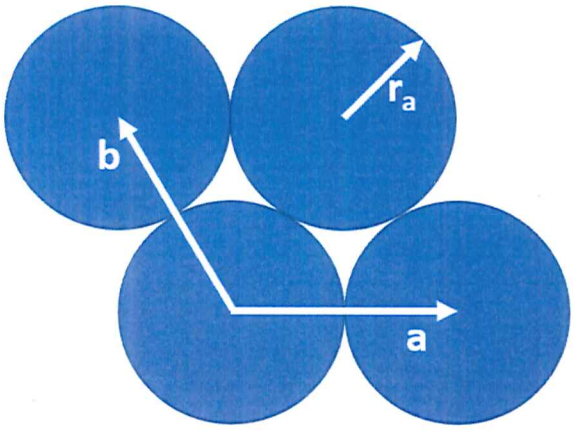
- $k_1 = k_2 = \frac{2\pi}{5a}$
- ✓ $k_1 = k_2 = \frac{2\pi}{3a}$
- $k_1 = -k_2 = \frac{\pi}{a}$
- $k_1 = k_2 = \frac{\pi}{a}$
- $k_1 = -k_2 = \frac{2\pi}{3a}$

Maximum marks: 1

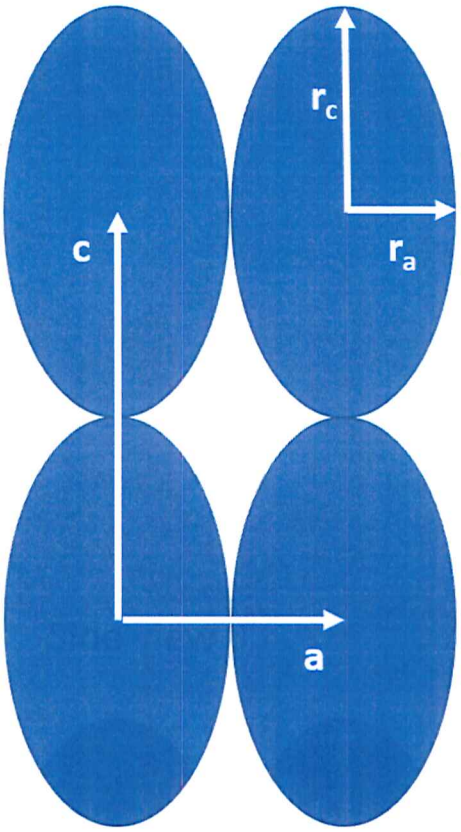
13 Task

A molecular crystal consists of densely packed compact spheroide-shaped molecules with radii $r_a = r_b < r_c$. There is one molecule per unit cell. The crystal structure is hexagonal, the molecules are oriented with their long axis along the unit cell *c*-axis, and $c = 2r_c$. In the (*a*,*b*)-plane, the molecules are hexagonally dense packed, with $a = b = 2r_a$.

View along *c* axis



Side view



What is the filling ratio (or "packing ratio") of the structure?

Select one alternative:

- 0.58
- √ 0.60
- 0.63
- 0.68
- 0.65

Maximum marks: 1

14 Task

In a diffraction experiment with monochromatic X-ray radiation, an unknown substance is measured. A point detector (sensor) is used to measure the intensity as function of scattering angle 2θ.

First, a scan is done of the scattering angle 2θ, giving a diffraction pattern $I(2\theta)$ consisting of many sharp Bragg peaks.

Second, the point detector is kept stationary at a position $2\theta_0$ which gave a strong measured intensity in the first scan. While the sample is rotated through $360°$, the scattered intensity measured by the detector exhibits only a few sharp diffraction points with low intensity in-between.

Which alternative can explain these observations?

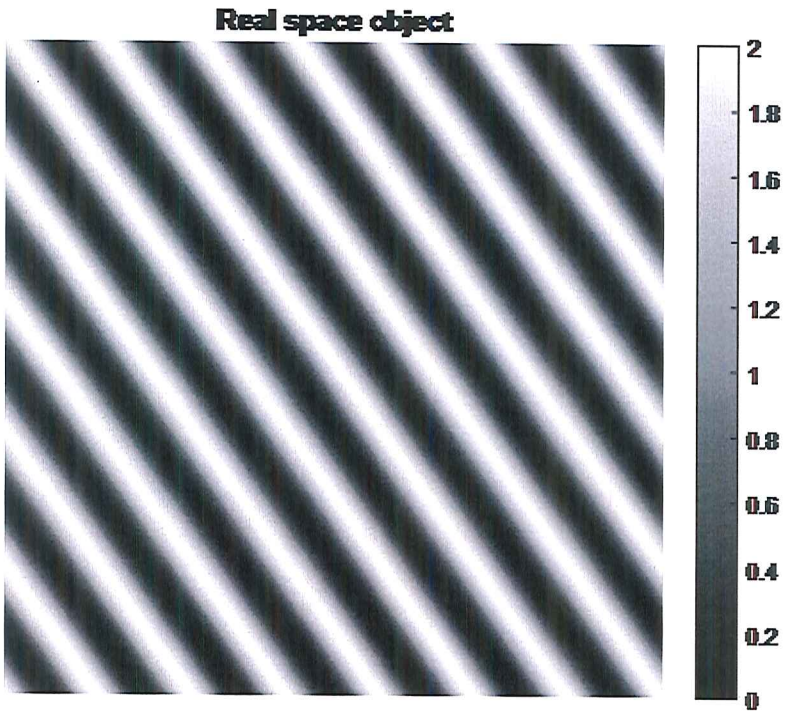
Select one alternative:

- The sample is a liquid.
- The sample is a gas.
- The sample is an isotropic crystalline powder.
- ✓ The sample is a single crystal.
- The sample is amorphous.

Maximum marks: 1

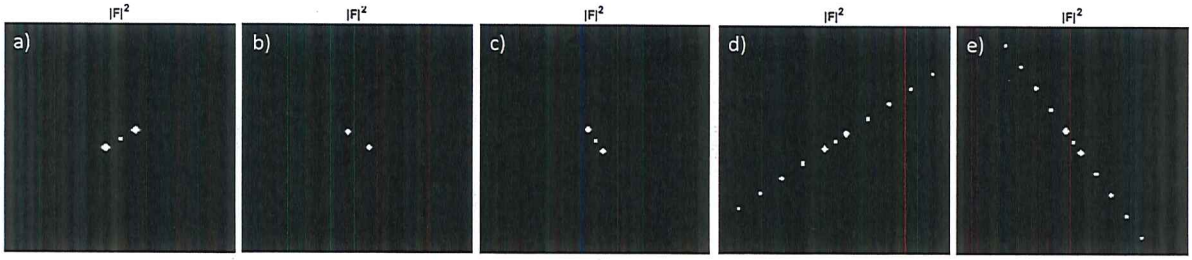
15 Task

A real space density distribution (extending to infinity in all directions) is shown as the "Real space object":



The density variation $\rho(x, y)$ is given by the square of a sinusoidal variation on a flat background, ensuring that the density is everywhere ≥ 0.

Which of the suggested patterns depicts the corresponding absolute square ("power spectrum") of the Fourier transform, i.e., $|FT(\rho)|^2$?



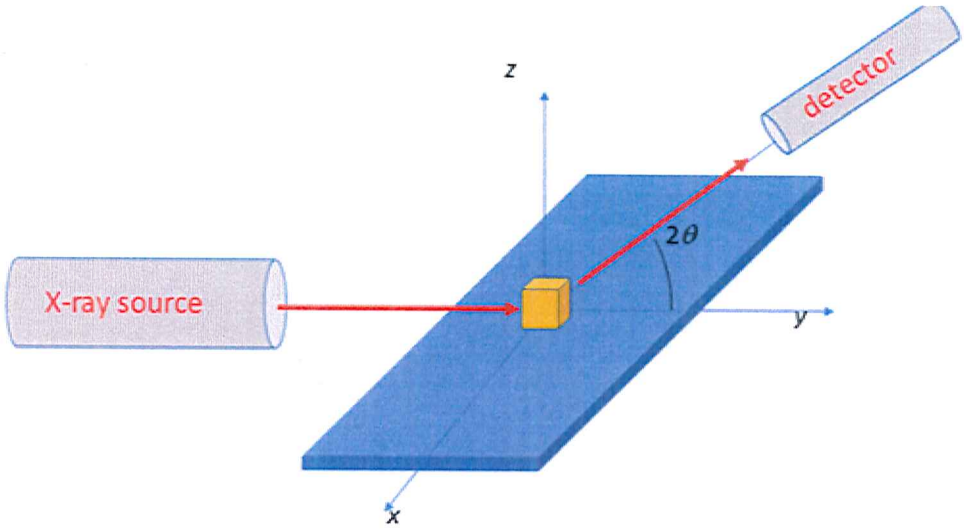
Select one alternative:

- pattern a)
- pattern b)
- pattern c)
- pattern d)
- pattern e)

Maximum marks: 1

16 Task

A single crystal is oriented in the orthogonal laboratory coordinate system $(\mathbf{x}, \mathbf{y}, \mathbf{z})$, where $\mathbf{z}$ is the vertical direction. The crystal belongs to the monoclinic crystal system with primitive translation vectors oriented with $\mathbf{a} = a\hat{\mathbf{x}}$, $\mathbf{b} = b\hat{\mathbf{y}}$ and $\mathbf{c} = c\cos(\beta)\hat{\mathbf{x}} + c\sin(\beta)\hat{\mathbf{z}}$. The incoming X-ray beam is directed along $\hat{\mathbf{y}}$. The point detector (sensor) is placed in the (*y*,*z*) plane, and is at an angle 2θ with respect to the horizontal, as shown in the figure.



Define a reciprocal coordinate system (*X*, *Y*, *Z*) with $\hat{\mathbf{X}}||\hat{\mathbf{x}}; \hat{\mathbf{Y}}||\hat{\mathbf{y}}; \text{and } \hat{\mathbf{Z}}||\hat{\mathbf{z}}$.
Which expression gives a correct representation of the reciprocal lattice vector $\mathbf{G}_{100}$?
Select one alternative:

- $\mathbf{G}_{100} = \frac{2\pi}{\lambda} < 0, \cos 2\theta - 1, \sin 2\theta >$
- $\mathbf{G}_{100} = 2\pi < \frac{1}{a}\sin\beta, 0, \frac{1}{c}\cos\beta >$
- The question cannot be answered without further information.
- ✓ $\mathbf{G}_{100} = \frac{2\pi}{a} < 1, 0, -\cot(\beta) >$
- $\mathbf{G}_{100} = \frac{2\pi}{a} < \sin\beta, 0, -\cos\beta >$

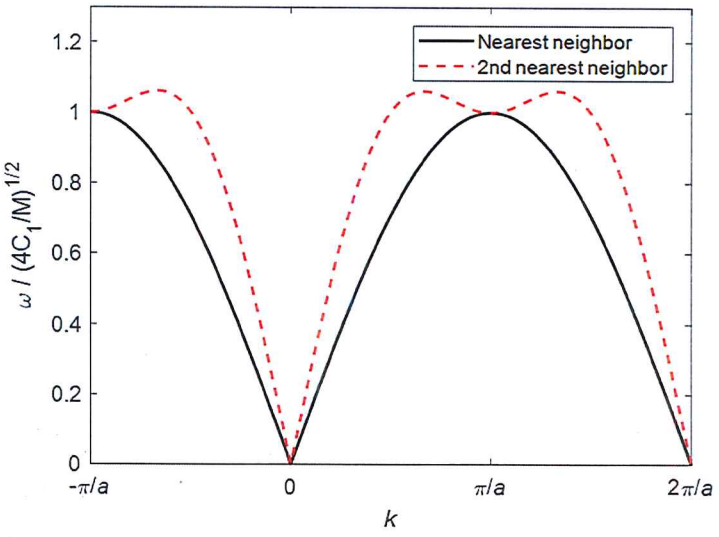
Maximum marks: 1

17 Task

It can be shown that the dispersion relation for phonons on a 1D string with next nearest neighbors included is given by

$$\omega^2 = \frac{4}{M}(C_1 \sin^2 \frac{ka}{2} + C_2 \sin^2 ka).$$

Here, M is the mass of the atoms, C_1 and C_2 are force constants, a denotes the interatomic spacing and k is the wave vector.
Now assume $C_2 = ½\ C_1$. The dispersion relation is plotted in the figure (red broken line). For comparison, the standard nearest neighbor dispersion relation with the same values for C_1 and M is also given.



What is the limiting value of the group velocity for large wavelengths?

Select one alternative:

- 0
- $a\sqrt{\frac{C_1}{3M}}$
- $3a\sqrt{\frac{C_2}{M}}$
- $\frac{2\pi a^2}{L}\sqrt{\frac{3C_1}{M}}$, where L is the length of the object
- $a\sqrt{\frac{3C_1}{M}}$

Maximum marks: 1

18 Task

In a rectangular 2D lattice with a = 6.90 Å and b = 4.50 Å, three phonons each having wavevector $\mathbf{k}$ = <0.80, 0.90> Å^{-1} merge to become one phonon. (Assume that the energy is conserved).

What is the reduced zone scheme representation of the **k**-vector of the resulting phonon?

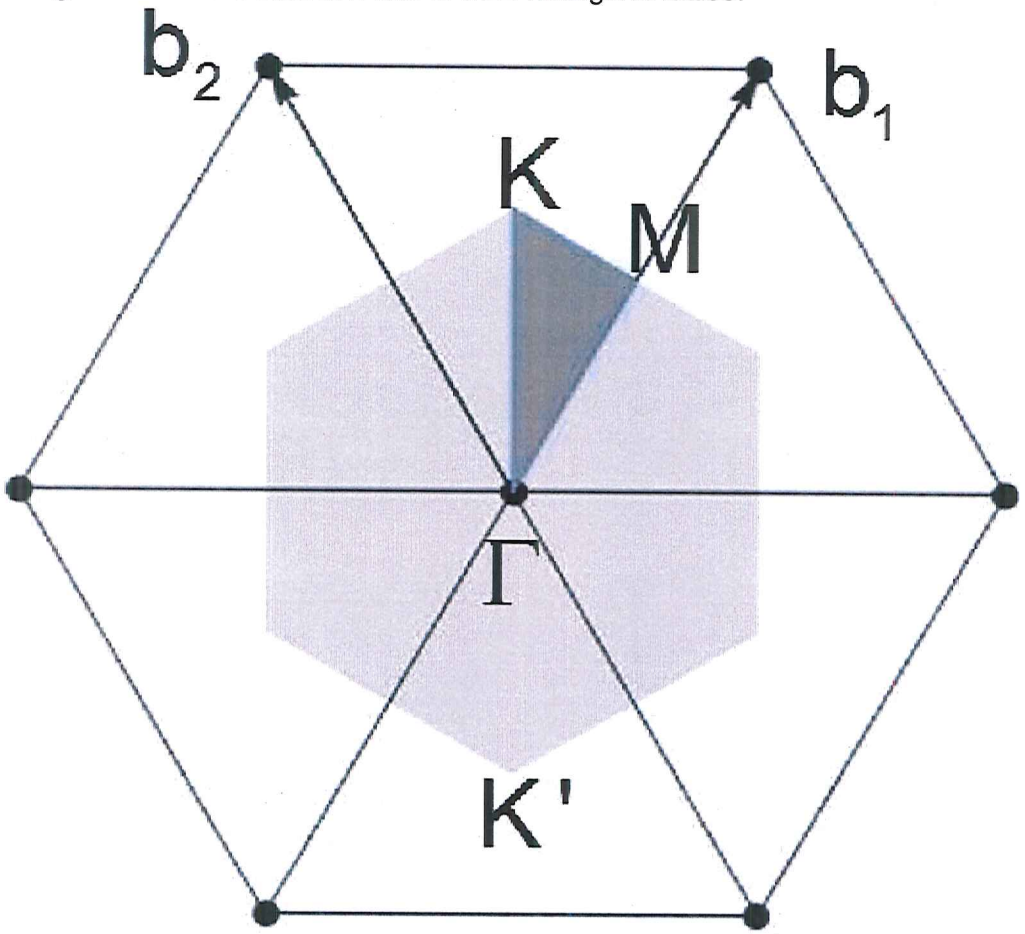
Select one alternative:

- $\mathbf{k} = \mathbf{0}$ Å^{-1}
- $\mathbf{k} =< -0.22, 0.43 >$ Å^{-1}
- $\mathbf{k} =< -0.44, -0.51 >$ Å^{-1}
- $\mathbf{k} =< 0.33, 0.10 >$ Å^{-1}
- ✓ $\mathbf{k} =< -0.33, -0.10 >$ Å^{-1}

Maximum marks: 1

19 Task

The figure shows the Brillouin zone of a 2D hexagonal lattice.



According to the free electron Fermi model, what is the energy of electrons at the *M* point in reciprocal space (see sketch)?
Assume a = 2.9 Å.
(Take the energy to be zero at the Brillouin zone center).

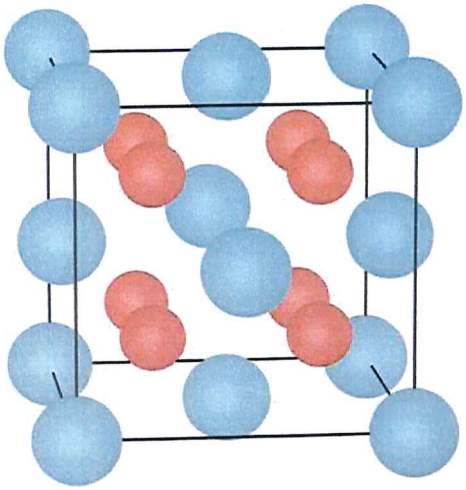
Select one alternative:

- 3.3 eV
- 2.0 eV
- 4.0 eV
- ✓ 6.0 eV
- 8.0 eV

Maximum marks: 1

20 Task

A unit cell with the generic structure AB_2 is face centered cubic (FCC), with A^{2+} in the corners and the face centres, and B^- on the space diagonals at a distance of 1/4 of the diagonal from all the corners.



Which alternative gives a correct description of the basis (associated with each Bravais lattice point) for this FCC unit cell?

Select one alternative:

- The basis is:
  A^{2+} at (1,0,0)
  B^- at (-1/4, -1/4, -1/4) and (1/4, 1/4, 1/4)

- The basis is:
  A^{2+} at (0,0,0)
  B^- at (1/4, 1/4, 1/4)

- The basis is:
  A^{2+} at (0,0,0), (1/2, 1/2, 0), (1/2, 0, 1/2) and (0, 1/2, 1/2)
  B^- at (1/4, 1/4, 1/4), (1/4, 3/4, 1/4), (3/4, 1/4, 1/4), (3/4, 3/4, 1/4); and (1/4, 1/4, 3/4), (1/4, 3/4, 3/4), (3/4, 1/4, 3/4), (3/4, 3/4, 3/4)

- The basis is:
  A^{2+} at (0,0,0)
  B^- at (-1/4, -1/4, -1/4) and (3/4, 3/4, 3/4)

- The structure is centred, so the basis is extinct.

Maximum marks: 1

21 Task

A 1D electronic wave function is given by $\psi_k(x) = \exp(ikx)\exp(-B\sin^2(\sqrt{\kappa}x))$, where B and $\kappa$are constants.
Which of the expressions for κ below satisfies the requirements for $\psi_k(x)$ to describe an electronic state in a periodic potential with repetition distance *a*?

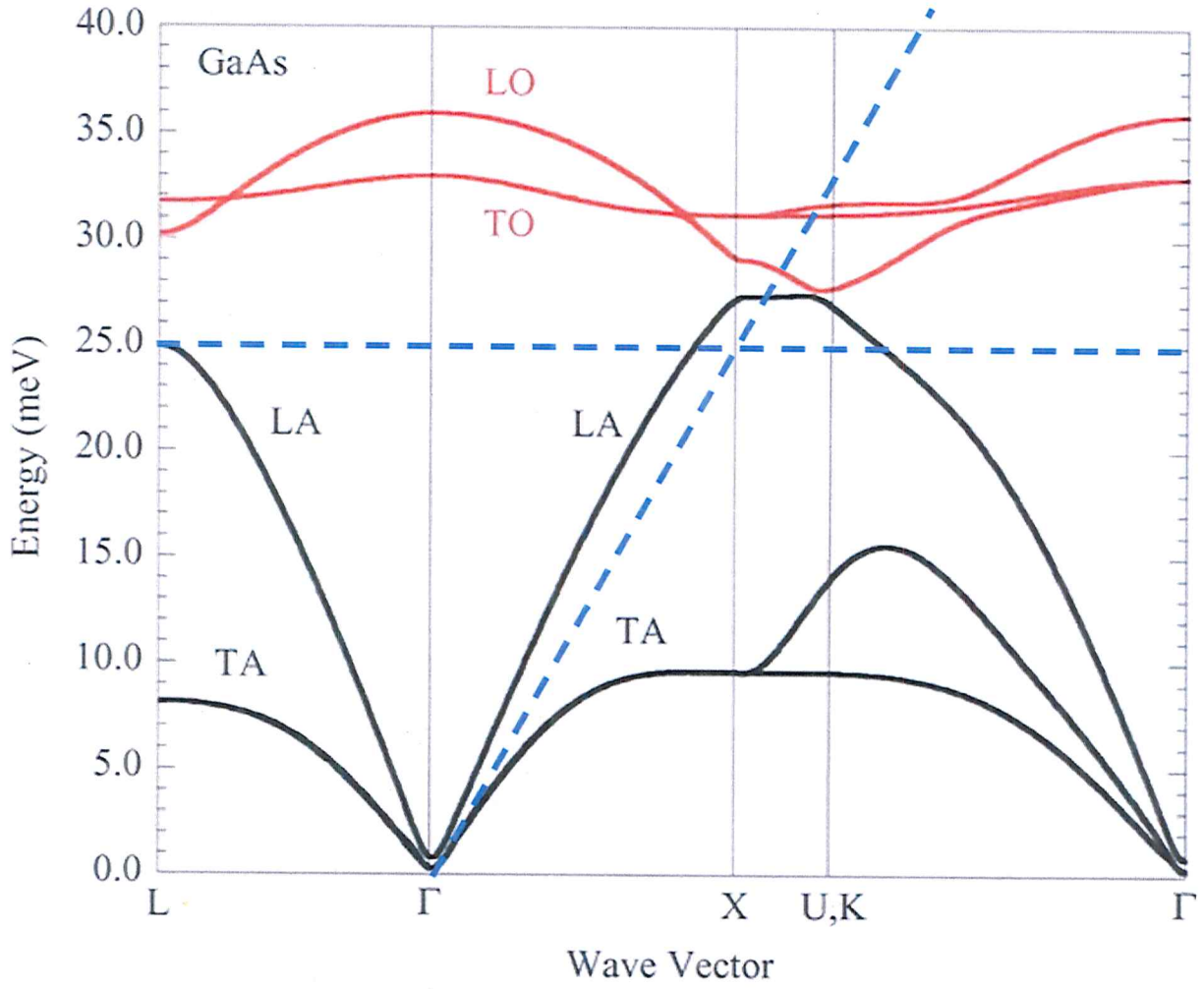
Select one alternative:

- All alternatives satisfy the requirements.
- $\kappa = 2\pi/a$
- $\kappa = (\pi/a)^2$ ✓
- $\kappa = (2\pi/a)^2$ ✓
- $\kappa = \pi/a$

By error, both alternatives are correct.

Maximum marks: 1

22 Task



The figure shows the dispersion relations for lattice vibrations in an fcc semiconductor, which is cubic with a = 5.65 Å. Note that the X-point of the fcc structure is at $2\pi/a$.

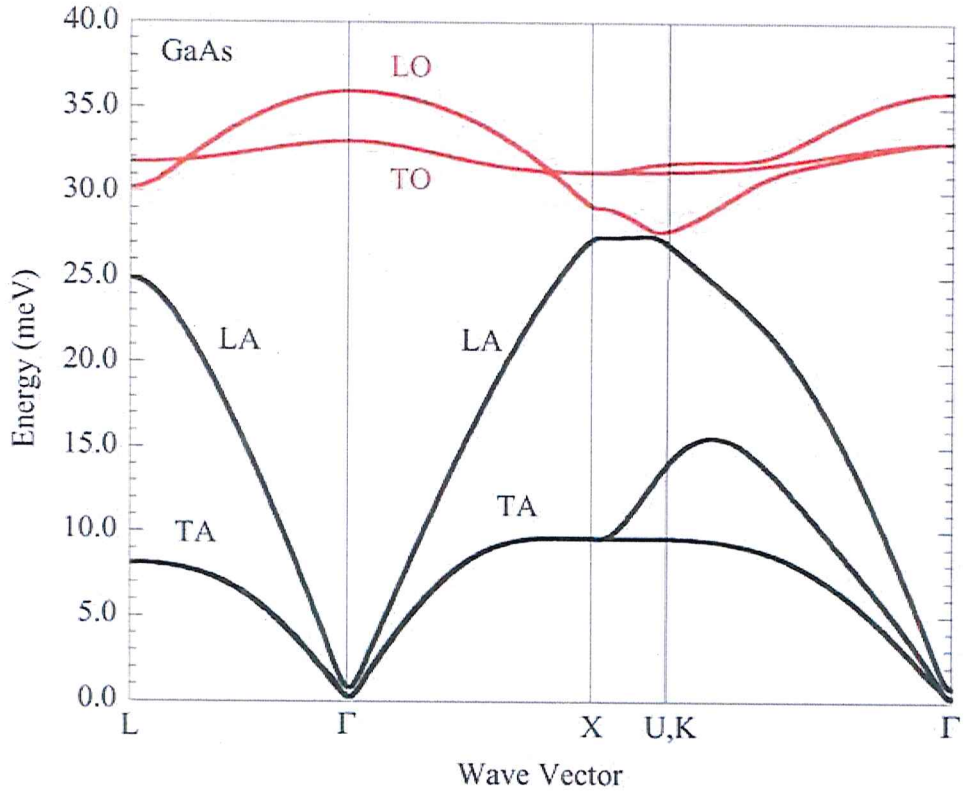
Find the approximate speed of sound corresponding to the TA mode, using the indicated intersection at E = 25.0 meV.

Select one alternative:

- 3810 m/s
- 3610 m/s
- ✓ 3410 m/s
- 3310 m/s
- 4010 m/s

Maximum marks: 1

23 Task



The figure shows the dispersion relations for lattice vibrations in GaAs, which is cubic with a = 5.65 Å. Find the approximate wave length (in units of micrometer) of the most strongly absorbed electromagnetic radiation by LO phonons.

Select one alternative:

- 0.99
- 0.53
- 0.64
- 4.4
- √ 35

Maximum marks: 1

24 Task

A cubic crystal structure containing two elements with atomic form factors f_A and f_B has structure factor given by

$S_{hkl} = f_A(1 + (-1)^{h+k+l}) + f_B \exp(\frac{\pi}{2} i(h+k+l)) \cdot [1 + (-1)^{h+k} + (-1)^{k+l} + (-1)^{l+h}]$

.

Which expression is proportional to the scattered *intensity* in the special case of *h, k* and *l* all odd? Assume $f_A, f_B \in \mathbb{R}$.

Select one alternative:

- 0
- $16 f_B^2$
- $(f_A + f_B)^2$
- $4i(-1)^n f_B^2, n \in \mathbb{Z}$
- $(f_A + 4f_B)^2$

Maximum marks: 1

25 Task

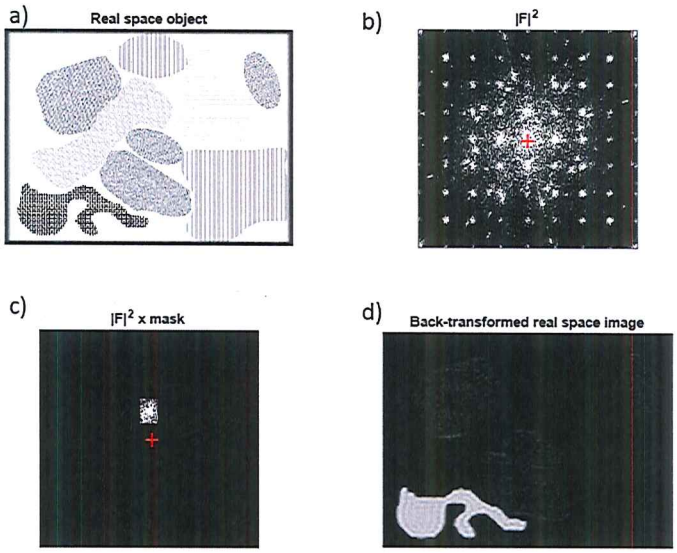


Figure illustrating Fourier filtering. a) Artificially generated object. b) Fourier transform squared of the object shown in (a). c) Illustration of a mask applied to the Fourier transform.
d) Resulting image after inverse Fourier transformation of the masked signal.
The red crosses mark the origin of Fourier space ($k_x = k_y = 0$).

Which of the following statements is *incorrect*?
Select one alternative:

- The mask transmits spatial frequencies that are found (almost) exclusively in the part of the object lighting up in fig. d).
- ✓ The process in the figure corresponds to bright-field imaging.
- The process in the figure corresponds to dark-field imaging.
- Points far away from the origin of Fourier space (cf. fig. b), correspond to plane waves with high spatial frequency.
- In general, a reciprocal lattice is the Fourier transform of a real space lattice.

Maximum marks: 1

26 Task

Diamond has a Debye temperature of about 1860 K. Estimate the heat capacity of diamond at 140 K using the Debye formalism.

Select one alternative:

- 1.2 J mol^{-1} K^{-1}
- √ 0.8 J mol^{-1} K^{-1}
- 24.9 J mol^{-1} K^{-1}
- 1.7 J mol^{-1} K^{-1}
- 6.1 J mol^{-1} K^{-1}

Maximum marks: 1