NTNU – Trondheim
Norwegian University of
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Department of Physics

Examination paper for TFY4225 Nuclear and Radiation Physics

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Examination date: 18.12.2014

Examination time (from-to): 9.00-13.00

Permitted examination support material (code C):

- **Simple specified calculator**
- **Barnett & Cronin: Mathematical Formulae**
- **Rottmann: Matematische Formelsammlung**

Other information: Each sub-question (1a, 1b, 3 etc) carries equal weight in the evaluation. Exam might be answered in English or Norwegian.

Language: English

Number of pages (front page excluded): 3

Number of pages enclosed: 2

Checked by:

Date Signature

CONSTANTS

Speed of light	c	2.99792458×10^{8} m/s
Charge of electron	e	1.602189×10^{-19} C
Boltzmann constant	k	1.38066×10^{-23} J/K
		8.6174×10^{-5} eV/K
Planck's constant	h	6.62618×10^{-34} J · s
		4.13570×10^{-15} eV · s
	$\hbar = h/2\pi$	1.054589×10^{-34} J · s
		6.58217×10^{-16} eV · s
Gravitational constant	G	6.6726×10^{-11} N · m^2/kg^2
Avogadro's number	N_A	6.022045×10^{23} mole^{-1}
Universal gas constant	R	8.3144 J/mole · K
Stefan-Boltzmann constant	σ	5.6703×10^{-8} W/m^2 · K^4
Rydberg constant	R_∞	1.0973732×10^{7} m^{-1}
Hydrogen ionization energy		13.60580 eV
Bohr radius	a_0	5.291771×10^{-11} m
Bohr magneton	μ_B	9.27408×10^{-24} J/T
		5.78838×10^{-5} eV/T
Nuclear magneton	μ_N	5.05084×10^{-27} J/T
		3.15245×10^{-8} eV/T
Fine structure constant	α	1/137.0360
	hc	1239.853 MeV · fm
	$\hbar c$	197.329 MeV · fm
	$e^2/4\pi\epsilon_0$	1.439976 MeV · fm

PARTICLE REST MASSES

	u	MeV/c^2
Electron	5.485803×10^{-4}	0.511003
Proton	1.00727647	938.280
Neutron	1.00866501	939.573
Deuteron	2.01355321	1875.628
Alpha	4.00150618	3727.409
$\pi^{\pm}$	0.1498300	139.5669
π^{0}	0.1448999	134.9745
μ	0.1134292	105.6595

CONVERSION FACTORS

1 eV = 1.602189×10^{-19} J

1 b = 10^{-28} m^2

1 u = 931.502 MeV/c^2
= 1.660566×10^{-27} kg

1 Ci = 3.7×10^{10} decays/s

Upper region of the periodic table of the elements:

1 H							2 He
3 Li	4 Be	5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg	13 Al	14 Si	15 P	16 S	17 Cl	18 Ar

Atomic masses of some isotopes:

^{18}N: 18.014079 u
^{18}O: 17.999161 u
^{18}F: 18.000938 u
^{18}Ne: 18.005708 u
^{18}Na: 18.02597 u

Order of the lowest shells in the Woods-Saxon with spin orbit coupling model: $1s_{1/2}$, $1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$, $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$, $1g_{9/2}$

The reduced de-Brogli wavelength:

$$\lambda\!\!\!^{-} = 1/k \text{ where } k = p/\hbar$$

Remember:

- **Relevant information that might become useful in solving the problems are stated on page 1 of the exam !!**
- **Each subproblem (1a, 2b, 3 etc) carries equal weight in the evaluation.**

Problem 1

^{18}F is the most used radioisotope for positron emission tomography (PET).

1 a)

- Write down the full reaction for β^+ decay of ^{18}F and the expression for the reaction energy Q.
- Calculate the value of Q.
- What other decay-process is possible and why?

1 b) Spin-parity state of ^{18}F is 1^+.

- Discuss this value with relation to the shell model.
- Determine the spin-parity state of the daughter nucleus in the above decay.
- Classify the transition in terms of degree of allowedness/forbiddenness and Fermi vs Gamow-Teller type.

Problem 2

^{18}F is usually produced in the following nuclear reaction:

$$^{18}O(p,n)^{18}F$$

with protons accelerated in a cyclotron.

2 a)

- Describe how the cyclotron works (at non-relativistic particle velocities).
- Derive the expression for the cyclotron frequency. The centripetal force is given by mv^2/r
- Give an expression for the final proton kinetic energy.
- If the cyclotron has radius 79 cm, what is the required magnetic field to reach 30 MeV for the protons?

2 b)

- Set up the expression for energy conservation of the nuclear reaction stated above, and express it as a function of the reaction energy Q.
- Calculate the value of Q.
- Assume the target is at rest, derive an expression for the kinetic energy available for the reaction. The Center of Mass (CM) reference frame is defined by $\sum p_{i,CM} = 0$

2 c)

- Calculate the semi-classical reaction cross section for the nuclear reaction above, for protons at 30 MeV. Use R_0 = 1.2 fm (=1.2 10^{-15} m).
- Calculate the Coulomb potential energy at closest hard sphere distance between the target and projectile, and compare this to the reaction energy Q.
- How much will the Coulomb repulsion change the cross section according to the classical hard sphere model?

Problem 3

In a typical PET-exam the patient is injected with an activity of 400 MBq of ^{18}F .

- Calculate the committed effective dose for the patient. Assume 100 % probability for β^+ decay, uniform distribution of radiation in the body and no physiological clearance. Halflife of ^{18}F is 109 minutes.
- Compare the resulting dose to the recommended maximum annual dose for the general public.

Problem 4

4 a)

- Describe the main principles of PET imaging.
- Discuss possible advantages compared to single photon emission computed tomography (SPECT).

4 b)

- Describe the possible interaction processes between the 511 keV photon and matter.
- The following data of a PET-detector element is given:
  - scintillator:
    - linear attenuation coeff: 50 m^{-1}
    - thickness 2.5 cm.
    - Energy conversion factor: 38000 photons/MeV.
  - PMT:
    - geometric efficiency: 0.2
    - photo-electron efficiency: 0.9
- Calculate the energy resolution of this detector. (FWHM = 2.355 σ).

Problem 5

5 a) Analyse the momentum transfer from a moving heavy charged particle to an electron initially at rest, as result of the Coulomb force: $\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{1}{r^2}\hat{r}$

- Show that the following expression holds for the energy transferred in the collision (Q), and explain the terms:

$$Q = \left(\frac{Ze^2}{4\pi\varepsilon_0}\frac{2}{bV}\right)^2 \frac{1}{2m}$$

5 b)

- Use the above equation to explain the socalled Bragg-peak.
- Why is the Bragg-peak relevant with respect to radiation therapy?