NTNU – Trondheim
Norwegian University of
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Department of Physics

Examination paper for TFY4225 Nuclear and Radiation Physics

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Examination date: 05.12.2016

Examination time (from-to): 9.00-13.00

Permitted examination support material (code C):

- **Simple specified calculator**
- **Barnett & Cronin: Mathematical Formulae**
- **Rottmann: Matematische Formelsammlung**

Other information: Each sub-question (1, 2a, 2b etc) carries equal weight in the evaluation. Exam might be answered in English or Norwegian.

Language: English

Number of pages (front page excluded): 9

Checked by:

Date Signature

Note: Important information is given in tables, figures and equations after the problem text! Please browse through all sides before you start to work on the problems.

Problem 1

The nuclear shell model was a major step forward in nuclear physics.

- Describe the key ingredient of the model and how this affected the output.
- What were the two main success-criteria for the model?
- Explain what is meant by the "extreme independent particle model".
- Give one example situation where the "extreme independent particle model" does not work.

Problem 2

^{11}C can be used in positron emission tomography and can be produced in a cyclotron via the following reaction:

$$^{14}N(p,\alpha)^{11}C$$

The mass excess values for the particles involved are: $^{14}N : 3074\mu u, ^1H : 7825\mu u, ^4He : 2603\mu u, ^{11}C : 11434\mu u$.

Problem 2a

- Calculate the Q-value for the above nuclear reaction.
- Derive the expression for the required minimum (threshold) kinetic energy T_{th} of the proton in order for the reaction to occur.
- Calculate the numeric value of T_{th}.

Problem 2b

- Calculate the Coulomb barrier for the reaction (use $R_0 = 1.6 fm$).
- Relate your results in 2a and 2b to the observed reaction cross-section as shown in figure 1, in particular the range 0-6 MeV.

Problem 2c

Production of ^{11}C:

- Set up the differential equation for the number of ^{11}C product nuclei $N_1(t)$, given a constant reaction rate R.
- Show that the expression for $N_1(t)$ given on the formula sheet is a solution to your equation, and calculate the irradiation time required to reach 90 % of maximum activity.

Problem 2d

The decay scheme of ^{11}C is shown in figure 2.

- In the Fermi theory for beta-decay, explain the approximation behind what is called an *allowed* transition. Show that this approximation is justified.
- Explain/derive the selection rules for angular momentum and parity in the allowed approximation and explain why beta decay of ^{11}C is an allowed transition.
- What is the difference between a Fermi type and a Gamow-Teller type transition? Can you tell which type the ^{11}C beta decay is? Justify your answer.

Problem 2e

A patient is injected with 1 GBq of ^{11}C for a PET exam. Assume even distribution of activity in the body and no biological clearance. Photon attenuation length in soft tissue @ 511 keV is 36 cm.

- Calculate the whole body effective dose to the patient. If you need to make any further simplifying assumptions, please justify/explain them.

Problem 3

In figure 3 an example of a chain of events in a small volume of mass $dm = 1\mu g$ is shown. A photon of energy $E_\gamma = 662keV$ is Compton-scattered and an electron is released with initial kinetic energy $T_e = 100keV$. This electron is scattered and a bremsstrahlung photon with energy $E_b = 20keV$ is emitted and leaves the volume. Via further interactions, the electron comes to rest within the volume.

- Calculate the energy imparted ε and KERMA K for this example.

Problem 4

In the lab-assignments of this course we used two different detectors: a) NaI scintillator detector and b) HPGe (high-purity Germanium) semi-conductor detector.

- Describe the chain of events in the NaI scintillator detector from the initial interaction between the photon and the scintillator material to a count in the spectrum shown on the computer screen.
- Describe the chain of events in the HPGe detector.
- Describe how the output spectra from the two detectors will look. What is the key physical origin of the difference between the two spectra?

Problem 5

The following 9 topics of "Sources of Radiation" were covered in the first project work of this course:

1. Natural Radioactivity on Earth.
2. Radon Gas: focus on Norway.
3. Cosmic Radiation.
4. Medical Imaging Exams.
5. Chernobyl: Local Consequences and Europe in General.
6. Chernobyl: Consequences in Norway.
7. Fukushima Accident.
8. Low Dose Risk: Linear-No-Threshold Model.
9. Low Dose Risk: Hormesis Model.

Choose one of the above topics and describe what was presented on that topic.

Problem 6

The neutron absorption and scattering cross section for light water (H_2O) and heavy water (D_2O) is given in table 1. With respect to the neutron cycle in a thermal fission reactor, which type of water will give the highest neutron multiplication factor, given all other things equal? Justify your answer.

Table 1: Neutron absorption and scattering cross sections

Material	σ_a (b)	σ_s (b)
H_2O	0.66	49.2
D_2O	0.001	10.6

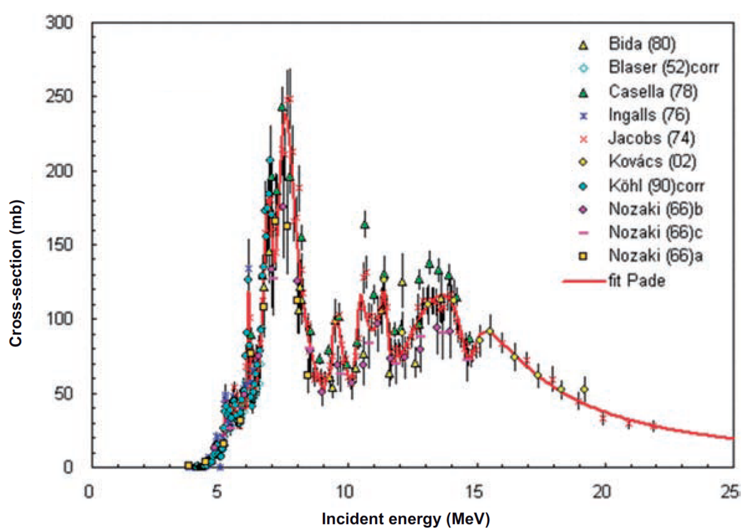

Figure 1: Reaction cross section for ${}^{14}N(p,\alpha){}^{11}C$.

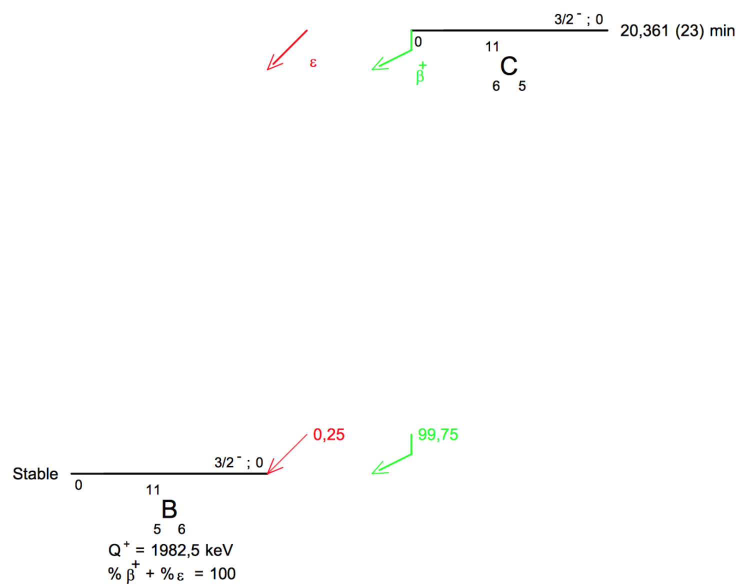

Figure 2: Decay scheme of ^{11}C.

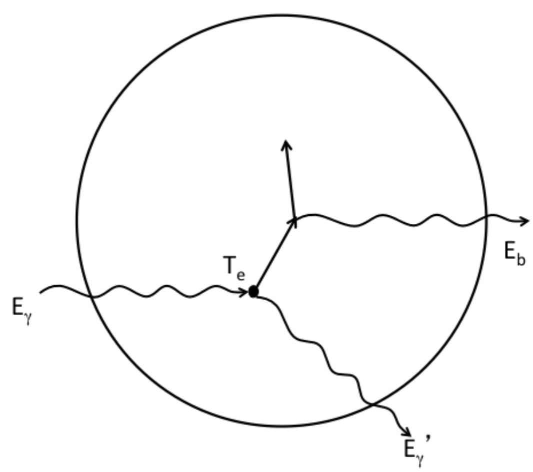

Figure 3: Example radiation interactions in a small volume.

CONSTANTS

Speed of light	c	2.99792458×10^{8} m/s
Charge of electron	e	1.602189×10^{-19} C
Boltzmann constant	k	1.38066×10^{-23} J/K
		8.6174×10^{-5} eV/K
Planck's constant	h	6.62618×10^{-34} J · s
		4.13570×10^{-15} eV · s
	$\hbar = h/2\pi$	1.054589×10^{-34} J · s
		6.58217×10^{-16} eV · s
Gravitational constant	G	6.6726×10^{-11} N · m²/kg²
Avogadro's number	N_A	6.022045×10^{23} mole⁻¹
Universal gas constant	R	8.3144 J/mole · K
Stefan-Boltzmann constant	σ	5.6703×10^{-8} W/m² · K⁴
Rydberg constant	R_∞	1.0973732×10^{7} m⁻¹
Hydrogen ionization energy		13.60580 eV
Bohr radius	a_0	5.291771×10^{-11} m
Bohr magneton	μ_B	9.27408×10^{-24} J/T
		5.78838×10^{-5} eV/T
Nuclear magneton	μ_N	5.05084×10^{-27} J/T
		3.15245×10^{-8} eV/T
Fine structure constant	α	1/137.0360
	hc	1239.853 MeV · fm
	$\hbar c$	197.329 MeV · fm
	$e^2/4\pi\epsilon_0$	1.439976 MeV · fm

PARTICLE REST MASSES

	u	MeV/c^2
Electron	5.485803×10^{-4}	0.511003
Proton	1.00727647	938.280
Neutron	1.00866501	939.573
Deuteron	2.01355321	1875.628
Alpha	4.00150618	3727.409
$\pi^{\pm}$	0.1498300	139.5669
π^{0}	0.1448999	134.9745
μ	0.1134292	105.6595

CONVERSION FACTORS

1 eV = 1.602189×10^{-19} J

1 u = 931.502 MeV/c^2
= 1.660566×10^{-27} kg

1 b = 10^{-28} m²

1 Ci = 3.7×10^{10} decays/s

Figure 4: Physical constants.

Appendix: Selected expressions

$$
\begin{aligned}
F &= \frac{|q_1 q_2|}{4\pi\epsilon_0 r^2} \\
P &= \frac{q_1 q_2}{4\pi\epsilon_0 r} \\
\vec{F} &= q\vec{E} + q\vec{v} \times \vec{B}
\end{aligned}
$$

$$
\begin{aligned}
\vec{p} &= m\vec{v} \\
\vec{L} &= \vec{r} \times \vec{p} \\
F &= ma \\
F &= \frac{mv^2}{r}
\end{aligned}
$$

$$
\begin{aligned}
U(\vec{r}) &= -\int_{\infty}^{\vec{r}} \vec{F} \cdot d\vec{r} \\
\Delta\vec{p}(t) &= \int_0^t \vec{F} d\tau
\end{aligned}
$$

$$
\begin{aligned}
T &= \frac{1}{2}mv^2 \\
p &= \frac{mv}{\sqrt{1 - v^2/c^2}} \\
E &= \sqrt{p^2c^2 + m^2c^4} \\
T &= E - mc^2 \\
E_0 &= mc^2 \\
\lambda &= \frac{h}{p} \\
\Delta p \Delta x &\leq \frac{\hbar}{2} \\
\Delta E \Delta t &\leq \frac{\hbar}{2} \\
\text{Plane wave } \psi(x) &= e^{i\vec{p}\cdot\vec{r}/\hbar} \\
\langle l^2 \rangle &= \hbar^2 l(l+1) \\
\langle l_z \rangle &= \hbar m_l
\end{aligned}
$$

$$
\begin{aligned}
B &= a_v A - a_s A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_{sym} \frac{(A-2Z)^2}{A} + \delta \\
Q &= \sum_i m_i c^2 - \sum_f m_f c^2
\end{aligned}
$$

$$
\begin{aligned}
N(t) &= N_0 e^{-\lambda t} \\
A(t) &\equiv \lambda N(t) \\
t_{1/2} &= \frac{ln2}{\lambda} = \tau ln2 \\
N_1(t) &= \frac{R}{\lambda_1}(1 - e^{-\lambda_1 t}) \\
N_2(t) &= N_0 \frac{\lambda_1}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t})
\end{aligned}
$$

$$
\begin{aligned}
\lambda &= \frac{2\pi}{\hbar}|V_{fi}|^2 \rho(E_f) \\
V_{fi} &= \int \psi_f^* V \psi_i d\nu \\
\lambda &= \frac{g^2 |M_{fi}|}{2\pi \hbar^7 c^3} \int_0^{p_{max}} F(Z', p) p^2 (Q - T_e)^2 dp
\end{aligned}
$$

$$
\begin{aligned}
\pi(ML) &= (-1)^{L+1} \\
\pi(EL) &= (-1)^L
\end{aligned}
$$

$$
\begin{aligned}
\sqrt{T_b} &= \frac{\sqrt{m_a m_b T_a}\cos\theta \pm \sqrt{m_a m_b T_a \cos^2\theta + (m_Y + m_b)\left[m_Y Q + (m_Y - m_a) T_a\right]}}{m_Y + m_b} \\
\sigma_{sc} &= \sum_{l=0}^{\infty} \pi \lambda\!\!\bar{}\,(2l+1)|1 - \eta_l|^2 \\
\sigma_r &= \sum_{l=0}^{\infty} \pi \lambda\!\!\bar{}\,(2l+1)(1 - |\eta_l|^2) \\
-\frac{dE}{dx} &= \left(\frac{ze^2}{4\pi\epsilon_0}\right)^2 \frac{4\pi Z \rho N_A}{A m v^2}\left[ln\left(\frac{2mv^2}{I}\right) - ln(1 - \beta^2) - \beta^2\right] \\
n &= \frac{\rho N_A}{A} \\
N &= N_0 e^{-\mu x} \\
\mu &= n\sigma \\
\sigma &= \sigma_{PE} + Z\sigma_C + \sigma_{PP}
\end{aligned}
$$

$$
\begin{aligned}
D &\equiv \frac{d\bar{\varepsilon}}{dm} \\
\varepsilon &= R_{in} - R_{out} + \sum Q \\
K &= \frac{dD_{tr}}{dm}
\end{aligned}
$$

$$
\begin{aligned}
K_C &= E\Phi\left(\frac{\mu_{en}}{\rho}\right) \\
D(r_T, T_D) &= \sum_{r_S} \tilde{A}(r_S, T_D) S(r_T \leftarrow r_S) \\
S(r_T \leftarrow r_S) &= \frac{1}{M(r_T)} \sum_i E_i Y_i \phi(r_T \leftarrow r_S, E_i) \\
\tilde{A}(r_S, T_D) &= \int_0^{T_D} A(r_S, t) dt \\
E &= \sum_T w_T \sum_R w_R D_R(r_T, T_D)
\end{aligned}
$$