TFY4225 Nuclear and Radiation Physics
Exam

Autumn 2023

Problem 1

The ground state of ^{40}K has nuclear spin 4 and negative parity. In 90 % of the cases this nuclei disintegrates by β- to the ground state of ^{40}Ca, and in 10 % of the cases by β+ and/or EC to an excited state of ^{40}Ar, which subsequently is de-excited to the ground state by γ emission with energy 1460 keV. The half-life of 40K is 1.25 10^9 years.

a) Write the definition of the Q value of a nuclear reaction, and derive the formula for the Q value of β+ disintegration, expressed by atomic masses, and also expressed by mass excess values (as will be used for calculations in the present problem). (6p)
Solution: Q-value for X(a,b)Y:

$$Q = (\sum m_i - \sum m_f)c^2 = [(m_X + m_a) - (m_b + m_Y)]c^2 \tag{1}$$

β^+-decay: ${}^A_Z X \rightarrow^A_{Z-1} Y + \beta^+ + \nu$

$$Q_{\beta+} = [(m_N({}^A_Z X) - m_N({}^A_{Z-1} Y) - m(\beta^+) - m(\nu)]c^2 (NuclearMass) \tag{2}$$

Assume $m(\nu) = 0$ and look at atomic mass:

$$Q_{\beta+} = [(m_a({}^A_Z X) - Zm_e - (m_a({}^A_{Z-1} Y) - (Z-1)m_e) - m_e]c^2 \tag{3}$$

$$Q_{\beta+} = [(m_a({}^A_Z X) - m_a({}^A_{Z-1} Y) - (Zm_e - (Z-1)m_e) - m_e]c^2 \tag{4}$$

$$Q_{\beta+} = [(m_a({}^A_Z X) - m_a({}^A_{Z-1} Y) - 2m_e]c^2 \tag{5}$$

Expand with mass excess $\Delta_{A,Z} = m_a({}^A_Z X) - A$

$$Q_{\beta+} = [(\Delta_{A,Z} + A - (\Delta_{A,Z-1} + A) - 2m_e]c^2 = [(\Delta_{A,Z} - (\Delta_{A,Z-1}) - 2m_e]c^2 \tag{6}$$

b) How do you interpret the fact that ^{40}K disintegrates by both β- and β+/EC processes? (4p)
Solution: A is even so we can have both odd-odd and even-even nuclei. Due to the pairing term and its different values for odd-odd and even-even, we will get two parabolas in the mass vs Z diagram. ^{40}K is an odd-odd nucleus and the most stable of the odd-odd A=40 nuclei, but with a larger mass than both Ar and Ca why it is energetically allowed for both β^- and β^+/EC

c) Calculate the Q values for disintegration of ^{40}K by β-, β+, and EC, respectively. Determine whether the transition to the excited level of ^{40}Ar takes place by β+ or EC. The following mass excess values are given (in **μu** units): ^{40}Ar: -37617; ^{40}K: -36001; ^{40}Ca:

-37409. (8 p)

Solution: β^-: $^{40}K \rightarrow ^{40} Ca + \beta^- + \overline{\nu}$

$$Q_{\beta^-} = [(\Delta_{^{40}K} - \Delta_{^{40}Ca}]c^2 = [-36001 - (-37409)]\mu u \cdot c^2 = 1.311 MeV \quad (7)$$

β^+: $^{40}K \rightarrow ^{40} Ar + \beta^+ + \nu$

$$Q_{\beta^+} = [(\Delta_{^{40}K} - (\Delta_{^{40}Ar}) - 2m_e]c^2 \quad (8)$$

$$Q_{\beta^+} = [(-36001 - (-37617))\mu u - 2m_e]c^2 = [1616 \cdot 10^{-6} uc^2 - 2m_e c^2] = \quad (9)$$

$$Q_{\beta^+} = 1616 \cdot 10^{-6} \cdot 931.5 - 2 \cdot 0.511 (MeV) = 0.483 MeV \quad (10)$$

β^+: $^{40}K \rightarrow ^{40} Ar + \nu$

$$Q_{EC} = [(\Delta_{^{40}K} - (\Delta_{^{40}Ar})]c^2 = 1.505 MeV \quad (11)$$

Since the excited level has an energy $E = 1460 keV > Q_{\beta^+}$, the decay by β^+ is not possible to this level and the decay of ^{40}K to ^{40}Ca is by EC.

d) On average a human body of 70 kg consists of 189 g potassium, of which 0.012 % is ^{40}K. Calculate the annual effective dose to the human body due to the content of 40K. Assume that the absorbed fraction of the emission of quantum energy 1460 keV is 30 % for the whole body as both source and target organ. You can also assume that the ^{40}K content in the body is constant due to daily intake of K. (12p)

Solution: Use the MIRD-formalism for internal dosimetry. The S-function represents the absorbed dose in target organ T per disintegration in the source organ S :

$$S(r_T \leftarrow r_S) = \frac{1}{M_T} \sum_i E_i Y_i \Phi(r_T \leftarrow r_S) \quad (12)$$

where the sum is taken over all decay branches/emitted particles in one disintegration. E_i is the (average) energy carried by particle i, Y_i is the yield, or fraction of particles emitted per disintegration, and $\Phi(r_T \leftarrow r_S)$ is the absorbed fraction in target organ T for particle i originating in source organ S.

The total number of disintegrations in source organ S over a time period T_D is:

$$\tilde{A}(r_S, T_D) = \int_0^{T_D} A(r_S, t) dt \quad (13)$$

where A is the activity. The absorbed dose in the target organ T is:

$$D_T(r_T, T_D) = \sum_{r_S} \tilde{A}(r_S, T_D) S(r_T \leftarrow r_S) \quad (14)$$

where the sum is taken over all source organs. The equivalent dose H in the target organ becomes:

$$H_T(r_T, T_D) = \sum_R w_R D_T(r_T, T_D) \quad (15)$$

And finally the whole body effective dose D:

$$D = \sum_T w_T H_T(r_T, T_D) \quad (16)$$

In our case, we have : 189 g K in the body of which 0.012% is ^{40}K or $0.189 \cdot 0.00012 = 22.68 \cdot 10^{-3}$ g ^{40}K The number of ^{40}K atoms in the body becomes: (M: molar mass)

$$N = N_A \frac{m}{M} = 6.022 \cdot 10^{23} \cdot \frac{22.68 \cdot 10^{-3}}{39.1} = 3.493 \cdot 10^{20} \quad (17)$$

Radiation	w_R	k Fraction of decay	E(keV)	ϕ
β^-	1	0.9	1311/3	1
ν	0	0.1		0
γ	1	0.1	1460	0.3

Table 1: Energies and weighing factors

Since we have radiation with different energies and absorbed energy fractions we get table 1:

Since we have the same weighing factor w_T we can calculate the S function

$$S(r_T \leftarrow r_S) = \frac{1}{M_T}\sum_i E_i Y_i \Phi(r_T \leftarrow r_S) \quad (18)$$

$$= \frac{1}{M_T}[437keV \cdot 0.9 \cdot 1 + 1460keV \cdot 0.1 \cdot 0.3] \quad (19)$$

$$= \frac{437.1keV}{M_T} \quad (20)$$

We calculate the total number of disintegrations in the body as a whole in one year:

$$\tilde{A}_{body} = \int_0^{1year} A(^{40}K)e^{-\lambda t}dt \quad (21)$$

$$= \int_0^{1year} \lambda N(^{40}K)e^{-\lambda t}dt \quad (22)$$

However $N(^{40}K)$ and $A(^{40}K)$ is constant so the activity can be approximated with

$$\tilde{A}_{body} = \lambda NT = \frac{ln2}{T_{1/2}}NT \quad (23)$$

$$= \frac{0.692}{1.25 \cdot 10^9 y}3.493 \cdot 10^{20} \cdot 1y \quad (24)$$

$$= 1.93 \cdot 10^{11} \quad (25)$$

The dose can be calculated by multiplying the S function and the activity and adjusting the units to Joule:

$$D = \tilde{A}S(r_T \leftarrow r_S) = 1.93 \cdot 10^{11} \cdot \frac{437.110^{11}}{70} \cdot 1.602 \cdot 10^{-19}[J/kg = Sv] \quad (26)$$

$$= 2 \cdot 10^{-4}[Sv] = 0.2mSv \quad (27)$$

The annual effective dose due to ^{40}K for a 70kg human is 0.2 mSv.

Problem 2

a) Naturally occurring uranium is a mixture of the ^{238}U (99.28%) and ^{235}U (0.72%) isotopes. How old must the material of the solar system be if one assumes that at its creation both isotopes were present in equal quantities? The mean life are τ (^{235}U) = 1×10^9 years and τ (^{238}U) = 6.6×10^9 years. (6p)

Solution: We have for the mean life the decay $N(t) = N_0 e^{-t/\tau}$ The ratio of the two isotopes is given by:

$$\alpha = \frac{N_1(t)}{N_2(t)} = \frac{N_{0,1}e^{-t/\tau_1}}{N_{0,2}e^{-t/\tau_2}} = \frac{e^{-t/\tau_1}}{e^{-t/\tau_2}} = e^{(1/\tau_2 - 1/\tau_1)t} \quad (28)$$

We get the time:

$$t = \frac{ln\alpha}{(1/\tau_2 - 1/\tau_1)} = \frac{ln(99,28/0.72)}{(1/1\cdot 10^9 - 1/6.6\cdot 10^9)} = 5.8\cdot 10^9 [years] \tag{29}$$

b) You produce a radioactive isotope with half life T with a constant rate (Q) in an accelerator. Given that you want to obtain 3/4 of the maximum number of radioactive nuclei (or activity), how long do you have to run the production? (6p)
Solution: Assuming an infinite number of target nuclei and constant particle flux from the accelerator the production rate can be considered constant (Q). The produced nuclei decay according to the decay law. The change in the number of radioactive nuclei will be governed by:

$$\frac{dN(t)}{dt} = Q - \lambda N \tag{30}$$

Solving this equation gives:

$$N(t) = \frac{Q}{\lambda}(1 - e^{-\lambda t}) = \frac{Q}{\frac{ln2}{T_{1/2}}}(1 - e^{-\frac{ln2}{T_{1/2}}t}) \tag{31}$$

We want to know when N(t) reaches 75% of the maximum value ($= 0.75N_{max} = 0.75\frac{Q}{\frac{ln2}{T_{1/2}}}$):

$$0.75 = 1 - e^{-\frac{ln2}{T_{1/2}}t} \tag{32}$$

$$e^{-\frac{ln2}{T_{1/2}}t} = 0.25 \tag{33}$$

$$-\frac{ln2}{T_{1/2}}t = ln0.25 \tag{34}$$

$$t = -ln0.25\frac{T_{1/2}}{ln2} = 2T_{1/2} \tag{35}$$

The production must run for 2 half lifes.

Problem 3

a) Show how the mass formula can be used to obtain an expression for the value of Z corresponding to the most stable nuclide at a given value of A. (8p)

Solution: The semi-empirical mass formula:

$$m(^A_ZX) = Zm(^1H) + (A-Z)m_n - \frac{1}{c^2}\{a_vA - a_sA^{2/3} - a_C\frac{Z(Z-1)}{A^{1/3}} - a_{sym}\frac{(A-2Z)^2}{A} + \delta\} \tag{36}$$

where we have used $N = A - Z$. The most stable nucleus for a given value of A is the one with the lowest mass $m(^A_ZX)$. To find an expression for this, we must differentiate $m(^A_ZX)$ with respect to Z and set this equal to zero:

$$\frac{\partial m(^A_ZX)}{\partial Z} = m(^1H) - m_n + \frac{1}{c^2}\{a_C\frac{(2Z-1)}{A^{1/3}} - a_{sym}\frac{4(A-2Z)}{A}\} \tag{37}$$

$$\frac{\partial m(^A_ZX)}{\partial Z} = 0 \tag{38}$$

$$Z_{min} = \frac{[m_n - m(^1H)]c^2 + a_CA^{-1/3} + 4a_{sym}}{2a_CA^{-1/3} + 8a_{sym}A^{-1}} \tag{39}$$

b)What is the most stable isotope for A=99 from the SEMF. (4p)
Solution: Use the result from a):

$$Z_{min} = \frac{[\Delta_n - \Delta(^1H)] + a_C A^{-1/3} + 4a_{sym}}{2a_C A^{-1/3} + 8a_{sym}A^{-1}} \quad (40)$$

$$Z_{min} = \frac{8.071 - 7.289 + 0.72 \cdot 99^{-1/3} + 4 \cdot 23}{2 \cdot 0.72 \cdot 99^{-1/3} + 8 \cdot 23 \cdot 99^{-1}} = 42.7 \approx 43 \quad (41)$$

$$(42)$$

Z=43 is Tc. In reality, this is not a stable isotope. ^{99}Ru has a slightly lower mass and is the only stable isotope with A=99. But that was not the question.

c) What is the most stable isotope for A=100. Discuss your result.(8p)
Solution: Use the result from a):

$$Z_{min} = \frac{[\Delta_n - \Delta(^1H)] + a_C A^{-1/3} + 4a_{sym}}{2a_C A^{-1/3} + 8a_{sym}A^{-1}} \quad (43)$$

$$Z_{min} = \frac{8.071 - 7.289 + 0.72 \cdot 100^{-1/3} + 4 \cdot 23}{2 \cdot 0.72 \cdot 100^{-1/3} + 8 \cdot 23 \cdot 100^{-1}} = 43.2 \approx 43 \quad (44)$$

$$(45)$$

Z=43 is Tc, but we have not taken the pairing term into account. ^{100}Tc is an odd-odd nucleus and the pairing term has a negative value causing an offset for the odd Z mass parabola. In this case leading to that ^{100}Tc will have a higher mass than the neighbouring ^{100}Ru which is stable.

Problem 4

Calculate the Q values for the following fission reactions and determine the number of neutrons emitted.

a) $^{235}U(n,f)^{137}Cs +^{96} Rb + xn$ (6p)
Solution: We have 236 nucleons on the left side and 233 + y on the right side, thus x=3.
$^{235}U(n,f)^{137}Cs +^{96} Rb + 3n$

$$Q = (\sum m_i - \sum m_f) \cdot c^2 \quad (46)$$

Use mass excess Δ and mass number A, A is conserved so only Δ is needed:

$$Q = \Delta(^{235}U) + \Delta(n) - \Delta(^{137}Cs) - \Delta(^{96}Rb) - 3\Delta(n) \quad (47)$$

$$Q = 40918 + 8071 + (-86546) + (-61354) - 3 \cdot 8071 = 172676(keV) \quad (48)$$

or 172.7 MeV

b) $^{235}U(n,f)^{141}Cs +^{93} Rb + yn$ (6p)
Solution: We have 236 nucleons on the left side and 234 + y on the right side, thus x=2.
$^{235}U(n,f)^{141}Cs +^{93} Rb + 2n$

$$Q = \Delta(^{235}U) + \Delta(n) - \Delta(^{141}Cs) - \Delta(^{93}Rb) - 2\Delta(n) \quad (49)$$

$$Q = 40918 + 8071 + (-74977) + (-72620) - 2 \cdot 8071 = 180444(keV) \quad (50)$$

or 180.4 MeV

c) The reaction products are unstable, reason what type of radiation you would expect from these. (6p)
Solution: ^{235}U has a surplus of neutrons (neutron-rich) the products will therefore also be neutron-rich and in this case far from the stability line. They will then decay by β^- towards the stability line.

Problem 5

You have been tasked to find the best radiotherapy for a tumour on the adrenal gland by the kidney. You have two alternatives x-ray therapy and Proton beam. Discuss the pros and cons with each method. (10p)
Solution: should consist of a discussion on the dose imparted on the target and the dose on surrounding healthy tissue. The discussion must include the Bragg peak. The discussion should also mention the generation of X-rays and protons.

Multiple Choice Qustions

Which of the following statements is **false**

- The heaviest stable nucleus that we know today is 208Pb.
- ^{208}Pb is a doubly magic nucleus.
- ^{208}Pb is produced in the decay of heavier unstable nuclei
- The nuclear spin of ^{208}Pb is 2 since it has an unpaired neutron and an unpaired proton **False**
- ^{212}Po decays to ^{208}Pb

The Q-values for 64 Cu are for β-,EC an α, 579 keV, 1674keV and -6199 keV, respectively.
Which one of the following statements is **false**.

- ^{64}Cu can decay by beta- decay
- ^{64}Cu can decay by EC decay
- ^{64}Cu can decay by beta+ decay
- ^{64}Cu is stable **False**
- ^{64}Cu can not decay by alpha decay

Which of the following statements is **false**.

- You can determine the energy of the radiation with a Geiger-Müller counter **False**
- Scintillator detectors must include a device that detects photons in the visible or UV range.
- It is possible to use organic liquids in scintillator detectors.
- Semiconductor detectors will normally have the best energy resolution
- Neutrons are normally detected with the use of materials that is rich in low_Z elements

Which of the following statements is **false**.

- Cells are more sensitive to radiation during mitosis
- Most damage to the DNA is due to indirect effects of radiation, i.e. creation of radicals through radiolysis
- Only the total determines the cell survival **False**
- LET (linear Energy Transfer) decrease with increasing kinetic energy for the particles used

- An example of a stochastic effect of radiation is cancer.

Which statement is **true**?

- Boron is used as a moderator in nuclear reactors.
- Steel shields gamma rays better than Lead.
- Natural (non-enriched) uranium can be used as fuel in nuclear reactors. **True**
- It is possible to build a functional atomic bomb with ^{232}Th.