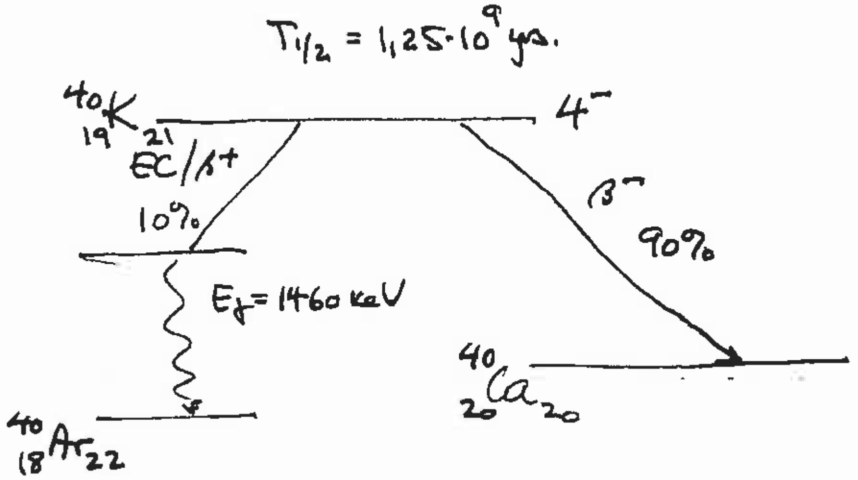
Answers to Exam TFY 4225 H10 (01.12.2010)

Problem 1.



a) Explain spin/parity 4^- of ^{40}K ground state.

$^{40}_{19}K_{21}$ is an odd-odd nuclide.

Spin and parity of the ground state is therefore determined by the combination of spin and parity of the single (odd) valence proton (no. 19) and the single (odd) valence neutron (no. 21).

20 is a magic number in the shell model, corresponding to filled shells: $(1s\,1/2)^2(1p\,3/2)^4(1p\,1/2)^2(1d\,5/2)^6(2s\,1/2)^2(1d\,3/2)^4$

$\Rightarrow$ Proton 19 is unpaired in state $(1d\,3/2)$, or rather, there is a single vacancy (position 20) in $(1d\,3/2)$ i.e. $(1d\,3/2)^{-1}$

$\Rightarrow$ Proton 19 represents spin/parity $\underline{3/2^+}$ $(\pi = (-1)^2 = +1)$

Neutron 21 is ~~a~~ single in state $1f\,7/2$, representing spin/parity $\underline{\frac{7}{2}^-}$ $\left(\pi = (-1)^3 = -1,\ l=3 \text{ for } f\right)$

Total spin $\vec{I} = \vec{j}_p + \vec{j}_n \Rightarrow |j_p - j_n| \le I \le |j_p + j_n|$

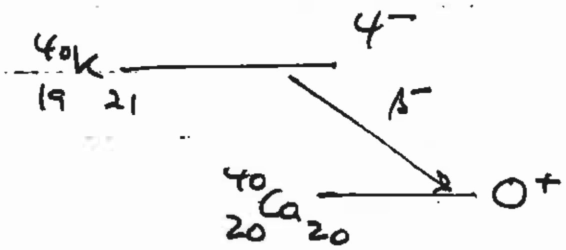
$$\left|\frac{3}{2}-\frac{7}{2}\right| \le I \le \left|\frac{3}{2}+\frac{7}{2}\right|$$

$\underline{2 \le I \le 5}$ i.e. $I=4$ is a possible value

Parity: $\underline{\pi = \pi_{P19}\cdot\pi_{N21}} = (+1)(-1) = \underline{-1 \text{ i.e. neg. parity}}$

⇒ Conclusion: 4^- is a possible ground state of ^{40}K.

β^- - transition



The ground state of $^{40}_{20}Ca_{20}$ is a 0^+ state since ^{40}Ca is an even-even nucleus.
Thus, the β^- transition is a $4^- \to 0^+$ transition.

$\Delta\pi$ = Yes $\Rightarrow$ $L = 1, 3, \ldots$ (only odd numbers)

$$|I_i - I_f| \le |\vec{L} + \vec{S}| \le |I_f + I_i|$$
$$|4-0| \le |\vec{L}+\vec{S}| \le |4+0|$$
$$\Rightarrow \underline{|\vec{L} + \vec{S}| = 4}$$

For odd L, this is only possible for $L=3$ and $S=1$.

The β^- decay is 3rd forbidden, pure Gamow-Teller.

For $\log(f t_{1/2})$ values $t_{1/2}$ must be in seconds, and represent the partial half life corresponding to the β^- transition having branching ratio $b = 0.9$.

$$\Rightarrow \lambda_{\beta^-} = b \cdot \lambda_{tot} \quad \Rightarrow t_{1/2,\beta^-} = \frac{\ln 2}{\lambda_{\beta^-}} = \frac{\ln 2}{b \cdot \lambda_{tot}} = \frac{1}{b} \cdot t_{1/2}$$

$$t_{1/2\,\beta^-} = \left(\frac{1}{0.9} \cdot 1.25 \cdot 10^9 \cdot 365 \cdot 24 \cdot 3600\,s\right) = 4{,}38 \cdot 10^{16}\,s$$

$$\log(f t_{1/2\,\beta^-}) = \log f + \log(4{,}38 \cdot 10^{16}) = 1{,}5 + 16{,}6 = 18.1$$

The $\log(f t_{1/2})$ value of 18.1 is in reasonable agreement with the typical values of ~16 for 3rd forbidden transitions. (2nd forbidden ~12, 4th forbidden ~21)

b) Q-value for the nuclear reaction $X(a,b)Y$

$$Q = \left(\sum m_i - \sum m_f\right)c^2 = \left[(m_X + m_a) - (m_b + m_Y)\right]c^2$$

For radioactive processes, there is no incoming particle a.

β^+-decay: $\quad {}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}X' + \beta^+ + \nu$

$$Q_{\beta^+} = \left[m_N({}^{A}_{Z}X) - m_N({}^{A}_{Z-1}X') - m(\beta^+) - m(\nu)\right]c^2$$

Assume $m(\nu) \simeq 0$; $m(\beta^+) = m_e$.

$$Q_{\beta^+} = \left[m_a({}^{A}_{Z}X) - Zm_e - \left(m_a({}^{A}_{Z-1}X') - (Z-1)m_e\right) - m_e\right]c^2$$

$$Q_{\beta^+} = \left[m_a({}^{A}_{Z}X) - m_a({}^{A}_{Z-1}X') + (Z-1-Z-1)m_e\right]c^2$$

$$\underline{\underline{Q_{\beta^+}}} = \left(m_a({}^{A}_{Z}X) - m_a({}^{A}_{Z-1}X') - 2m_e\right)c^2 = \underline{\underline{(m_P - m_D - 2m_e)c^2}}$$

Expressed by mass excess $\quad {}^{A}_{Z}\Delta = m({}^{A}_{Z}X) - A$:

$$\underline{\underline{Q_{\beta^+}}} = \left[{}^{A}_{Z}\Delta + A - ({}^{A}_{Z-1}\Delta + A) - 2m_e\right]c^2 = \underline{\underline{(\Delta_P - \Delta_D - 2m_e)c^2}}$$

Interpretation of the fact that ${}^{40}_{19}K_{21}$ decays by both β^- and β^+/EC:

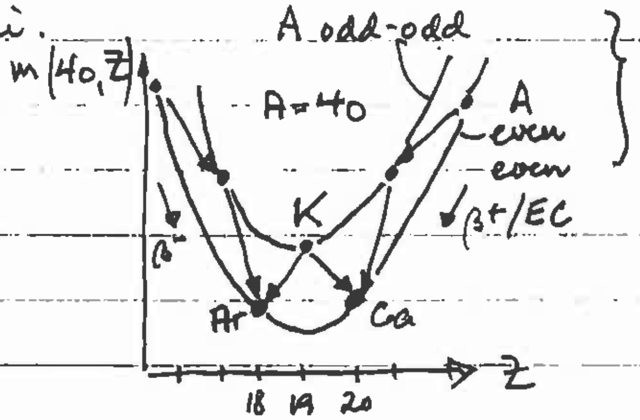
In the semi-empirical mass formula for $m({}^{A}_{Z}X)$:

$$m({}^{A}_{Z}X) = Zm({}^{1}H) + (A-Z)m_n - B/c^2$$

the formula for the total binding energy contains a pairing term δ, which is:

$$\delta = \begin{cases} >0 & \text{for even-even nuclei} \\ 0 & \text{for odd A nuclei (even-odd or odd-even} \\ <0 & \text{for odd-odd nuclei.} \end{cases}$$

For a fixed value of A, the expression for $m(A,Z)$ will be a second-order polynomial in Z, and for A even (as for ^{40}K) there will be one parabola for odd-odd nuclei, and another for even-even nuclei.



due to the δ-term in B.

Evidently, ^{40}K is the most stable of $A=40$ odd-odd nuclides (very long half-life, $1{,}25\cdot 10^9$ yrs!). Nuclides at the min. point of the odd-odd parabola can decay by ~~both~~ either β^- or β^+/EC to alternative daughter nuclides of lower mass, in this case $^{40}_{18}Ar$ or $^{40}_{20}Ca$ (by β^+/EC or β^-, respectively).

c) β^- : $\quad ^{40}K \rightarrow {}^{40}Ca + \beta^- + \bar{\nu}$

$$Q_{\beta^-} = (\Delta_P - \Delta_D)c^2 = [-36001 - (-37409)]\,\mu u \cdot c^2$$

$$\underline{\underline{Q_{\beta^-}}} = 1408 \cdot 10^{-6}\, uc^2 = 1408\cdot 10^{-6} \cdot 931{,}5 \text{ MeV} = \underline{\underline{1{,}311 \text{ MeV}}}$$

β^+ : $\quad ^{40}K \rightarrow {}^{40}Ar + \beta^+ + \nu$

$$Q_{\beta^+} = (\Delta_P - \Delta_D - 2m_e)c^2 = \left[\left(-36001 - (-37617)\right)\mu u - 2m_e\right]c^2$$

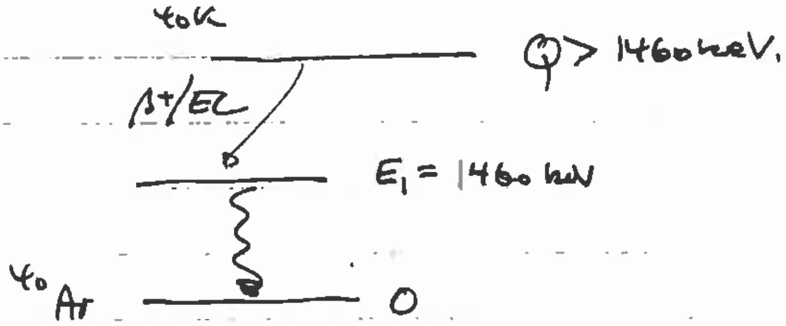
$$Q_{\beta^+} = 1616\cdot10^{-6}\,uc^2 - 2m_ec^2 = \left[1616\cdot10^{-6}\cdot931{,}5 - 2\cdot0{,}511\right]\text{MeV}$$

$$Q_{\beta^+} = (1{,}505 - 1{,}022)\ \text{MeV} = 0{,}483\ \text{MeV} = \underline{\underline{483\ \text{keV}}}.$$

EC: $\quad {}^{40}K + e \rightarrow {}^{40}Ar + \nu$

$$\underline{\underline{Q_{EC}}} = (\Delta_P - \Delta_D)c^2 = 1{,}505\ \text{MeV} = \underline{\underline{1505\ \text{keV}}}.$$

β^+/EC disintegration goes to an excited state of energy $E_1 = 1460$ keV.



Since the excited level E_1 has energy $E_1 = 1460\ \text{keV} > Q_{\beta^+}$, the decay to this excited level is not possible by β^+.

$\Rightarrow$ The decay of ${}^{40}K$ to ${}^{40}Ar$ is by EC.

d) 0,27 % (by weight) of K in the human body, of which 0,012 % is ^{40}K.

Specific effective energy for radiation protection

$$SEE = \frac{1}{m_T} \sum w_i k_i E_i \varphi_i (T \leftarrow S)$$

m_T : mass of body (= 70 kg)
w_i : radiation weighting factor (= 1 for $\gamma, \beta^{\pm}$)
k_i : yield of radiation of type i (fraction / disintegration)
E_i : average quantum energy of radiation i.
φ_i : absorbed fraction of energy emitted from source organ and absorbed in target organ.

Body content of ^{40}K : $m = 70\,kg \cdot \frac{0,27}{100} \cdot \frac{0.012}{100} = \underline{\underline{22,68 \cdot 10^{-6}\,kg}}$

Number of ^{40}K atoms in body : $N = N_A \cdot \frac{m}{M}$ ← molar mass

$$N = 6,022 \cdot 10^{26} \frac{1}{kmol} \cdot \frac{22,68 \cdot 10^{-6}\,kg}{39,1\,kg/kmol}$$

$$\underline{N = 3,493 \cdot 10^{20}}$$

Radiation from ^{40}K : $\left(\bar{E} \approx \frac{1}{3} E_{max} \text{ for } \beta^- \right)$

	w_R	k	E (keV)	φ	$w_i k_i E_i \varphi_i$ [keV/dis]
β^-	1	0.9	$\frac{1311}{3}$	1	393,3
ν from EC		0.1	45	0	0
γ	1	0.1	1460	0.3	43,8
				$\Sigma =$	437,1 keV/dis

$$SEE_T = \frac{1}{m_T} \sum w_i k_i E_i \varphi_i (T \leftarrow S) = \frac{437{,}1 \text{ keV/dis} \cdot 1{,}602 \cdot 10^{-19} \frac{J}{eV}}{70 \text{ kg}}$$

$$SEE_T = 10{,}0 \cdot 10^{3} \cdot 10^{-19} \frac{J}{kg \cdot dis} = 1 \cdot 10^{-15} \frac{J}{kg \cdot dis}$$

Units of SEE are $\frac{J}{kg \cdot dis}$, corresponding to $\frac{Gy}{dis}$, but here, the radiation weighting factor w [Sv/Gy] is included, thus the units of SEE_T will be Sv/dis

i.e. $\underline{\underline{SEE_T = 1 \cdot 10^{-15} \text{ Sv/dis}}}$

Yearly effective dose : $E = w_T \tilde{A} \cdot SEE_T$ $\qquad w_T = 1{,}0$ for whole body

$$\tilde{A} = \int_0^{1yr} A(^{40}K)\, dt \qquad \text{i.e. number of disintegrations in 1yr.}$$

$$A(^{40}K) = \lambda \cdot N \qquad \lambda = \frac{\ln 2}{T_{1/2}} = \frac{0{,}693}{1{,}25 \cdot 10^9 \cdot 365 \cdot 24 \cdot 3600}$$

$$\tilde{A} \approx A(^{40}K) \cdot T \qquad \text{if } A(^{40}K) \approx \text{constant during time } T.$$

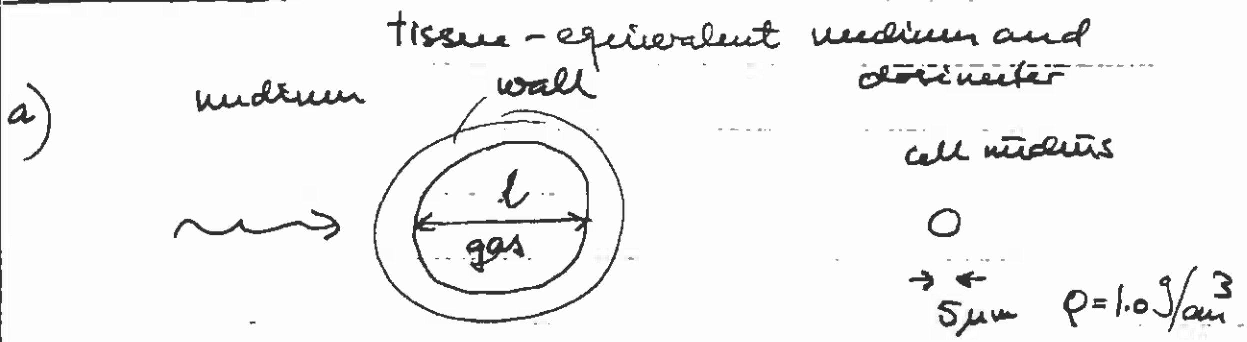
$$\Rightarrow \tilde{A} = \lambda \cdot N \cdot T = \frac{0{,}693}{1{,}25 \cdot 10^9 yr} \cdot 3{,}493 \cdot 10^{20} \cdot 1yr = \underline{1{,}93 \cdot 10^{11} \text{ dis.}}$$

Annual effective dose: $E = w_T \tilde{A} \cdot SEE_T = 1 \cdot 1{,}93 \cdot 10^{11} \text{ dis} \cdot 1 \cdot 10^{-15} \text{ Sv/dis}$

$$\Rightarrow E = 0{,}2 \text{ mSv}$$

Problem 2.

a)



Equal energy deposition from one electron track through dosimeter and through cell nucleus.

$$\Rightarrow \delta\varepsilon_{gas} = \left(-\frac{dE}{dx}\right)_{col,\,gas} \cdot l_{gas} = \delta\varepsilon_{cell} = \left(-\frac{dE}{dx}\right)_{col,\,cell} \cdot l_{cell}$$

Average mass collision stopping power for spectrum of secondary particles (electrons)

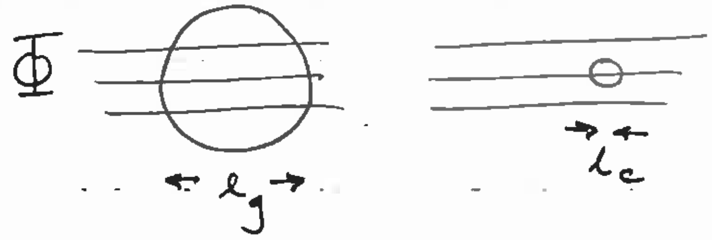
$$\frac{S_{col}}{\rho} = \frac{1}{\rho}\left(-\frac{dE}{dx}\right)_{col.}$$

$$\Rightarrow \left(\frac{S_{col}}{\rho}\right)_{gas} \cdot \rho_{gas} \cdot l_{gas} = \left(\frac{S_{col}}{\rho}\right)_{cell} \cdot \rho_{cell} \cdot l_{cell}$$

Tissue-equivalent dosimeter $\Rightarrow \left(\frac{S_{col}}{\rho}\right)_{gas} \approx \left(\frac{S_{col}}{\rho}\right)_{cell}$

$$\Rightarrow \rho_{gas} = \frac{\rho_{cell}}{l_{gas}} \cdot l_{cell} = \frac{1\,g/cm^3}{2\cdot 10^{-2}\,m} \cdot 5\cdot 10^{-6}\,m = 2{,}5\cdot 10^{-4}\,\frac{g}{cm^3}$$

Both volumes assumed of equal shape (spherical)



Same secondary particle fluence through gas cavity and cell nucleus.

Specific energy $z = \dfrac{\Sigma \delta \varepsilon}{m}$

$$\Sigma \delta \varepsilon = \underbrace{\left(-\frac{dE}{dx}\right) \cdot l_{ce}}_{\substack{\delta \varepsilon \\ \text{per track}}} \cdot \underbrace{l^2 \cdot \Phi}_{\text{number of tracks}}$$

$$m = \rho \cdot l^3 \quad \left(\text{or } \rho \cdot \tfrac{4}{3}\pi \left(\tfrac{l}{2}\right)^3 \text{ for sphere}\right)$$

$$\Rightarrow \quad z = \frac{\Sigma \delta \varepsilon}{m} = \frac{\left(-\frac{dE}{dx}\right)_{ce} l^3 \cdot \Phi}{\rho \cdot l^3} = \left(\frac{S_c}{\rho}\right) \cdot \Phi$$

If Φ is assumed equal in gas cavity and cell nucleus:

$$\Phi = \frac{z_{gas}}{\left(\frac{S_c}{\rho}\right)_{gas}} = \frac{z_{cell}}{\left(\frac{S_c}{\rho}\right)_{cell}}$$

$$\Rightarrow \quad z_{gas} = \frac{\left(\frac{S_c}{\rho}\right)_{gas}}{\left(\frac{S_c}{\rho}\right)_{cell}} \cdot z_{cell}.$$

Since $\left(S_c/\rho\right)_{gas} = \left(\frac{S_c}{\rho}\right)_{cell}$ for tissue-equivalent dosimeters, this means that $\underline{\underline{z_{gas} = z_{cell} \quad q.e.d.}}$

Geometrical factors of average cord length and volume related to linear dimension l, will be equal for the two volumes, and therefore cancel in the final expression $\underline{z_{gas} = z_{cell}}$

b) Necessary exposure time for dose 1 Gy.

$$D \overset{CPE}{=} K_c = \Psi\left(\frac{\mu_{en}}{\rho}\right) = \dot{\Phi}\cdot T\cdot h\nu\left(\frac{\mu_{en}}{\rho}\right)$$

$$\Rightarrow T = \frac{D}{\dot{\Phi}\cdot h\nu\left(\frac{\mu_{en}}{\rho}\right)}$$

$$T = \frac{1\ \text{Gy}}{10^{10}\,\frac{1\ \text{MeV}}{\text{cm}^2\cdot\text{s}}\cdot 1{,}602\cdot 10^{-13}\,\frac{\text{J}}{\text{MeV}}\ 0{,}0309\,\frac{\text{cm}^2}{\text{g}}}$$

$$\underline{\underline{T}} = \frac{1\ \text{J/kg}\cdot 10^{-3}\,\text{kg}\cdot\text{s}}{0{,}0495\cdot 10^{-3}} = \underline{\underline{20{,}2\,\text{s}}}$$

c) Graph: see fig (a) page 10 of Suppl. Dosimetry (enclosed)

Expression for dose valid for the whole interval

$$\underline{\underline{D = \bar{z}(D)\cdot p(D)}}$$

probability of dose deposit when dose is set at D.

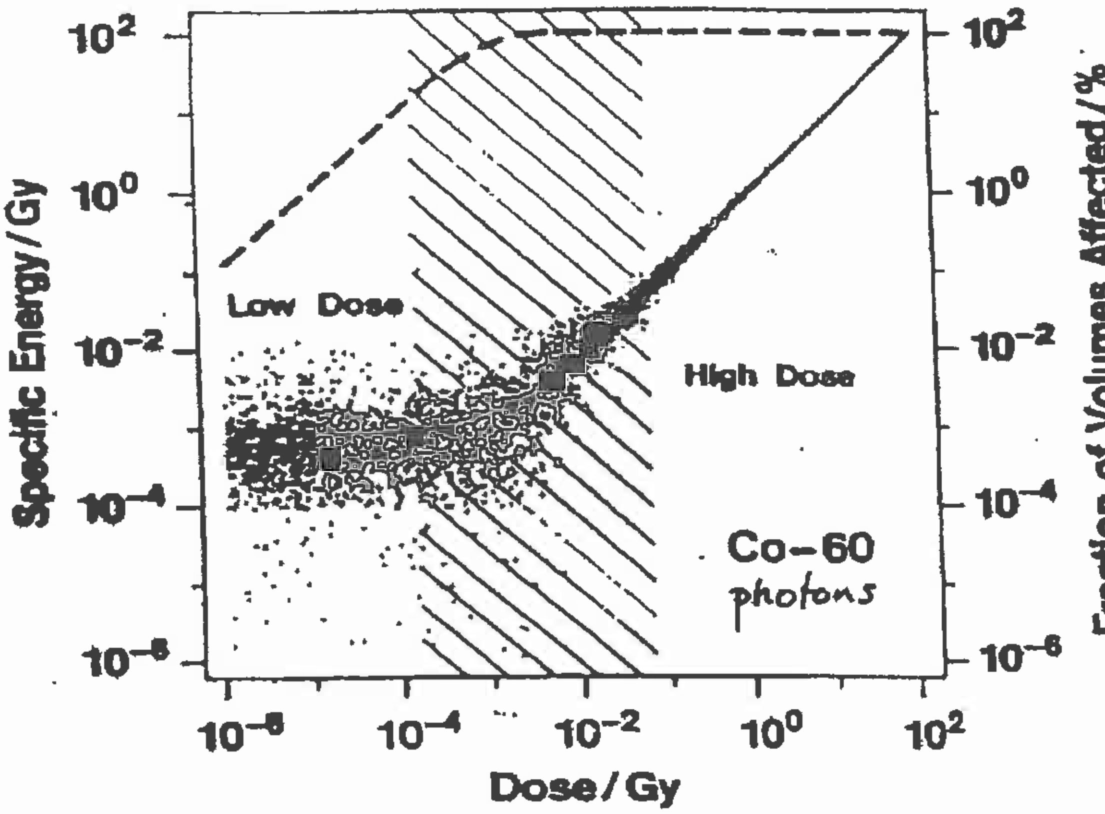
average specific energy at dose D

d) Graph: see fig (b) page 10 of Suppl. Dosimetry (enclosed)

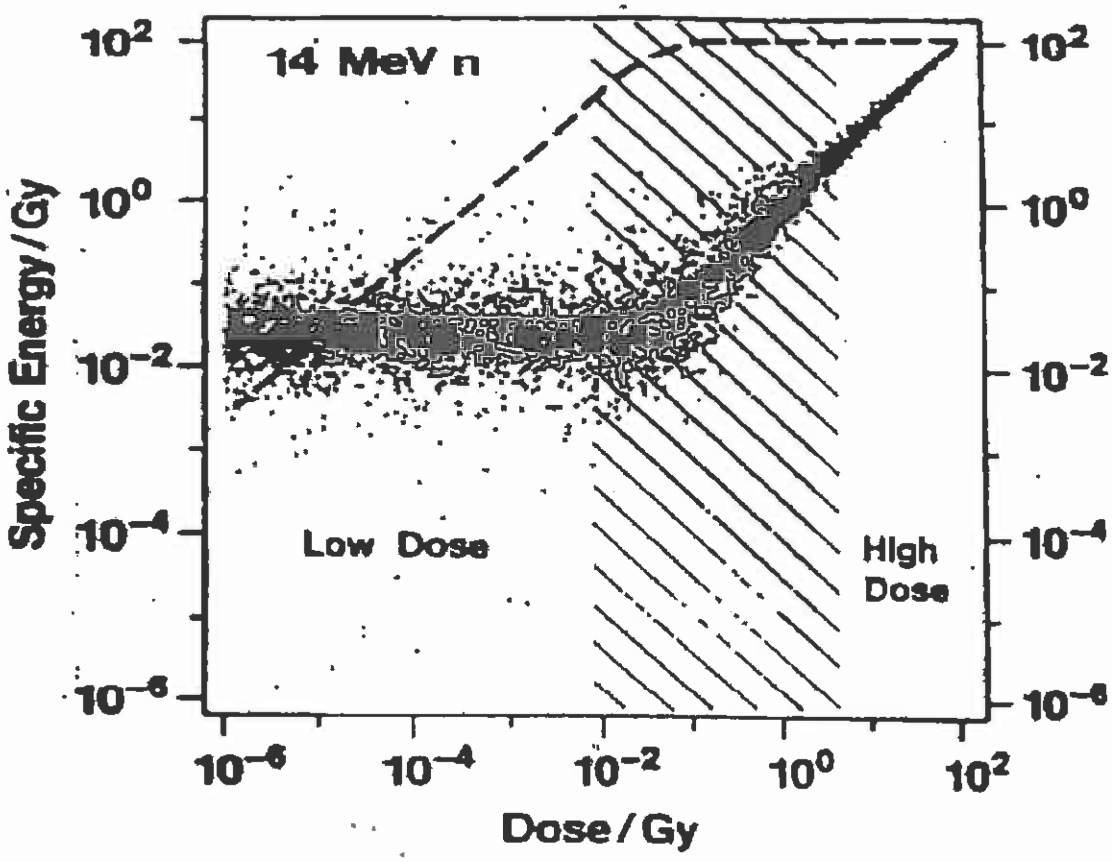
Comment: see comments page 9 of Suppl. Dosimetry (enclosed)

The two figures on the following page show measurements of specific energy by a micro-dosimetric detector in a situation where the exposure is gradually decreased so that very low values of dose will be given. Low doses are achieved by decreasing the exposure time and/or increasing the distance between the detector and the point source giving off the radiation. At low doses the stochastic nature of energy deposition will become apparent, and eventually the dose will be so low that not all exposure events result in an energy deposit in the detector. In the figure panels, the left ordinate shows specific energy z, the right ordinate shows the probability (in per cent) of an exposure causing an energy deposit in the detector (from secondary electron(s) passing through the detector).

Notice that as long as this probability p is 100 per cent, specific energy z equals dose D, as indicated by the diagonal segment of the graph. For lower doses, specific energy reaches a constant plateau, whereas the probability of a "hit" (i.e. an energy deposit in the detector) decreases.



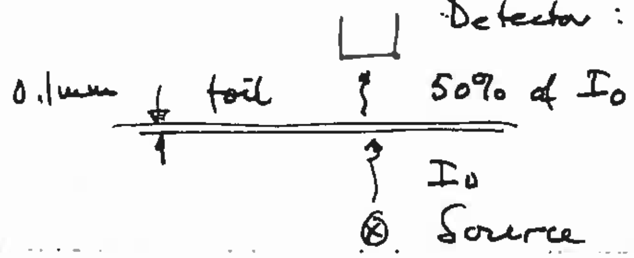
(a)



(*b*)

The main conclusion from these two panels is that each type of radiation has a characteristic value for its single-hit energy deposition in a small volume like the nucleus of a biological cell. For Co-60 photons this value is indicated as 1 mGy, whereas for neutrons which have protons as their secondary charged particles, this value is 50 mGy, i.e. 50 times larger.

Problem 3.



Detector : Counting Efficiency $\varepsilon = 0.01 = 1\%$ in absence of foil

Negligible background

a) Required activity of the source for controlling thickness to within 5% variation. 1s counting time

5% change in thickness $\Rightarrow \geq 2\sigma$ change in count rate.

Change Δx in thickness leads to change in intensity :

$$I(x_0+\Delta x) = I_0 e^{-\mu(x_0+\Delta x)} = I(x_0)e^{-\mu\Delta x} \approx I(x_0)(1-\mu\Delta x)$$

if $\mu\Delta x \ll 1$.

Determines μ from $\frac{I(x_0)}{I_0} = 0.5$

$\Rightarrow \quad I_0 e^{-\mu x_0} = 0.5 \cdot I_0 \quad \Rightarrow \quad \mu x_0 = \ln\frac{1}{0.5} = 0.693$

$\Rightarrow \quad \mu = \frac{0.693}{x_0} = \frac{0.693}{0.1\,\mathrm{mm}} = 6{,}93\ \mathrm{mm}^{-1}$

5% change in $x_0 \Rightarrow \quad \Delta x = 0{,}05 \cdot x_0 = 0{,}05 \cdot 0.1\,\mathrm{mm} = 5\cdot 10^{-3}\,\mathrm{mm}$

Sets intensity equal to count rate : $\quad r \equiv I$

$$r(x_0+\Delta x) = r(x_0)\left[1-\mu\Delta x\right] = r(x_0)\left[1 - 6{,}93\tfrac{1}{\mathrm{mm}}\cdot 5\cdot 10^{-3}\,\mathrm{mm}\right]$$

$$r(x_0+\Delta x) = r(x_0)\left[1 - \underbrace{0{,}03465}_{\ll 1}\right]$$

not a very good approximation.

$$|\Delta r| = \left| r(x_0)\left[e^{-\mu\Delta x} - 1\right]\right| = r(x_0)\left[e^{-0{,}03465} - 1\right] = \underline{0{,}03406 \cdot r(x_0)}$$

Requirement: $|\Delta r| \geq 2\sigma_r$

Standard deviation in count rate: σ_r

$$r = \frac{n}{t} \quad \leftarrow \text{negligible background} \qquad \text{Poisson distribution} \Rightarrow \sigma_n = \sqrt{n}$$

$$\mathrm{Var}(r) = \frac{1}{t^2} \cdot \mathrm{Var}(n) = \frac{n}{t^2} = \frac{r}{t} \Rightarrow \sigma_r = \sqrt{\frac{r}{t}}$$

$$\Rightarrow \Delta r = 0{,}03406 \cdot r(x_0) \geq 2\sqrt{\frac{r(x_0)}{t}}$$

$$(0{,}03406)^2 r^2(x_0) \geq 4\,\frac{r(x_0)}{t}$$

$$\Rightarrow \underline{r(x_0) \geq} \frac{4}{(0{,}03406)^2 \cdot \underbrace{t}_{=1\,\mathrm{s}}} = \underline{3448 \text{ cps.}}$$

$$\underline{r(x=0)} = I_0 = \frac{r(x_0)}{0.5} = \frac{3448 \text{ cps}}{0.5} = \underline{6896 \text{ cps}}$$

Required minimum activity of source: A_0

$$\text{From } r(x=0) = \varepsilon A_0 \Rightarrow A_0 = \frac{r(x=0)}{\varepsilon} = \frac{6896 \text{ cps}}{0.01 \text{ cps/Bq}}$$

$$\Rightarrow \underline{\underline{A_0}} = 689{,}6 \text{ kBq} \approx \underline{\underline{690 \text{ kBq}}}.$$

b) Expression for counting efficiency:

$$\varepsilon = f \cdot p \cdot \Omega \cdot k$$

k : yield of photons of energy E_γ per disintegration of source

Ω : solid angle ~~seen~~ by detector, as fraction of 4π.

p : probability of photon interacting with detector material
$p = (1 - e^{-\mu_D x_D})$ where μ_D is attenuation coeff. of photons of energy E_γ in detector material and x_D is thickness of detector.

f : full-energy fraction of registered photon spectrum, with ROI cut around peak at energy E_γ.