Answers to Exam H12. 12.12.2012.

Problem 1.

a) EC disintegration:

$$ {}^{A}_{Z}X_{N} + e \rightarrow {}^{A}_{Z-1}Y_{N+1} + \nu $$

$$ Q = \left(\sum m_{initial} - \sum m_{final}\right)c^2 $$

$$ Q = c^2\Big\{ \underbrace{\underset{\text{atomic mass}}{m({}^{A}_{Z}X)} - Zm_e}_{\simeq \text{ nuclear mass}} + \Big(m_e - \underset{\text{binding energy of electron}}{\frac{E_B}{c^2}}\Big) - \Big[m({}^{A}_{Z-1}Y) + (Z-1)m_e\Big] - \underset{m_\nu \simeq 0}{m_\nu} \Big\} $$

$$ Q = \Big[m({}^{A}_{Z}X) - m({}^{A}_{Z-1}Y) \Big] + \Big(-Z+1+Z-1\Big)m_e - \frac{E_B}{c^2}\Big] c^2 $$

$$ Q = \Big[m({}^{A}_{Z}X) - m({}^{A}_{Z-1}Y) \Big]c^2 - E_B. $$

If transition goes to an excited state Y^* of the daughter nuclide:

$$ \Rightarrow Q^* = \Big[m({}^{A}_{Z}X) - m({}^{A}_{Z-1}Y^*) \Big]c^2 - E_B. $$

where $m({}^{A}_{Z-1}Y^*) = m({}^{A}_{Z-1}Y) + \frac{E_{ex}}{c^2}$, $E_{ex} = E_2, E_3$

Transition to $E_3 = 2340$ keV:

$$Q_3 = (m_P - m_D)c^2 - E_3 - E_B$$

parent daughter

$$(m_P - m_D) = \left[\left(m\left({}^A_Z X\right) - A\right) - \left(m\left({}^{A}_{Z-1} Y\right) - A\right)\right] = \Delta_P - \Delta_D$$

$$\Rightarrow (m_P - m_D) = (\Delta_{Bi\text{-}207} - \Delta_{Pb\,207}) = [-21545 - (-24119)]\,\mu u$$

$$(m_P - m_D) = \Delta m = 2574\,\mu u$$

Capture of K-electrons: $E_K(\text{Bi}207) = E_K(Z=83) = 90.5$ keV.

$$Q_{3K} = 2574 \cdot 10^{-6} u \cdot 931{,}502 \frac{\text{MeV}}{u} - 2340\text{ keV} - 90{,}5\text{ keV} = -33\text{ keV}.$$

EC transition to E_3 by K-electron capture is not possible

Capture of L-electrons: $E_L(\text{Bi}207) = E_L(Z=83) = 15.2$ keV.

$$Q_{3L} = \underbrace{2574 \cdot 10^{-6} u \cdot 931{,}502 \frac{\text{MeV}}{u}}_{2397.7\text{ keV}} - 2340\text{ keV} - 15.2\text{ keV} = 43\text{ keV}$$

EC transition to E_3 by L-electron capture is possible since $Q_{3L} > 0$.

Transition to level $E_2 = 1630$ keV :

$$Q_2 = \Delta m c^2 - E_2 - E_B \qquad E_B = \begin{cases} E_K = 90.5 \text{ keV} \\ E_L = 15.2 \text{ keV} \end{cases}$$

$Q_{2K} = (2397.7 - 1630 - 90.5)$ keV $= \underline{677 \text{ keV}}$) : > 0

$Q_{2L} = (2397.7 - 1630 - 15.2)$ keV $= \underline{750 \text{ keV}}$) : > 0

EC transition to E_2 is possible by both K and L electron capture.

b) γ - transitions.

γ_1 : $\frac{5}{2}^- \longrightarrow \frac{1}{2}^-$

Parity : $\Delta\pi$ = No $\Rightarrow$ Possible multipoles : M1, E2, M3, ...

$$|I_f - I_i| \leq \sigma L \leq |I_f + I_i|$$
$$\left|\tfrac{1}{2} - \tfrac{5}{2}\right| \leq \sigma L \leq \left|\tfrac{1}{2} + \tfrac{5}{2}\right|$$
$$2 \leq \sigma L \leq 3$$

Lowest possible multipole will dominate, i.e. E2

Probability of $M(L+1) \ll$ Probability of $E(L)$, therefore no contribution from M3.

γ_2: $\underline{\frac{13}{2}^+ \longrightarrow \frac{5}{2}^-}$

Parity: $\Delta\pi$ = Yes $\Rightarrow$ Possible multipoles: E1, M2, E3, M4, -

$$\left|\frac{5}{2}-\frac{13}{2}\right| \leq \Delta L \leq \left|\frac{5}{2}+\frac{13}{2}\right|$$
$$4 \leq \Delta L \leq 9$$

Lowest multipole dominates i.e. M4, possibly with some contribution from E5 since E(L+1) is not too improbable compared to M(L)

γ_3: $\underline{\frac{7}{2}^- \longrightarrow \frac{5}{2}^-}$

Parity: $\Delta\pi$ = No $\Rightarrow$ Possible multipoles: M1, E2, M3, ...

$$\left|\frac{5}{2}-\frac{7}{2}\right| \leq \Delta L \leq \left|\frac{5}{2}+\frac{7}{2}\right|$$
$$1 \leq \Delta L \leq 6$$

Lowest multipole dominates i.e. M1, possibley with some contribution from E2.

γ_2 is of significant higher order than the two other transition. Thus, E_2 has longer halflife than E_1 and E_3 $\Rightarrow$ E_2 is the metastable state

c) Conversion electrons in parallel to γ.

$E_{\gamma i} = \Delta E = E_i - E_j$

here, the recoil energy of the daughter nucleus is neglected

Conversion electron by deexcitation from E_1:

$E_{1K} = (E_1 - E_0) - E_K(Z=82) = (570 - 0 - 88.0)\,\text{keV} = 482\ \text{keV}$

binding energy in Pb-207

$E_{1L} = (E_1 - E_0) - E_L(Z=82) = (570 - 0 - 14.7)\,\text{keV} = 555\ \text{keV}.$

CE from E_2:

$E_{2K} = E_2 - E_1 - E_K(Z=82) = (1630 - 570 - 88.0)\,\text{keV} = 972\ \text{keV}$

$E_{2L} = E_2 - E_1 - E_K(Z=82) = (1630 - 570 - 14.7)\,\text{keV} = 1045\ \text{keV}.$

CE from E_3:

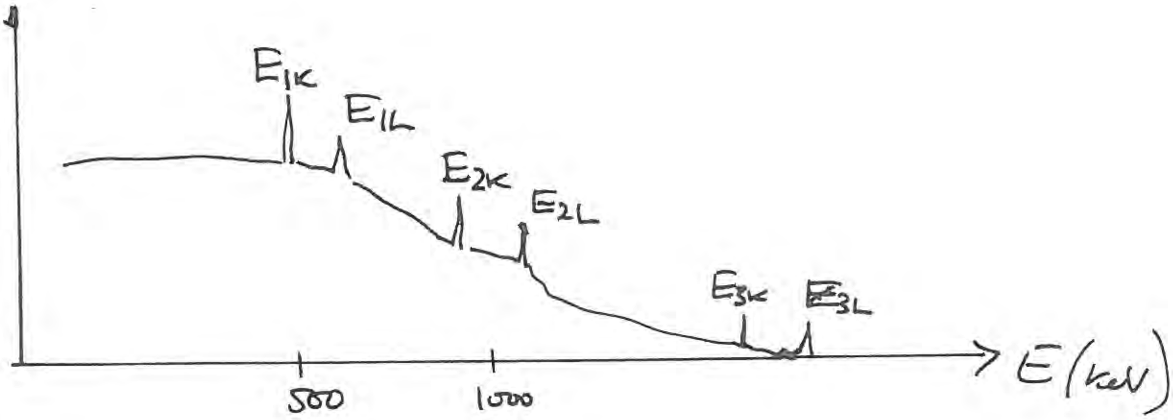
$E_{3K} = E_3 - E_1 - E_K(Z=82) = (2340.0 - 570 - 88.0)\,\text{keV} = 1682\ \text{keV}$

$E_{3L} = E_3 - E_1 - E_L(Z=82) = (2340.0 - 570 - 14.7)\,\text{keV} = 1755.6\,\text{keV}.$

Principle of determining stopping power of electrons (500 keV, 1000 keV) in e.g. aluminium.

Conversion electrons from E_1 and E_2 have suitable energies.

Spectrum of emitted electrons



Determine shift (to the left) in peak positions between spectrum 1 acquired without material between source and detector, and spectrum 2 acquired after material of thickness ΔX is inserted between source and detector. Otherwise same detection geometry.

Energy decrement after passage through material thickness ΔX:

$-\Delta E = E(\text{spectrum } 2) - E(\text{spectrum } 1)$ recorded for peak E_{1K} and peak E_{2K}.

Estimate of stopping power $S_{col} = \left(\frac{-\Delta E}{\Delta X}\right)$ for peak $E_{1K} \simeq 500$ keV and peak $E_{2K} \simeq 1000$ keV.

Suitable detector for β-spectroscopy will be a semiconductor diode detector of sufficient thickness to obtain full energy deposition from the most energetic electrons in the emission.

d) Nucleon configuration in ground state:

$^{207}_{83}Bi_{124}$

83 P : Filling all shells $1s\,1/2$ ... through $1h\,11/2$, (total of 82)
Single proton (no 83) in $1h\frac{9}{2}$ i.e. $(1h\frac{9}{2})^1$

124 N : Filling all shells $1s\,1/2$ ... through $2f\,5/2$ (total of 124)

Pairwise spin cancellation for all nucleons/states, except for single proton in state $(1h\,9/2)$, which therefore will determine net spin and parity of the ground state.

Spin : $9/2$, parity $(-1)^l = (-1)^5 = -1$ i.e. $9/2^-$

$^{207}_{82}Pb_{125}$

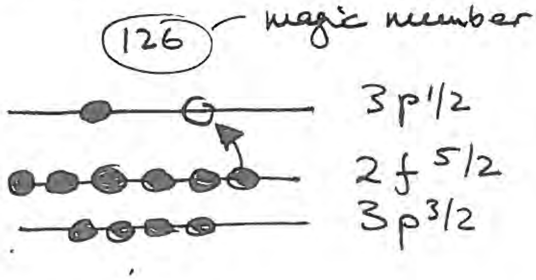
82 P : Filled shells $1s\,1/2$ through $1h\,11/2$ (total of 82)

125 N : Filled shells $1s\,1/2$ through $2f\,5/2$ (total of 124), and last neutron (no. 125) single in state $(3p\,1/2)^1$.

Pairwise spin cancellation except for single neutron (valence neutron) in state ($3p\,1/2$).

<u>Spin : $1/2$, parity $(-1)^{\ell} = (-1)^{1} = -1$ i.e. $1/2^{-}$</u>

<u>Excited state E_1 of Pb207 : $\frac{5}{2}^{-}$</u>



total of 125 neutrons in Pb207.

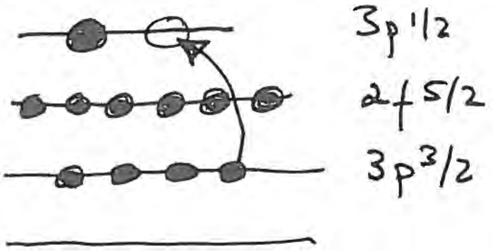
<u>Excite one neutron from $2f\,5/2$ to $3p\,1/2$</u>

$\Rightarrow$ Spin cancellation in $3p\,1/2$,

single vacancy in $2f\,5/2$ gives spin $\frac{5}{2}$,

parity $(-1)^{\ell}$ $\ell = 3$ for f $\Rightarrow \pi = -1$ <u>i.e $\frac{5}{2}^{-}$ q.e.d.</u>

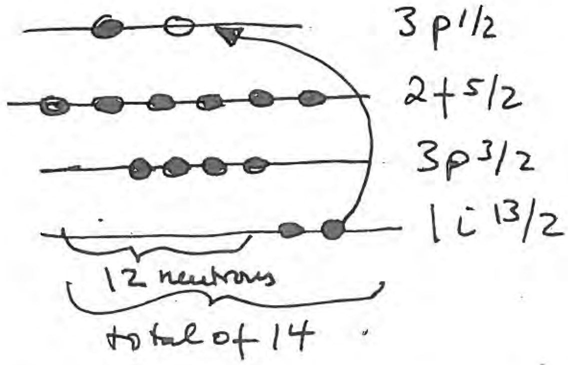
Excited state $\frac{3}{2}^{-}$ of Pb207:



Excite one neutron from $3p\,3/2$ to $3p\,1/2$.

Single vacancy in $3p\,3/2$ determines spin and parity <u>i.e. $3/2^{-}$ q.e.d.</u>

Excited state E_2 of Pb207 : $\frac{13}{2}^+$



Excite one neutron from $1i\frac{13}{2}$ to $3p^1/2$

$\Rightarrow$ spin cancellation in $3p^1/2$, single vacancy in $1i\frac{13}{2}$ contributes spin $\frac{13}{2}$, parity $(-1)^l$ for $l=6 \Rightarrow +1$

i.e. Excited state E_2 is $\frac{13}{2}^+$ q.e.d.

Excited state E_3 is $\frac{7}{2}^-$

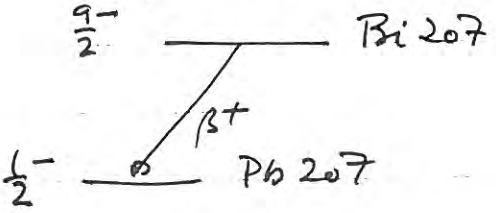
Excite one neutron from $2f\frac{7}{2}$ to $3p^1/2$

$\Rightarrow$ Spin cancellation in $3p^1/2$,

Single vacancy in $2f\frac{7}{2}$, $\Rightarrow$ spin $\frac{7}{2}$,

parity $(-1)^l$ for $l=3 \Rightarrow \pi=-1$ i.e. E_3 state is $\frac{7}{2}^-$ q.e.d

e) Possible β^+ emission from ground state of Bi 207 to ground state of Pb 207



Classification of β^+ disintegration:

No parity change $\Rightarrow$ Possible L-values $L = 0, 2, 4, \cdots$

$(\vec{L}+\vec{S})$ is total angular momentum represented by the (β^+, ν) particle system.

$$|I_f - I_i| \leq |\vec{L}+\vec{S}| \leq |I_f + I_i|$$

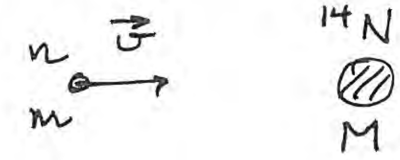
$$\left|\frac{1}{2} - \frac{9}{2}\right| \leq |\vec{L}+\vec{S}| \leq \frac{9}{2} + \frac{1}{2}$$

$$\underline{\underline{4 \leq |\vec{L}+\vec{S}| \leq 5}} \quad , \underline{L \text{ even.}}$$

Lowest possible value of L is $L=4$, which in combination with both $S=0$ (Fermi) and $S=1$ (Gamow/Teller) satisfy the above requirement.

<u>i.e. 4th forbidden transition of mixed Fermi and G/T type.</u>

Problem 2.

a) 

M at rest in lab system.

$$\Rightarrow \underline{T_{lab} = T_n = \frac{1}{2} m v^2}$$

Velocity of CM (coordinate) system:

$$\underline{\vec{v}_{CM}} = \frac{\sum \vec{p}_i}{\sum m_i} = \frac{m\vec{v}}{(m+M)} = \underline{\frac{m}{m+M} \cdot \vec{v}}$$

Velocities in CM coordinate system:

$$\underline{\vec{v}_m'} = \vec{v} - \vec{v}_{CM} = \frac{m+M-m}{m+M} \cdot \vec{v} = \underline{\frac{M}{m+M} \cdot \vec{v}}$$

$$\underline{\vec{v}_M'} = 0 - \vec{v}_{CM} = \underline{-\frac{m}{m+M} \cdot \vec{v}}$$

Total kinetic energy in CM system (i.e. "collision energy"):

$$T_c = \sum T_i' = \frac{1}{2} m (v_m')^2 + \frac{1}{2} M (v_M')^2 = T'$$

$$T_c = \frac{1}{2} m \left(\frac{M}{m+M}\right)^2 v^2 + \frac{1}{2} M \left(\frac{m}{m+M}\right)^2 v^2 = \frac{1}{2} m v^2 \cdot \frac{M}{m+M}$$

$$\underline{\underline{T_c = T_n \cdot \frac{M}{m+M} \qquad q.e.d.}}$$

(It is easily seen that kinetic energy <u>of</u> the center of mass, $T_{CM} = \frac{1}{2}(m+M) v_{CM}^2 = T_n \frac{m}{m+M}$, thus total energy $T = T' + T_{CM} = \frac{1}{2} m v^2 = T_{lab}$.)

b) $\quad {}^{14}N + n \rightarrow {}^{15}N^*$

at rest in CM system.

Conservation of total energy in CM coordinate system:

$$m({}^{14}N) + m_n + T_c/c^2 = M({}^{15}N^*) = M({}^{15}N) + E^*/c^2$$

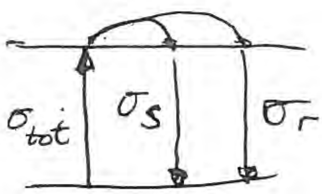
$$\Rightarrow E^* = \left[m({}^{14}N) + m_n - m({}^{15}N)\right]c^2 + T_c$$

Excitation energy of ${}^{15}N^*$: $\quad \left(T_c = 1.779 \cdot \frac{14}{14+1}\ \mathrm{MeV}\right)$

$$E^* = \left[(14{,}003074 + 1.008665 - 15{,}000109) \cdot 931{,}5 + \frac{14}{14+1} \cdot 1.779\right] \mathrm{MeV}$$

$$\underline{\underline{E^* = (10.8333 + 1.6604)\ \mathrm{MeV} = 12.49\ \mathrm{MeV}.}}$$

c) Branching for reactions, i.e. interactions different from elastic scattering (s)



$$y_r = \frac{\sigma_r}{\sigma_{tot}} \qquad \sigma_{tot} = \sigma_s + \sigma_r$$

$$\sigma_r = \sigma_{tot} - \sigma_s = 40b - 24b = 16b.$$

$$\underline{y_r} = \frac{\sigma_r}{\sigma_{tot}} = \frac{16b}{40b} = \underline{0.4 \quad \text{i.e. } 40\%}$$

<u>40% of interactions will be something else than elastic scattering.</u>

Problem 3.

a) Specific effective energy:

$$SEE = \frac{1}{m_T} \sum_i k_i \bar{E}_i \varphi_i (T \leftarrow S)$$

m_T : mass of target organ T. (here T = whole body)
k_i : yield of radiation type i from source S.
$\bar{E}_i$: average quantum energy of radiation i.
(= E_γ for gamma, $\approx \frac{1}{3} E_{max}$ for beta)
φ_i : absorbed fraction of radiation i in target T.
For α and β : $\varphi = 0$ if $S \not\equiv T$
$\varphi = 1$ if $S \equiv T$
For γ : φ must be known (here 0.75 for 662 keV)

	k_i	$\bar{E}_i$	φ_i	$k_i \bar{E}_i \varphi_i$
β_1	94.4 %	514/3 keV	1	161.7 keV
γ	94.4 %	662 keV	0.75	468.7 keV
β_2	5.6 %	1176/3 keV	1	21.9 keV.
				Σ = 652.4 keV/dis.

$$SEE = \frac{1}{70\,kg} \cdot 652.4\ keV/dis \cdot 1{,}602 \cdot 10^{-16} \frac{J}{keV}.$$

$$SEE = 14.9 \cdot 10^{-16} \frac{J}{kg \cdot dis.} = 1.5 \cdot 10^{-15}\ Gy/dis.\quad q.e.d.$$

b) Committed effective dose after intake of radioactivity A_0 at time $t=0$:

$$\underline{E(50)} = \sum_T \underbrace{w_T}_{\text{tissue weighting factor}} \underline{H_T(50)} = \sum_T w_T \sum \underbrace{w_R}_{\text{radiation weighting factor.}} \cdot \underline{D_{T,R}(50)}$$

$$\underline{D_{T,R}(50) = \int_0^{50\,yrs} \dot{D}_{T,R}(t)\,dt = \int_0^{50} SEE \cdot A_0 e^{-\lambda_{eff} \cdot t}\,dt.}$$

Here, $w_T = 1{,}0$ (whole body as source and target organ)
$w_R = 1$ (β and γ radiation)
$\lambda_{eff} = \lambda_{rad} + \lambda_{biol}$, $\lambda = \ln 2 / T_{1/2}$.

$T_{1/2\,biol} \ll T_{1/2\,rad}$ and $T_{1/2\,biol} \ll 50$ yrs

$$\Rightarrow \underline{\lambda_{eff} \simeq \lambda_{biol}} \text{ and } \int_0^{50\,yrs} e^{-\lambda_{eff} \cdot t}\,dt \simeq \int_0^{\infty} e^{-\lambda_{eff} \cdot t}\,dt = \underline{\frac{1}{\lambda_{eff}}}.$$

With the above simplifications:

$$e(50) = \frac{E(50)}{A_0} = \frac{1}{A_0} \cdot w_T w_R \cdot SEE \cdot A_0 \cdot \frac{1}{\lambda_{biol}}.$$

$$\Rightarrow T_{1/2\,biol} = \frac{\ln 2}{\lambda_{biol}} = \frac{\ln 2 \cdot e(50)}{w_T w_R \cdot SEE}$$

$$T_{1/2\,biol} = \frac{0.693 \cdot 13 \cdot 10^{-6}\,Sv \frac{1}{10^3\,Bq}}{1.0 \cdot 1\,Sv/Gy \cdot 1.5 \cdot 10^{-15}\,\frac{Gy}{Bq \cdot s}} = 6 \cdot 10^6\,s$$

$$\underline{\underline{T_{1/2\,biol}}} = \frac{6 \cdot 10^6\,s}{24 \cdot 3600\,s/day} = \underline{\underline{69 \text{ days.}}} \quad (\ll 30\,yrs)$$