Exam 2015 TFY4225

Problem 1.

To answer this problem, one of the following applications should be discussed:

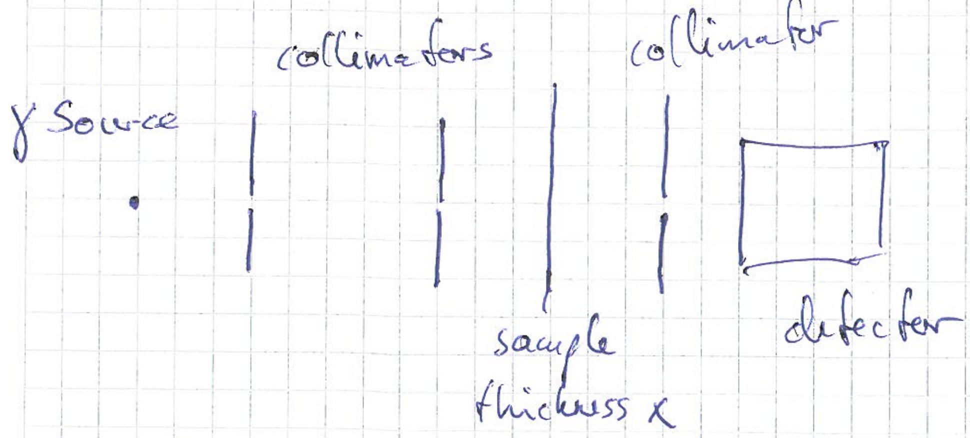
- Tracing
- Gauging
- Material Modification
- Sterilization and Food preservation
- Radioactive dating
- Neutron Activation analysis
- Rutherford Backscattering
- Particle-induced x-ray emission
- Accelerator Mass Spectroscopy

The above topics were presented to the class by groups of 2-3 students. Before the exam, students were told they would be able to answer question on exam only based on the application they pesented themselves.

Two parts of the question:

- physical principle must be explained satisfactory
- Some applications should be described.

Problem 2.



Measurement procedure:

1. Decide on a fixed integration time on the detector, make sure to get sufficient number of counts on the detector to get high accuracy of count. Arrival of photons follow Poisson statistics, so that std $= \sqrt{n}$.
2. Make a measurement without sample. Note # counts
3. Put in a known thickness of the material, repeat measurement of # counts.
4. Put in increasing thickness x of sample material and repeat meas. for each.

Analysis:

1. Plot # counts on a semilog y-scale as function of sample thickness x.
2. Fit a slope to the data. This slope is the linear attenuation coeff. μ.

Problem 2 cont.

Buildup:

Due to Compton-scattering photons will be scattered out of the primary beam, but continue to propagate through the material.

In effect the beam will broaden out laterally and the photon flux will not be attenuated according to the linear attenuation coefficient.

In order to remove the effect of buildup, a final collimator is inserted just in front of the detector. This ensures that the scattered part of the beam does not contribute significantly to the # counts.

Problem 3.

a) $$Q = \sum_{\text{before}} m_i c^2 - \sum_{\text{after}} m_i c^2$$

$$= m_P c^2 - m_D c^2 - m_\alpha c^2$$

Parent nucleus — Daughter — α-particle

Energy conservation:

$$T_P + m_P c^2 = T_D + T_\alpha + m_D c^2 + m_\alpha c^2$$

$$\underline{Q = T_D + T_\alpha} \quad , \; T_P = 0 \qquad (1)$$

Momentum conservation:

$$0 = \vec{p}_D + \vec{p}_\alpha$$

$$\underline{m_D v_D = m_\alpha v_\alpha} \qquad (2)$$

From (2): $2 m_D T_D = 2 m_\alpha T_\alpha$

$$T_D = \frac{m_\alpha}{m_D} T_\alpha$$

From (1): $Q = \frac{m_\alpha}{m_D} T_\alpha + T_\alpha$

$$\underline{T_\alpha = \frac{Q}{1 + \frac{m_\alpha}{m_D}}}$$

3b) $$m_A = m_N + Z \cdot m_e - \sum_i B_i$$

atomic mass — nuclear mass — free electron mass — electron binding energy

Q-value:

$$Q = \left[m_{P,N} - m_{D,N} - m_\alpha \right] c^2$$

$$= \Big[m_A\left({}^{232}_{90}Th\right) - 90 \cdot m_e + \sum_{i,\,Th} B_i$$

$$- m_A\left({}^{228}_{88}Ra\right) + 88\, m_e - \sum_{Ra} B_i$$

$$- m_A\left({}^{4}_{2}He\right) + 2 \cdot m_e - \sum_{He} B_i \Big] c^2$$

$$\simeq \left[m_A\left({}^{232}Th\right) - m_A\left({}^{228}Ra\right) - m_A\left({}^{4}He\right) \right] c^2$$

where we have assumed that the difference in electron binding energy is negligible.

3b forts.

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mass excess $\equiv m_A - A = me$

$\rightarrow Q = [me(^{232}Th) + 232 - me(^{228}Ra) - 228$

$- me(^4He) - 4] c^2$

$= [me(^{232}Th) - me(^{228}Th) - me(^4He)] c^2$

$= [38050 - 31064 - 2603] uc^2 \cdot 10^{-6}$

$= 4383 \cdot 10^{-6} uc^2$

$= 4383 \cdot 10^{-6} \cdot 931.502 \frac{MeV}{c^2} \cdot c^2$

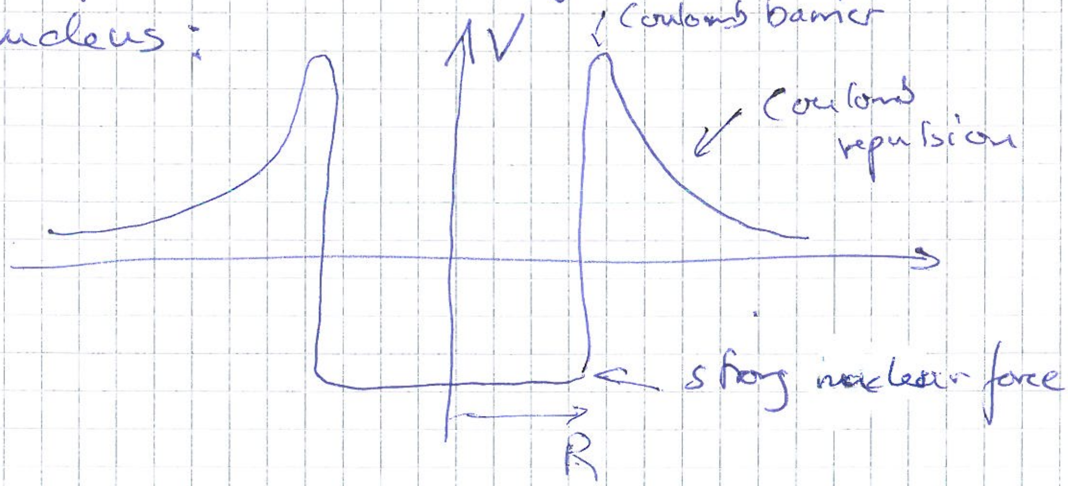
$= \underline{\underline{4.083 \text{ MeV}}}$

3c) Semi-classical theory of α-decay:

$$\lambda = P \cdot f \cdot e^{-2G}$$

The theory consists of these parts:

1. The daughter nucleus and α-particle is assumed to be pre-formed, i.e. instead of the parent nucleus, we assume an α-particle trapped in the potential well of the daughter nucleus:



The probability of this pre-formation is the factor P.

2. The α-particle will collide with the potential barrier f-times per time unit, where $f = \frac{1}{\tau}$, and τ is the time required to cross from one end to the other of the daughter nucleus:

$v_\alpha \cdot \tau = 2R$, v_α - velocity of α-particle

3. Each time the α-particle presents itself at the Coulomb-barrier, there is a certain probability of quantum tunneling to occur, given by the Gamow-factor e^{-2G}

The total rate of decay is then:

$$\underline{\lambda = P \cdot f \cdot e^{-2G}}$$

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3 d) This problem is mainly about applying a given equation and put in correct numbers and <u>units</u>.

P : Is assumed to be 1.

$$f = \frac{1}{\tau} = \frac{v_\alpha}{2R}$$

$\Rightarrow v_\alpha$: $T_\alpha = U + Q = 124\ MeV$

3d cont.

Non-relativistic calculation:

$$T_\alpha = \frac{1}{2} m_\alpha v_\alpha^2$$

$$v_\alpha = \sqrt{\frac{2T_\alpha}{m_\alpha}}$$

$$= \sqrt{\frac{2 \cdot 124\ \text{MeV}}{3727\ \text{MeV}/c^2}}$$

$$= \underline{c \cdot 0.26}$$

$$= \underline{\underline{7.7 \cdot 10^7\ \text{m/s}}}$$

Strictly speaking the above calculation should have been carried out using relativistic expressions, but since these are not given the student is not expected to do a relativistic calculation.

$$R = R_N(^{228}Ra) + R_N(^{4}He)$$

One can discuss whether the most correct radius should be the sum of the two nuclear radii, or simply the radius of the heavy nucleus. However, the above expression is what is given in Lilley.

$$R(^{228}Ra) = 1.2\,fm \cdot 228^{1/3} = \underline{7.33\ fm}$$

$$R(^{4}He) = 1.2\,fm \cdot 4^{1/3} = 1.90\,fm$$

$$\underline{R = 9.23\ fm}$$

$$\Rightarrow f = \frac{v_\alpha}{2R} = \underline{\underline{4.17 \cdot 10^{21}}}$$

3d cont 3.

$$G = \sqrt{\frac{2m}{\hbar^2 Q}} \cdot \frac{2Ze^2}{4\pi\varepsilon_0} \cdot \left[\cos^{-1}\left(\sqrt{\frac{Q}{B}}\right) - \sqrt{\frac{Q}{B}\left(1 - \frac{Q}{B}\right)} \right]$$

$$\sqrt{\frac{2m_\alpha}{\hbar^2 Q}} = \sqrt{\frac{2 \cdot 3727 \text{ MeV}/c^2}{\hbar^2 \; 4.08 \text{ MeV}}}$$

$$= \frac{1}{\hbar \cdot c} \cdot 42.73 = \frac{42.73}{197.33 \text{ MeV fm}}$$

$$= \underline{0.217 \; (\text{MeV fm})^{-1}}$$

$$\frac{2Ze^2}{4\pi\varepsilon_0} = 2 \cdot 88 \cdot 1.4399 \text{ MeV fm}$$

$$= \underline{253.4 \text{ MeV fm}}$$

$$\cos^{-1}\left(\sqrt{\frac{Q}{B}}\right) - \sqrt{\frac{Q}{B}\left(1 - \frac{Q}{B}\right)} :$$

$$B = \frac{2Ze^2}{4\pi\varepsilon_0 R} = 1.4399 \text{ MeV fm} \cdot \frac{88 \cdot 2}{9.23 \text{ fm}} = \underline{27.45 \text{ MeV}}$$

$$\frac{Q}{B} = \frac{4.083}{27.45} = \underline{0.149}$$

3 d conf 4

$$\cos^{-1}\sqrt{0.149} - \sqrt{0.149(1-0.149)} = \underline{0.819}$$

$$\Rightarrow G = 0.217 \ ^1/_{MeV\,fm} \cdot 253.4 \ MeV\,fm \cdot 0.819$$

$$= \underline{45.04}$$

$$\Rightarrow \lambda = 1 \cdot 4.17 \cdot 10^{21} \cdot e^{-2 \cdot 45.04} \ ^1/_s$$

$$= 3.15 \cdot 10^{-18} \ ^1/_s$$

$$t_{1/2} = \frac{\ln 2}{\lambda} = \underline{\underline{2.2 \cdot 10^{17} \ s}}$$

$$= \underline{\underline{7.0 \cdot 10^{9} \ years}}$$

The actual $t_{1/2}$ of ^{232}Th is $\underline{1.4 \cdot 10^{10}}$ years.

Problem 4.

Three things are required in order to solve this problem:

1. That the "compound nucleus" type of reaction is characterized by a reaction time much longer than the direct-collision interaction time.

2. That the life-time of the state is coupled to the energy-width by the uncertainty principle:

$$\Delta E \cdot \Delta \tau \geq \frac{\hbar}{2}$$

3. That the collision time can be estimated from the velocity of the projectile and the radius of the target nucleus:

$$v_n \, \tau_{coll} = 2R$$

Problem 4 cont.

Lifetime of the state:

$$\Delta\tau = \frac{\hbar}{2\cdot\Delta E} = \frac{6.58\cdot10^{-16}\ \text{eVs}}{0.13\ \text{eV}} = \underline{5.1\cdot10^{-15}\ \text{s}}$$

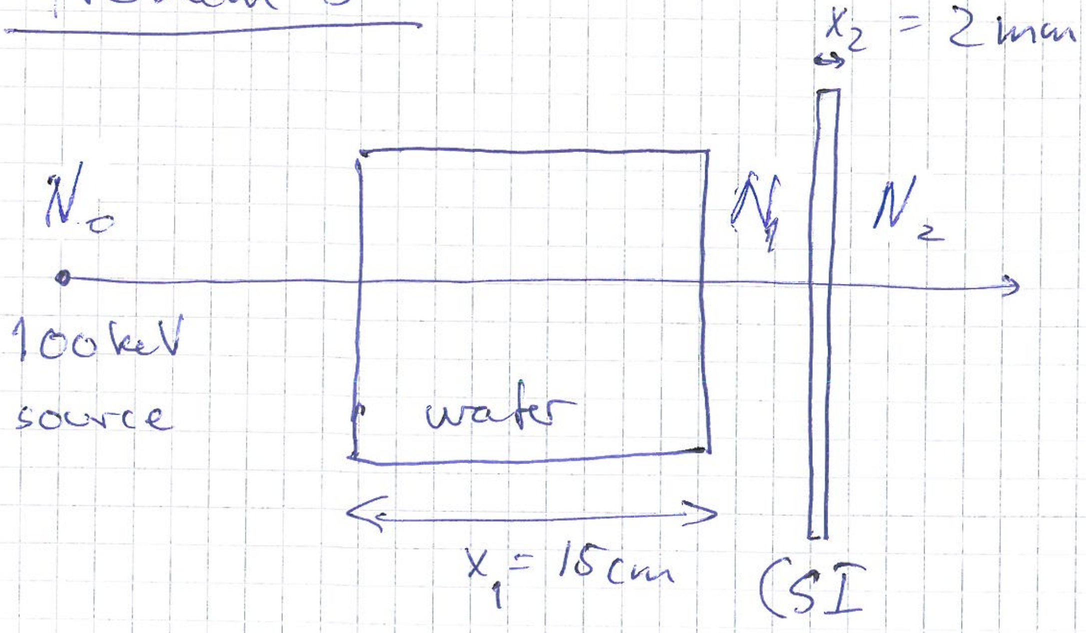
Collision time:

$$\tau_{coll} = \frac{2\cdot 1.2\ \text{fm}\cdot 113^{1/3}}{\sqrt{\frac{2\cdot 0.17\ \text{eV}}{939\ \text{MeV}/c^2}}} = \frac{11.6\ \text{fm}}{1.91\cdot10^{-5}\cdot c}$$

$$= \underline{2.0\cdot10^{-18}\ \text{s}}$$

The reaction time is three orders of magnitude longer than the collision time, so the reaction is a compound nucleus reaction.

Problem 5



We need to find N_0 such that

$$N_1 - N_2 = 100$$

$$N_1 = N_0 \, e^{-\mu_{water} \cdot 15\,cm}$$

$$N_2 = N_1 \cdot e^{-\mu_{CsI} \cdot 0.2\,cm}$$

$$N_1 - N_2 = N_1 \left(1 - e^{-\mu_{CsI} \cdot 0.2\,cm}\right)$$

$$= N_0 \, e^{-\mu_{water} \cdot 15\,cm} \left(1 - e^{-\mu_{CsI} \cdot 0.2\,cm}\right) = 100$$

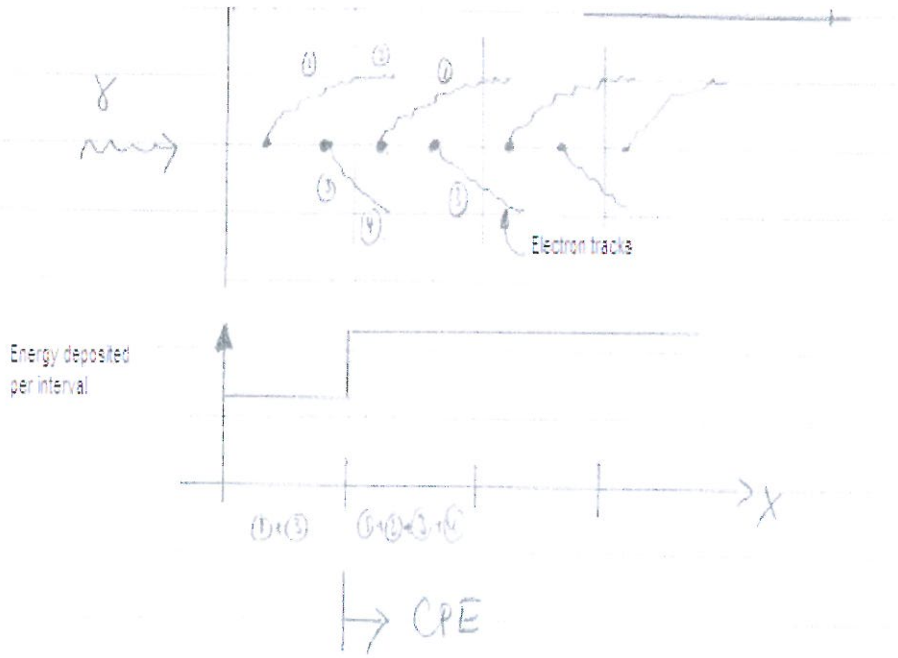
$$\underline{\underline{N_0 = 3585}}$$

Problem 6 ## 1.6 CPE (Charged particle equilibrium) 15

Dose can now be defined based on the mass energy absorption coefficient:

Dose: $D \stackrel{CPE}{=} \Psi\left(\frac{\mu_{en}}{\rho}\right), \quad \left[\frac{J}{kg}\right] = [Gy]$

Ψ is here the energy fluence of photons, i.e. $\Psi = \Phi \cdot h\nu$, where Φ is the particle fluence of photons. Due to conservation of linear momentum during the interactions, the track of each secondary electron will on average be in the forward direction, and the field of secondary charged particles therefore has a net forward direction. Close to the surface of the material the density of tracks in a material slice of thickness dx will therefore be less than in a slice further into the material. Charge particle equilibrium (here: electron equilibrium) can be illustrated to exist from a certain depth x in the material if equally many tracks that originate at lower x-values are found in the slice at depth x as those that exit from this slice into larger depths of the material. The figure below is meant to illustrate this. Actually, it is the dose contributions from those electron tracks gained and lost at depth x that must be equal for CPE to exist at depth x.



Problem 6 cont.

$$D \overset{CPE}{=} \Psi \cdot \frac{\mu_{en}}{\rho} = K_c \quad \text{collision kerma}$$

Ignoring photon attenuation; CPE assumed:

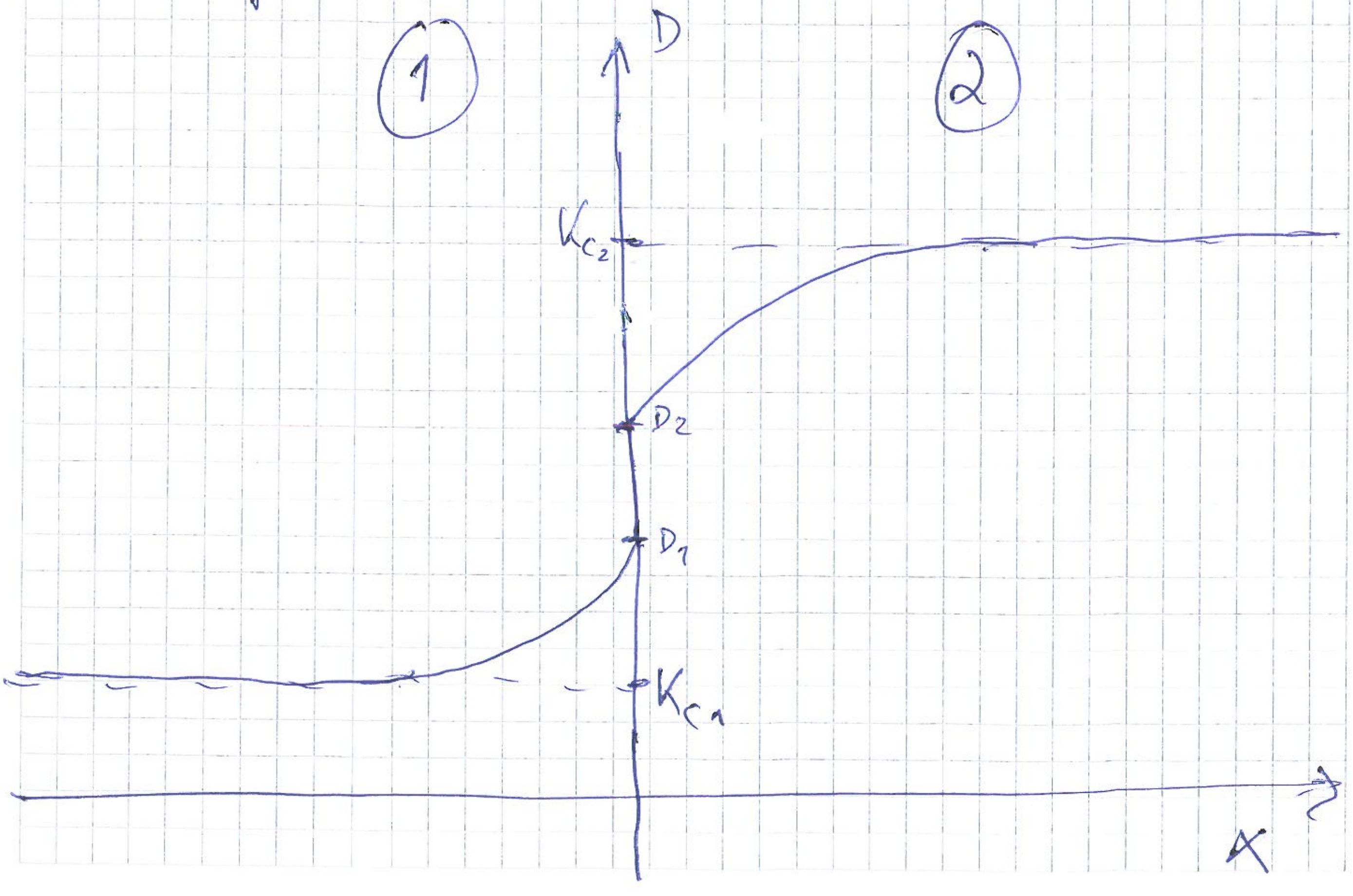
$$\left(\frac{\mu_{en}}{\rho}\right)_1 < \left(\frac{\mu_{en}}{\rho}\right)_2 \Rightarrow K_{c_1} < K_{c_2}$$

$$D_1 < D_2$$

At the boundary, charged particle flux is assumed to be continuous:

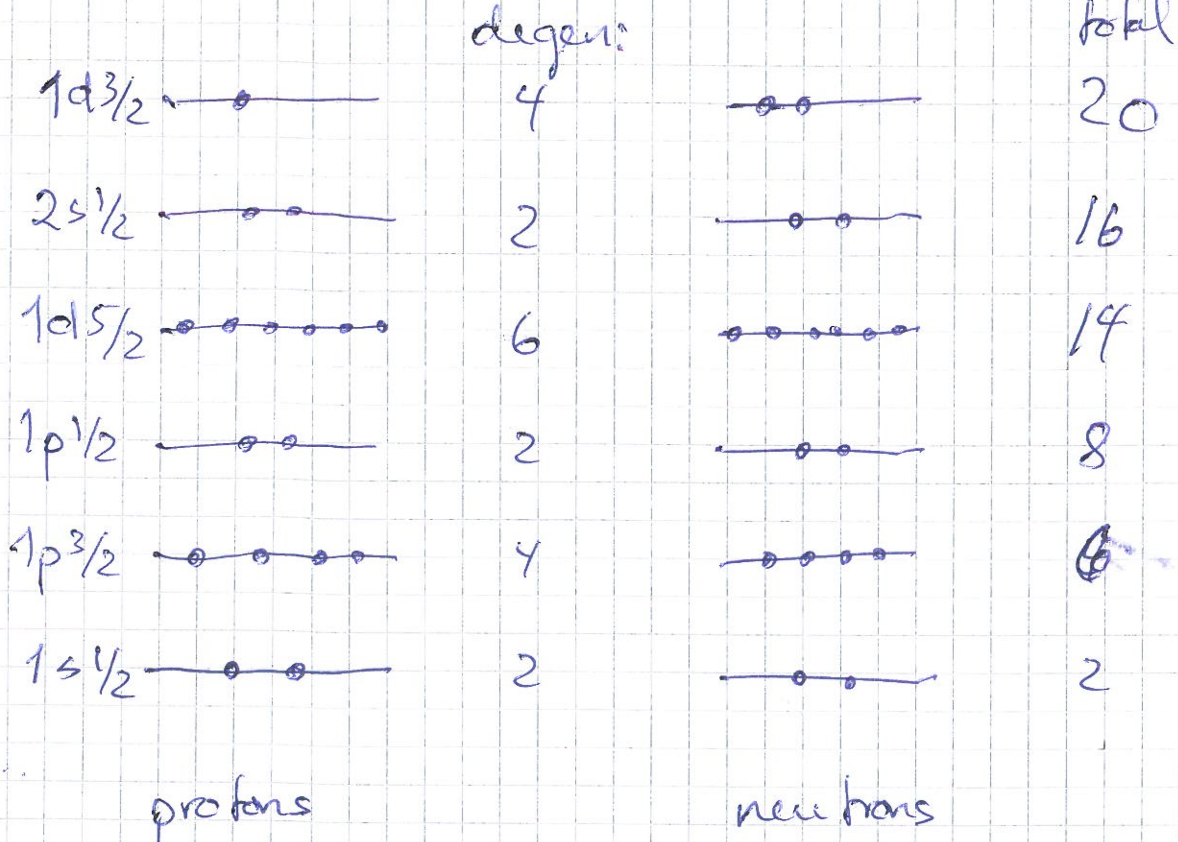
$$\frac{D_2}{D_1} = \frac{\left(\frac{S_c}{\rho}\right)_2}{\left(\frac{S_c}{\rho}\right)_1} \overset{\text{given}}{>} 1$$

Interface:



Problem 7.

$^{35}_{17}Cl$: 17 protons
18 neutrons.



Single unpaired proton is in state $1d\,3/2$

Single nucleon state interpretation of shell model then predicts the ground state of ^{35}Cl to be:

$$\underline{\underline{\frac{3}{2}^{+}}}$$

(parity found from $l = \overset{s}{0}, \overset{p}{1}, \overset{d}{2}\ \overset{f}{3}$

$\pi = (-1)^{l}$

Problem 7 cont.

Example

Possible excited states from single nucleon excitation:

proton excited from $2s\,1/2 \rightarrow 1d\,3/2$

$\Rightarrow$ single unpaired proton in $2s\,1/2$

$\Rightarrow$ $\underline{\underline{\frac{1}{2}^{+}}}$ nuclear state

Proton excited from $1d\,5/2 \rightarrow 1d\,3/2$

$\Rightarrow$ $\underline{\underline{\frac{5}{2}^{+}}}$

Proton excited from $1p\,1/2 \rightarrow 1d\,3/2$

$\Rightarrow$ $\underline{\underline{\frac{1}{2}^{-}}}$

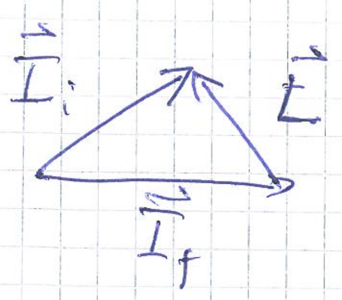
More possibilities exist.

Problem 7 cont.

De-excitation by gamma-emission.

$$\frac{1}{2}^{+} \rightarrow \frac{3}{2}^{+}$$

1. Conservation of angular momentum:



$\vec{L}$ = angular momentum carried by emitted photon

$$|I_i - I_f| \leq L \leq I_i + I_f$$

$$1 \leq L \leq 2 \quad \Rightarrow \quad \text{Lowest } L = 1$$

Conservation of parity:

$$\Pi_i = \Pi_f$$

$$\Pi\left(\tfrac{1}{2}^{+}\right) = \Pi\left(\tfrac{3}{2}^{+}\right) \cdot \Pi_{photon}$$

$$\Rightarrow \Pi_{photon} = \text{positive}$$

M-mode parity : $(-1)^{L+1}$
E-mode parity : $(-1)^{L}$

$\Big\}$ $\underline{\underline{M1}}$ lowest mode

Problem 7 cont.

M1 + E2 mode will be observed, since E-modes have higher probability than M-modes of the same polarity.

M1 + E2 are both high probability compared to higher order photons, so one would expect a relatively short half-life of such excited state of ^{35}Cl.