TFY4225 Nuclear and Radiation Physics Exam 2017: Suggested Solutions

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Problem 1

- Semi-empirical mass formula (SEMF)
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$$B = a_v A - a_s A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_{sym}\frac{(A-2Z)^2}{A} + \delta$$

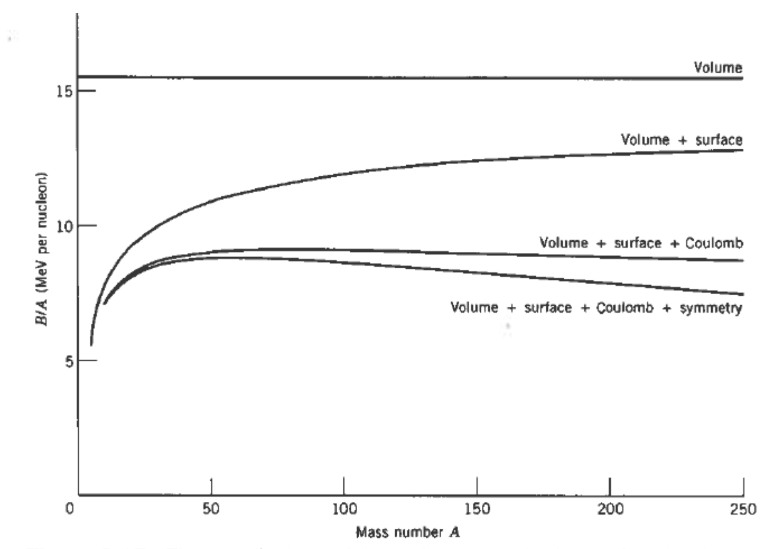


Figure 1: Contributions of the various terms in the SEMF to the binding energy per nucleon

- *Volume term*: When an assembly of nucleons of the same size is packed together into the smallest volume, each interior nucleon has a certain

number of other nucleons in contact with it. So, this nuclear energy is proportional to the volume.
Surface term: A nucleon at the surface of a nucleus interacts with fewer other nucleons than one in the interior of the nucleus and hence its binding energy is less. This surface energy term takes that into account and is therefore negative and is proportional to the surface area.
Coulomb term: The Coulomb term describes a decrease in the binding energy due to the Coulomb repulsion among protons in the nucleus. Thus for a given mass number A, it is less favorable to have a large number of protons. The Coulomb term dependence on A and Z is found from a simple model of the nucleus as a spherical charge.
Asymmetry term: Were it not for the Coulomb energy, the most stable form of nuclear matter would have the same number of neutrons as protons, since unequal numbers of neutrons and protons imply filling higher energy levels for one type of particle, while leaving lower energy levels vacant for the other type. This term is therefore based on that protons and neutrons occupy different quantum states and accounts for the fact that if N $\neq$ Z the energy of the nucleus increases and the binding energy decreases (Pauli exclusion principle).
Pairing term: This is a correction term that arises from the tendency of proton pairs and neutron pairs to occur. An even number of particles is more stable than an odd number.

Volume term is most important.

Problem 2

- The nuclear radius of the first nucleus is given by:

$$R_1 = r_0 A_1^{1/3}$$

  and the radius for the second nucleus:

$$R_2 = r_0 A_2^{1/3}$$

  Then

$$A_2 = A_1 (\frac{R_2}{R_1})^3 = 12(2^3) = 96$$

- The Rutherford-Geiger-Marsden experiment was to send alpha particles with high kinetic energy (about 5 MeV) through a thin gold film. The important (and at the time quite surprising) result was that some of the alpha particles were scattered through large scattering angles, even 180 degrees. From these experiments a new model of the atom was made: a very small massive nucleus with electrons around being so light that the alpha particles which were deflected met the full force of a bare nucleus. It was assumed that the nucleus carried a charge of +Ze. Using this theory, the force between the alpha particle (a helium nucleus) and a gold nucleus is the inverse-square law Coulomb force of electrostatic repulsion.

Problem 3

3a

- The ratio of the mass and proton numbers is Z/A $\approx$ 0.48. We know that for heavy stable nuclei this ratio is $\approx$ 0.41. Thus, we expect this isotope to be unstable and decay (by emitting protons).

- The Q value can be calculated from the mass differences:

$$Q = (m_{Eu} - m_{Sa} - m_p)c^2 = (121919.966/c^2 - 120980.755/c^2 - 938.28/c^2)c^2 MeV = 0.931\ MeV$$

- From the Q value calculated above we can find the speed by using the following equation: $\frac{1}{2}m_p v^2 = Q$, which can be written as

$$v = \sqrt{\frac{2 \cdot 0.931 MeV}{938.272 MeV/c^2}} = 0.045c\ m/s = 1.34 \cdot 10^7 m/s = 1.34 \cdot 10^{22}\ fm/s$$

In this calculation it is assumed that the proton has all the kinetic energy, while the daughter nucleus is still at rest. This is a good approximation given the masses.

A more precise calculation can be obtained if we consider conservation of momentum: $m_{Sa}v_{Sa} + m_p v = 0$, and find:

$$v = \sqrt{\frac{2Q}{m_p(1 + m_p/m_{Sa})}} = 0.045c\ m/s = 1.34 \cdot 10^{22}\ fm/s$$

The result is the same to the second decimal place.

The nuclear radius is given by $R = R_0 A^{1/3} = 1.2 \cdot 130^{1/3} = 6.079$ fm
Finally, the frequency is calculated: $f = \frac{v}{R} = 2.20 \cdot 10^{21} s^{-1}$

3b

- The Coulomb potential is given by:

$$V_C(R) = \frac{zZ'e^2}{4\pi\epsilon R}$$

where z = 1 and Z' = 62

The term $\frac{e^2}{4\pi\epsilon}$ is found in the attachment and is 1.439976 $MeV \cdot fm$
Then:

$$V_C(R) = \frac{1 \cdot 62 \cdot 1.439976 MeV \cdot fm}{6.079 fm} = 14.69\ MeV$$

- To find the distance R_C we equate the Coulomb potential to the Q value:

$$Q = \frac{zZ'e^2}{4\pi\epsilon R_C}$$

Then:

$$R_C = \frac{zZ'e^2}{4\pi\epsilon Q} = \frac{1 \cdot 62 \cdot 1.439976 MeV \cdot fm}{0.931 MeV} = 95.9\ fm$$

Alternatively:

$$R_C = \frac{zZ'e^2}{4\pi\epsilon Q} = R\frac{V_C(R)}{Q} = 6.079 fm \frac{14.69 MeV}{0.931 MeV} = 95.9\ fm$$

3c

- Since we want to estimate the tunnelling probability we can use the approximate expression $P_T = 4e^{-2\kappa L}$.
We first need to calculate κ:

$$\kappa = \frac{\sqrt{2m(V_H - Q)}}{\hbar} = \frac{\sqrt{2m_p(V_C/2 - Q)}}{\hbar} = \frac{\sqrt{2m_p c^2(V_C/2 - Q)}}{\hbar c}$$
$$= \frac{\sqrt{2 \cdot (938.28 MeV)(14.45/2 - 0.931) MeV}}{6.58217 \times 10^{-13} MeV \cdot c} = 0.55\ fm^{-1}$$

We then have

$$2\kappa L = 2 \times 0.55 \times (95.9 - 6.079)/2 = 49.4$$

Looking at the right figure, this corresponds to approximately $P_T = 4e^{-2\kappa L} \approx 4 \times 10^{-21}$.
(Or calculate this precisely; $P_T = 1.41 \times 10^{-21}$)
Since the tunnelling probability is very low, the approximation we did in considering $P_T = 4e^{-2\kappa L}$ is good.

- The decay rate is given by the product of the frequency at which the proton is at the potential barrier (or gets separated from the parent nuclide) times the probability of tunnelling through the barrier.
  Hence, the decay rate is given by

$$\lambda = fP_T = 2.2 \cdot 10^{21} s^{-1} \times 4 \times 10^{-21} = 8.8\ s^{-1}$$

  ($\lambda = 3.1\ s^{-1}$ if using $P_T = 1.4 \times 10^{-21} s^{-1}$)

  The half-life is:

$$t_{1/2} = \frac{ln2}{\lambda} = \frac{ln2}{8.8\ s^{-1}} = 0.079\ s$$

  (0.22 s if accurate)

Problem 4

- Dose from photons:

$$D_\gamma \quad = \quad \Psi_\gamma(\frac{\mu_{en}}{\rho})$$

  Dose from protons:

$$D_p \quad = \quad \Phi_p(\frac{S_c}{\rho})$$

- 1: The photon field is not significantly attenuated (can be neglected), 2: the range of electrons is short compared to the diameter of the volume V

- Doses must be equal:

$$\Psi_\gamma(\frac{\mu_{en}}{\rho}) \quad = \quad \Phi_p(\frac{S_c}{\rho})$$

  Fluence and energy fluence is related by:

$$\Psi_\gamma \quad = \quad h\nu\Phi_\gamma$$

  and

$$\Psi_p \quad = \quad T\Phi_p$$

  The fluence ratio is then:

$$\frac{\Phi_\gamma}{\Phi_p} \quad = \quad \frac{(\frac{S_c}{\rho})}{h\nu(\frac{\mu_{en}}{\rho})} = \frac{7.286}{1 \cdot 0.031} \approx 235$$

  Discussion: a factor 235 more photons are required to give the same dose as for protons.

Energy fluence:

$$\frac{\Psi_\gamma}{\Psi_p} = \frac{(\frac{S_c}{\rho})}{T(\frac{\mu_{en}}{\rho})} = \frac{7.286}{100 \cdot 0.031} \approx 2.35$$

Discussion: 1 MeV photons carry a factor 2.35 more energy in the radiation field than 100 MeV protons if they are to give the same dose.

Problem 5

See posters

Problem 6

6a

- See chapter 10.4 in Krane.
  Selection rules:

  $$|I_i - I_f| \leq L \leq I_i + I_f$$

  (no L = 0)

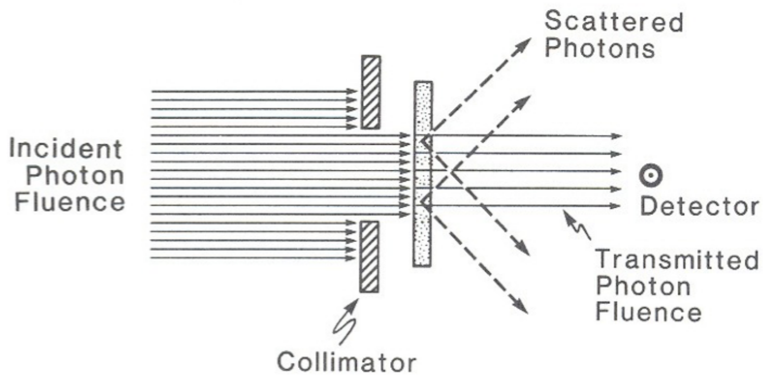
  Parity difference between initial and final states:
  $\Delta\pi$ = (-1)^L for electric transitions (EL)
  $\Delta\pi$ = (-1)$^{L+1}$ for magnetic transitions (ML)
  ($\Delta\pi$ = no: even electric, odd magnetic, $\Delta\pi$ = yes: odd electric, even magnetic)

- Angular momentum of gamma ray photon must be l = 2, 3, 4, 5 with even (+) parity.
  Possible transitions: E2, M3, E4, M5.
  Dominating transition: E2 (lowest permitted multipole usually dominates).

6b

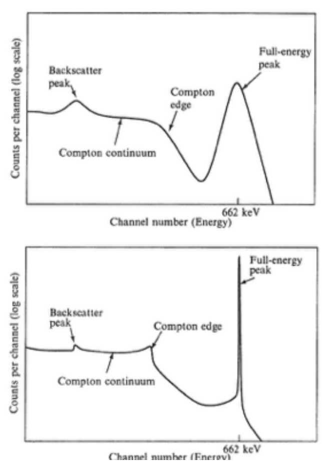
- Description is for example found in Lilley chapter 5.4.4.

- If the detector is far away: detects only primary photons, then it is possible to calculate the linear attenuation coefficient. If the detector is close: also detects scattered photons from the absorber.



- Possible detectors: gas detectors (ionization chamber, proportional counter, Geiger-Mueller), scintillation detectors (e.g., NaI), semiconductor detectors (e.g., germanium detectors).

  Example spectrum in Figure 6.8 in Lilley: An ideal detector would give



  a single, sharp full-energy peak for each gamma ray entering the detector. This is not possible because of background of Compton scattering which can mask other peaks. Small peaks: backscatter peaks from gamma rays that were Compton scattered by surrounding materials through large angles back to the detector. If the energy of the gamma ray exceeds 1.022 MeV it is possible for pair production to occur (”double-escape” and ”single-escape” peaks appear). But different detector arrangements give different detector performances. See more in Lilley chapter 6.5.1 and lab exercises.

6c

- Photoelectric effect, Compton effect, pair production
- Photoelectric: $\sim$ Z^5, Compton: independent (or very weak dependence), pair production: $\sim$ Z^3
- Use equations 5.16 and 5.17 in Lilley:

$$I = I_0 e^{-N\sigma x} = I_0 e^{-\mu x}, \mu_m = \mu/\rho$$

$\mu = \rho\mu_m = 0.141\ cm^{-1}$ and $e^{-\mu x} = 10^{-6} \ \rightarrow\ x = 98\ cm$