TFY4225 Nuclear and Radiation Physics
Exam 2018: Suggested Solutions

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Question 1

1a

${}^{40}_{19}K_{21}$ is an odd-odd nuclide. Spin and parity of the ground state is therefore determined by the combination of spin and parity of the single (odd) valence proton (number 19) and the single (odd) valence neutron (number 21).

20 is a magic number in the shell model, corresponding to filled shells:
$(1s_{1/2})^2, (1p_{3/2})^4, (1p_{1/2})^2, (1d_{5/2})^6, (2s_{1/2})^2, (1d_{3/2})^4$.

$\Rightarrow$ Proton 19 is unpaired in state $1d_{3/2}$
$\Rightarrow$ Proton 19 represents spin/parity $3/2^+ (\pi = (-1)^2 = +1$, where l = 2 for d)
$\Rightarrow$ Neutron 21 is unpaired in state $1f^{7/2}$
$\Rightarrow$ Neutron 21 represents spin/parity $7/2^- (\pi = (-1)^3 =$ -1, where l = 3 for f)

Total spin: $\vec{I} = \vec{j_p} + \vec{j_n} \Rightarrow |j_p - j_n| \leq I \leq |j_p + j_n| \Rightarrow |3/2 - 7/2| \leq I \leq |3/2 + 7/2|$
$\Rightarrow |2| \leq I \leq |5|$, i.e., 4 is a possible value.
Total parity: $\pi = \pi_{P19}\pi_{N21} =$ (+1)(-1) = -1, i.e. negative parity.

Conclusion: 4^- is a possible ground state of ${}^{40}K$.

1b

The ground state of ${}^{40}_{20}Ca_{20}$ is a 0^+ state since ${}^{40}Ca$ is an even-even nuclide. Thus, the β^- transition is a $4^- \rightarrow 0^+$ transition.
$\Delta\pi =$ yes $\Rightarrow L =$ 1, 3, ... (only odd numbers)
$|I_i - I_f| \leq |\vec{L} + \vec{S}| \leq |I_i + I_f|$ $|4 - 0| \leq |\vec{L} + \vec{S}| \leq |4 + 0| \Rightarrow |\vec{L} + \vec{S}| = 4$
For odd L, this is only possible for L = 3 and S = 1.

The β^- decay is 3rd forbidden, pure Gamow-Teller.

1c

Q value for nuclear reaction $X(a,b)Y$:

$$Q = (\sum m_i - \sum m_f)c^2 = [(m_x + m_a) - (m_b + m_Y)]c^2$$

For radioactive processes, there is no incoming particle a.

β^+ decay: ${}^A_Z X \rightarrow^A_{Z-1} X' + \beta^+ + \nu$

$$Q_{\beta^+} = [(m_N({}^A_Z X) - m_N({}^A_{Z-1}X') - m(\beta^+) - m(\nu))]c^2$$

We assume $m(\nu) = 0; m(\beta^+) = m_e$ and m_A denotes atomic masses:

$$\begin{aligned} Q_{\beta^+} &= [(m_A({}^A_Z X) - Zm_e - (m_A({}^A_{Z-1}X') - (Z-1)m_e) - m_e]c^2 \\ Q_{\beta^+} &= [(m_A({}^A_Z X) - m_A({}^A_{Z-1}X') + (Z-1-Z-1)m_e)]c^2 \\ Q_{\beta^+} &= [(m_A({}^A_Z X) - m_A({}^A_{Z-1}X') - 2m_e)]c^2 \end{aligned}$$

Expressed by mass excess, me, where $me({}^A X) = m_A({}^A X) - A$:

$$\begin{aligned} Q_{\beta^+} &= [me({}^A_Z X) + A - (me({}^A_{Z-1}X') + A) - 2m_e)]c^2 = [me({}^A_Z X) - me({}^A_{Z-1}X') - 2m_e)]c^2 \\ &= [(-36001 - (-37617)\mu u - 2m_e]c^2 = [1616 \cdot 10^-6 \cdot 931.5 - 2 \cdot 0.511]MeV \\ &= [1.505 - 1.022]MeV = \mathbf{0.483\ MeV} \end{aligned}$$

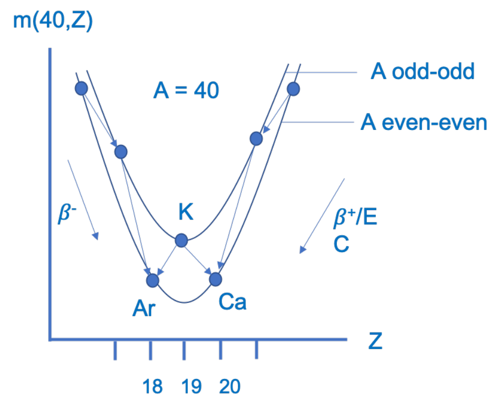
1d

In the semi-empirical mass formula:

$$\begin{aligned} m({}^A_Z X) &= Zm({}^1H) + (A-Z)m_n - \frac{B}{c^2} \\ &= Zm({}^1H) + (A-Z)m_n - \frac{1}{c^2}\{a_v A - a_s A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_{sym}\frac{(A-2Z)^2}{A} + \delta\} \end{aligned}$$

there is a pairing term, δ, which is:
> 0 for even-even nuclei
$= 0$ for odd A nuclei (even-odd or odd-even)
< 0 for odd-odd nuclei

For a fixed value of A, the expression for $m({}^A_Z X)$ will be a second-order polynomial in Z, and for A even (as for ${}^{40}K$) there will be one parabola for odd-odd nuclei, and another for even-even nuclei.
${}^{40}K$ is the most stable of A = 40 odd-odd nuclides (very long half-life; $1.25 \cdot 10^9$ years). Nuclides at the min. point of the odd-odd parabola can decay by either β^- or β^+/EC to alternative daughter nuclides of lower mass, in this case ${}^{40}_{18}Ar$ or ${}^{40}_{20}Ca$ (by β^+/EC or β^-).



Question 2

2a

We use the MIRD formalism for internal dosimetry. We start by calculating the total number of disintegrations in the body as a whole (for adults, we integrate over 50 years and we have A_0 = 500 MBq and $t_{1/2}$ = 1,8289 h = 6584 s):

$$\begin{aligned}
\tilde{A}_{body} &= \int_0^{50years} A_0 e^{-\lambda t} dt \\
&= \frac{A_0}{\lambda}(1 - e^{-\lambda \cdot 50years}) \\
&\approx \frac{A_0}{\lambda} = \frac{A_0 t_{1/2}}{ln2} \\
&= \frac{500MBq \cdot 6584s}{ln2} = 4.75 \cdot 10^{12}
\end{aligned}$$

The assumption "even distribution of activity" means that the activity in each organ is given by the fraction of the organ mass M_S to the total body mass M_{body}:

$$\tilde{A}(r_S) = \tilde{A}_{body} \frac{M_S}{M_{body}}$$

Next we calculate the S-function. There are two branches in this decay, positron emission (97 %) and electron capture (3 %). In the electron capture process, most of the reaction energy is carried away with the neutrino, which does not interact with the body. In other words, the absorbed fraction is close to zero, and we can ignore this branch. On the other hand, the positron emission deposit energy in the tissue through a two-step process; (i) first the kinetic energy of the emitted positron is deposited locally in the source organ; (ii) two annihilation photons are created and deposit their energy in the tissue according to the attenuation law. In total we get:

$$\begin{aligned} S(r_T \leftarrow r_S) &= \frac{1}{M_T}\sum_i E_i Y_i \Phi(r_T \leftarrow r_S) \\ &= \frac{1}{M_T}\left[211.3keV \cdot 0.97\delta_{ST} + 2\cdot 511keV \cdot 0.97 \cdot \frac{kM_T}{M_{body}}\right] \\ &= = \frac{205keV}{M_T}\delta_{ST} + \frac{496keV}{M_{body}} \end{aligned}$$

We use 634keV/3 = 211.3 keV as average positron kinetic energy (reaction energy $Q = 1656keV - 2\cdot 511keV = 634keV$ is shared between positron and neutrino). We have also assumed that the absorbed fraction is proportional to the mass fraction (=even absorption of radiation), with k as a general absorbed fraction for the body as a whole. At 511 keV the photon attenuation length is 36 cm, so a realistic absorption fraction is around 0.5. We put all this together:

$$\begin{aligned} E &= \sum_T w_T \sum_R w_R \sum_{r_S} \tilde{A}_{body}\frac{M_S}{M_{body}}\left[\frac{205keV}{M_T}\delta_{ST} + \frac{496keV}{M_{body}}\right] \\ &= \frac{\tilde{A}_{body}}{M_{body}}\sum_T w_T \sum_R w_R \left[205keV + \frac{496keV}{M_{body}}\sum_{r_S} M_S\right] \end{aligned}$$

We use M_{body} = 75 kg, $\sum_T w_T = 1$, $\sum_S M_S = M_{body}$, and w_R= 1 for both positrons and photons:

$$E = \frac{\tilde{A}_{body}\cdot 701keV}{M_{body}} = \frac{4.75\cdot 10^{12}\cdot 701\cdot 10^3 \cdot 1.602\cdot 10^{-19}}{75} = \mathbf{7.1\ mSv}$$

2b

The calculation of $\tilde{A}_{r_S}$ changes into:

$$\begin{aligned} \tilde{A}(brain) &= 0.6\cdot \tilde{A}_{body} = 0.6\cdot 4.75\cdot 10^{12} = 2.85\cdot 10^{12} \\ \tilde{A}(bladder) &= 0.4\cdot \tilde{A}_{body} = 0.4\cdot 4.75\cdot 10^{12} = 1.9\cdot 10^{12} \\ \tilde{A}(other) &= 0 \end{aligned}$$

The effective dose becomes:

$$\begin{aligned} E &= \sum_T w_T \sum_R w_R \sum_{r_S} \tilde{A}_{r_S}\left[\frac{205keV}{M_T}\delta_{ST} + \frac{496keV}{M_{body}}\right] \\ &= \sum_T w_T \left[\tilde{A}(brain) + \tilde{A}(bladder)\right]\left[\frac{205keV}{M_T}\delta_{ST} + \frac{496keV}{M_{body}}\right] \\ &= \frac{W_{brain}}{M_{brain}}\tilde{A}(brain)\cdot 205keV + \frac{W_{bladder}}{M_{bladder}}\tilde{A}(bladder)\cdot 205keV + \left[\tilde{A}(brain) + \tilde{A}(bladder)\right]\frac{496keV}{M_{body}} \end{aligned}$$

$$
\begin{aligned}
&= \frac{0.01}{2kg}2.85 \cdot 10^{12} \cdot 205keV + \frac{0.04}{0.1}1.9 \cdot 10^{12} \cdot 205keV + 4.75 \cdot 10^{12}\frac{496keV}{75kg} \\
&= 0.47mSv + 24.96mSv + 5.03mSv = \mathbf{30.5\ mSv}
\end{aligned}
$$

Most of the dose is caused by the positron energy deposition in the bladder. The result is an overestimation of the effective dose due to: 1) positron energy in the bladder is mostly absorbed by the urine, not by bladder tissue; 2) the effect of going to the toilet and clearing the bladder is not included (this can be done by reducing T_D for $\tilde{A}(bladder)$ to the time delay until a toilet visit).

Question 3

If we assume the target nuclei number is constant, we have the following expression for the number of nuclei of the produced isoptope N_1 as function of production rate R and time t:

$$
N = \frac{R}{\lambda}(1 - e^{-\lambda_1 t})
$$

We can use this expression to find the required time, τ_{90}, to reach 90 % of the maximum activity, since $A \propto N$:

$$
\begin{aligned}
(1 - e^{-\lambda_1 t}) &= 0.90 \\
\tau_{90} &= -\frac{ln(0.1)}{\lambda_1}
\end{aligned}
$$

Then;

$$
\begin{aligned}
\lambda_1 &= -\frac{ln(2)}{t_{1/2}} = 1.06 \times 10^{-4}s{-1} \\
\tau_{90} &= -\frac{ln(0.1)}{1.06 \times 10^{-4}}s{-1} = \mathbf{21723\ s}
\end{aligned}
$$

21723 s is about 6 hours. You will need to start production around 01:30.

Question 4

See posters about radioactive dating and accelerator mass spectrometry.

Question 5

The situation is described by a travelling wave before and after the wall (with the same wavelength λ, but different amplitude) and a decaying exponential in the wall.
The detector measures the transmitted flux. In the limit of small tunneling, we can write $\Phi_t \approx 4e^{-2\kappa L}\Phi_{inc}$, with $\kappa = \sqrt{\frac{2m(V-H)}{h^2}}$

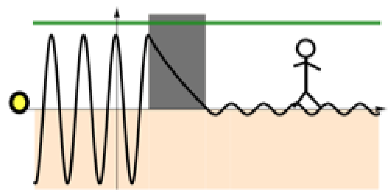


Figure 1: Sketch of neutron shielding

Question 6

6a

Put a sheet of paper over the source.

6b

Corrected rate = 1350 - 35 = 1315 min^{-1}
A = 1315/60 = **22 Bq**

6c

Find the area of the sphere: A = $4\pi r^2 = 4 \times \pi \times 0.1^2 = 0.126$ m^2
Total activity = $(0.126 \times 1315)/1.5 \times 10^{-4} = 1.01 \times 10^6$ min^{-1} = **16800 Bq**

6d

The tube is 4.5 times further away, so count rate is $1315/4.5^2 = 65$ min^{-1}
Displayed count rate = 65 + 35 = **100 min^{-1}**