TFY4225 Nuclear and Radiation Physics
Exam 2019: Suggested Solutions

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Question 1

1a

The key ingredient in the nuclear shell model was the spin-orbit coupling. The main potential function in the shell model consists of a central-symmetric part, but it was not until a small additional term consisting of a $\vec{l} \cdot \vec{s}$ term that the model was successful. The spin-orbit coupling results in a splitting of the original energy levels as determined from l and s into new levels based on the new quantum number for total angular momentum j.

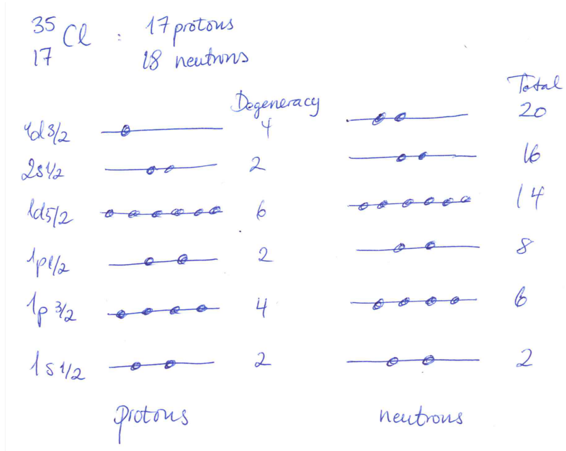
1b

The main success of the nuclear shell model was twofold:

- Its ability to predict the correct "magic numbers", which are the neutron and proton numbers for particularly stable isotopes.
- Its ability to predict the correct spin and parity for the ground state of most odd A nuclides. The model can also be used to predict spin-parity of possible excited states.

1c

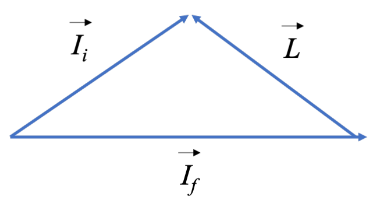
See the figure. A single unpaired proton is in state $1d_{3/2}$.
The parity is found from $\pi = (-1)^l, l = 0, 1, 2, 3 ...$
Single nucleon state interpretation of the shell model then predicts the ground state of ${}^{35}Cl$ to be $3/2^+$.
Examples of possible excited states from single nucleon excitation:
Proton excited from $2s_{1/2} \rightarrow 1d_{3/2}$: single unpaired proton is in $2s_{1/2}$ and the nuclear state is $1/2^+$.
Proton excited from $1d_{5/2} \rightarrow 1d_{3/2} : 5/2^+$
Proton excited from $1p_{1/2} \rightarrow 1d_{3/2} : 1/2^-$
More possibilities exist.



1d

De-excitation by gamma emission from $1/2^+$ to $3/2^+$.
We have conservation of angular momentum and $\vec{L}$ is the angular momentum carried away by emitted photon.

$$|I_i - I_f| \leq L \leq |I_i + I_f| \Rightarrow 1 \leq L \leq 2$$



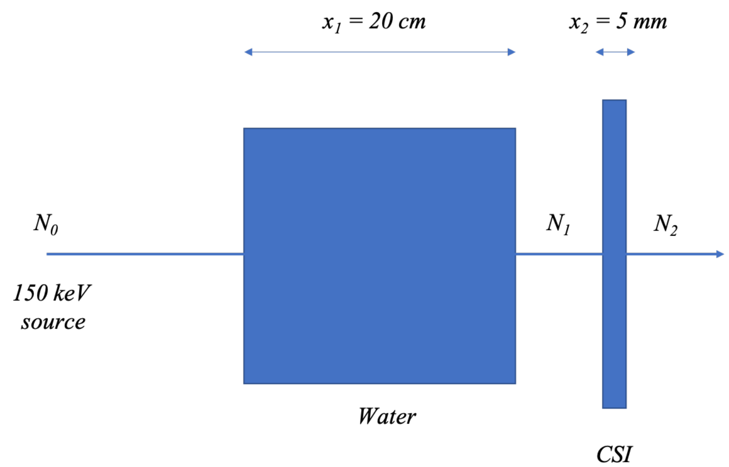
Conservation of parity:
$\pi_i = \pi_f$
$\pi_{1/2^+} = \pi_{3/2^+} \cdot \pi_{photon} \rightarrow \pi_{photon}$ is positive
M-mode parity: $(-1)^{L+1}$
E-mode parity: $(-1)^L$

M1 is the lowest mode. M1 + E2 will be ovserved since E-modes have higher probability than M-modes of the same polarity. M1 + E2 do both have high probability compared to higher order photons, so one would expect a relatively short half-life of such an excited state of ^{35}Cl.

Question 2

2a

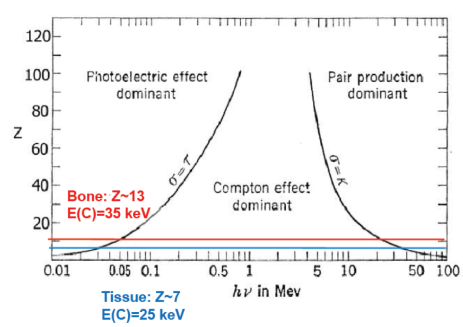
We need to find N_0 so that N_1 - N_2 is 100.
$N_1 = N_0 e^{-\mu_{water} \cdot 20cm}$ and $N_2 = N_1 e^{-\mu_{CSI} \cdot 0.5cm}$
$N_1 - N_2 = N_1(1 - e^{-\mu_{CSI} \cdot 0.5cm}) = N_0 e^{-\mu_{water} \cdot 20cm} \cdot (1 - e^{-\mu_{CSI} \cdot 0.5cm}) = 100$
$100 = N_0 e^{-0.227 \cdot 20} \cdot (1 - e^{-9.16 \cdot 0.5})$
$100 = N_0 \cdot 0.0106$
$N_0 = 9466 \approx 9500$ photons



2b

Compton effect, photoelectric effect, pair production

2c



Photoelectric effect: varies with $\sim Z^{4-5}$ and dominates in lower energy range (< 50 keV)
Compton effect: independent of Z, intermediate energy range (100 keV - 30 MeV)
Pair production: varies with $\sim Z^2$, high energy range (< 50 MeV) with threshold 1.022 MeV (two times 511 keV)

2d

$I = I_0 e^{-N\sigma x} = I_0 e^{-\mu x}$ and $\mu_m = \mu/\rho$
$\mu = \rho \cdot \mu_m = 2200 kg/m^3 \cdot 0.064 \cdot 10^{-3} cm^2/g = 0.1408 cm^{-1}$
Using $I/I_0 = e^{-\mu x}$ gives $10^{-6} = e^{-0.1408 \cdot x}$
$ln(10^{-6}) = -0.1408 \cdot x$
x = 98 cm concrete walls

Question 3

3a

Main principles of PET:

- PET is based on positron emission decay ($\beta +$) and annihilation when the positron interacts with an electron, followed by emission of two 511 keV photons
- The photons travel in opposite directions
- Coincident detection of both photons enable establishment of the so-called line of response (LOR), a line in space along the path where the annihilation must have taken place
- The detector ring is positioned around the subject (patient) and will detect many LORs over a give acquisition time
- Image reconstruction is performed by filtered back projection

3b

If we assume the target nuclei number to be constant, we have the following expression for the number of nuclei of the produced isotope N_1 as function of the production rate R and time t:

$N = \frac{R}{\lambda}(1 - e^{-\lambda_1 t})$.

We can use this expression to find the required time, τ_{95}, to reach 95% of the maximum activity, since $A \propto N$:

$(1 - e^{-\lambda_1 t}) = 0.95$
$\tau_{95} = -\frac{ln(0.05)}{\lambda_1}$
We have $\lambda_1 = \frac{ln(2)}{t_{1/2}} = \frac{ln(2)}{20.361} = 0.034$
$\tau_{95} = -\frac{ln(0.05)}{0.034} \approx 88min$

You have to start the production around 6:30.

3c

The Lorentz force in the circular orbit provides centripetal acceleration.
Thus, we have: $qvB = \frac{mv^2}{r}$, which we can rearrange to $v = \frac{qBr}{m}$

For a circle we have $d = 2\pi r$, therefore, the frequency is:
$\omega = \frac{v}{d} = \frac{\frac{qBr}{m}}{2\pi r} = \frac{qB}{2\pi m}$

For non-relativistic particles the kinetic energy is $T = \frac{1}{2}mv^2$, and since $v = \frac{qBr}{m}$, we have: $T = \frac{1}{2}m\frac{q^2B^2r^2}{m^2} = \frac{q^2B^2r^2}{2m}$

3d

With T = 30 MeV and r = 79 cm, then B will be 1.0 Tesla.

Question 4

4a

- The reactor was used to control the decay of U-235 and to use the heat from the decay to produce electric energy from steam of water
- U-235 becomes radioactive when hit by a neutron and releases three neutrons in a chain reaction. To slow down the neutrons and increase the interaction probability a moderator is used. Graphite was the moderator used in Chernobyl.
- The accident happened after a safety test. The reactor had been on, also producing a lot of xenon presumably without being aware of it. When starting the test the power decreased as part of the procedure, but then the xenon absorbed all neutrons and the power dropped more quickly than planned. Normally they would increase the power by pulling out a control rod (so that graphite was in the reactor). They were impatient and pulled out all rods fast. The fast increase in power caused water to vaporise and then xenon was used up. This lead to an unstable reactor where the rods were locked (so that graphite was not available), the reactor temperature increased and then exploded.

- Thyroid cancer. Because of exposure to radioactive I-131 from drinking milk coming from cows eating polluted grass. Children with iodine deficiency was particularly exposed due to higher absorbance of the mineral. The isotope has a half-life of only 8 days making the cancer appear fast.

4b

We assume even distribution of activity in the body. One reindeer steak provides a start activity of $A_0 = 0.250kg \cdot 3000Bq/kg = 750Bq$.

The total number of disintegrations per year in the body from eating one steak every week for 1 year (52 weeks) will be:

$\tilde{A}(peryear) = 52 \int_0^{\infty} A_0 e^{-\lambda t} dt = 52 \frac{750Bq \cdot 70days}{ln2} = 3.40 \cdot 10^{11}$

The decay process is β^- with Q = 1176 keV, 94% probability of going to the 2nd excited state of Ba-137 followed by gamma emission of 662 keV (yield 85%), while 6% goes directly to ground state. Since we can assume even distribution of activity and absorption in the body we can shortcut the organ summation and calculate the S-function for the whole body.

$$S = \frac{1}{M_{body}} [171keV \cdot 0.94 + 662keV \cdot 0.85 \cdot 0.5 + 392keV \cdot 0.06] = 9.9 \cdot 10^{-16} J/kg$$

using 0.5 for the absorbed fraction of photons and assuming that the average kinetic energy is 1/3 of the available reaction energy.

The annual whole body effective dose becomes:
$E = \tilde{A}(peryear) \cdot S = 3.40 \cdot 10^{11} \cdot 9.9 \cdot 10^{-16} J/kg = 0.34mSv$

4c

The annual limit for the general public is 1 mSv. You can safely enjoy a 250 g reindeer steak every Sunday.

Question 5

5a

The particle will be represented by a travelling wave from the left (the real part of the wavefunction is plotted in the figure) with an amplitude A for the incoming flux. Inside the barrier the wavefunction has a complex wave number $\kappa = \sqrt{2m(V_B - E)}/\hbar$, thus it is represented by a decaying exponential. The amplitude after the barrier is approximately $|A|^2 P_{tun}$.

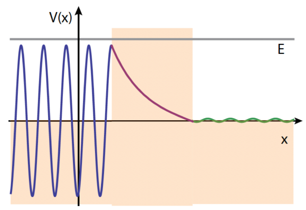


Figure 1: Scattering potential and wavefunction

5b

The approximate tunneling formula is given by $P_{tun} \approx 4e^{-2\kappa L}$
We have $\kappa = \sqrt{2m(V_B - E)}/\hbar = \sqrt{2mc^2(V_B - E)}/\hbar c = \sqrt{2 \cdot 4000MeV(85-5)MeV)}/200MeV \cdot fm = \sqrt{8000 \cdot 80}/200fm^{-1} = 4fm^{-1}$
Thus, the probability is:
$P_{tun} \approx 4e^{-2 \cdot 4fm^{-1}10fm} = 4e^{-80} \approx 4 \times 10^{-35}$
(as you could also have inferred from the graph)

5c

We can calculate $p(x)$ from the wavefunction amplitude as $p(x) = |\psi(x)|^2 = |A|^2$
and the particle velocity is given by $v = \frac{p}{m} = \frac{\hbar k}{m}$

Thus, we have $\Gamma = |A|^2 \frac{\hbar k}{m}$ from which we can calculate $|A|^2 = \frac{\Gamma m}{\hbar k}$

The wave number k is calculated from $\frac{\hbar^2 k^2}{2m} = E \to k = \sqrt{2mE}/\hbar = \sqrt{2mc^2 E}/(\hbar c) = \sqrt{2 \cdot 4000MeV \cdot 5MeV}/200MeV fm = 1fm^{-1}$.

Finally, $|A| = \sqrt{\frac{\Gamma mc^2}{\hbar ckc}} = \sqrt{\frac{6 \times 10^{21} s^{-1} \cdot 4000MeV}{200MeV fm \cdot 1 \cdot 3 \times 10^8 ms^{-1} 10^{15} m/fm}} = 2/\sqrt{10} fm^{-1/2}$

(Alternatively, you could have calculated the velocity from the kinetic energy $v = \sqrt{2E/m} = \sqrt{2 \times 5MeV c^2/4000MeV} = c/20$ and used this to calculate $|A|$.)

5d

Using the result from above, we obtain $\tau = \Gamma_{tun}^{-1} = \frac{1}{4} \times 10^{35} \cdot \frac{1}{6} \times 10^{-21} s = \frac{1}{24} \times 10^{14} s$ or about 130 thousand years.