TFY4225— Nuclear and radiation physics
Autumn 2020
Final exam problems

Problem 1: Detectors

a)

1. A scintillation detector with NaI(Tl) Crystal, suitable for gamma;
2. A Si semiconductor detector, suitable for alpha, beta, and low energy X-rays/gamma;
3. A He-3 gas detector, suitable for slow neutrons.

(Other reasonable choice of detectors and radiations are also acceptable)

b)

Travel distance d = 0.25 m
t = d/c = 0.25m/(3*10^8 m/s) = 8.33*10^-10 s = 0.83 ns.

c) Maximum difference in distance between the two photons: 0.3 m

(photon 1 travels 25 cm – 15 cm = 10 cm, photon 2 travels 25 cm + 15 cm = 40 cm)

difference in time: $\Delta t = 1$ ns.

d) Among the materials in the list the most suitable one for TOF detector is BaF_2. This is because it has the fastest decay constant of 0.8 ns (less than 1 ns).

e) Total number of photons emitted per second: 50,000,000.

50 MBq * 511 keV * 2 * 38 * 0.12 * 0.2 * 0.2 * (1/1024) = 9.3*10^6 counts/sec

In this calculation we did not take into account the geometry of the scanner and any attenuation between the point of annihilation and the detector. In reality only a small fraction of the emitted gamma photons will reach the detectors, and the difference of attenuation also need to be accounted for.

f)

Advantage: higher image quality (each event has to be a coincident event; random scatterings will most likely be rejected because the lack of coincidence of two photons).

Challenge: intrinsic resolution, the positron will travel a small distance before annihilation, making it difficult to improve resolution when using the same isotope.

(other reasonable answers with proper justification are also acceptable).

Problem 2: Dosimetry

a) Definition of Q: $Q = (m_{initial} - m_{final})*c^2$

As we see from the definition, Q value represents the mass energy difference between the initial and final species. In a nuclear reaction, Q-value can be positive, zero, or negative. If $Q > 0$, the reaction is said to be exoergic or exothermic; if $Q < 0$, the reaction is said to be endoergic or endothermic.

Explain why Cu64 can undergo b-, b+ and EC decays.

1. Cu-64 has 29 protons and 35 neutrons, following the energy level diagram of the nuclear shell model we can see the nucleus has one unpaired proton and one unpaired neutron. In beta decays either a proton is changed to a neutron, or a neutron is changed to a proton, and in the case of Cu-64, either reaction could happen.
2. The Q-value for Cu-64 decay to Ni-64 is 1675 keV, above the energy threshold for positron emission.

b)

β^-: $Q = 579.4\ keV$

$\beta+$: $Q = 1675\ keV - 2 * 511\ keV = 653\ keV$

Electron capture is more energetically economic than positron emission, even if Q is sufficient to emit positrons.

c)

MIRD formalism

370 MBq. Cu-64.

$t_{1/2} = 12.7$ hr. $= 45720$ Sec.

38.5% β^-, $Q^- = 579.4 keV$

17.5% β^+, $Q^+ = 1675 keV - (2 \cdot 511 keV) = 653 keV$

0.475% γ, $E_\gamma = 1346 keV$

43.5% EC

$$\tilde{A}_{body} = \int_0^{50yrs} A_0 e^{-\lambda t} dt$$

This is the total number of disintegrations.

$$= \frac{A_0}{\lambda}(1 - e^{-\lambda \cdot 50yrs})$$

$$\approx \frac{A_0}{\lambda} = \frac{A_0 t_{1/2}}{\ln 2}$$

$$= \frac{370 \times 10^6 Bq \cdot 45720 sec}{\ln 2}$$

$$= 2.44 \times 10^{13} \text{ disintegrations.}$$

Even distribution $\Rightarrow$ activity in each organ is given by the mass fraction of the organ to the total body mass.

$$\tilde{A}(r_S) = \tilde{A}_{body} \cdot \frac{M_S}{M_{body}}$$

Now we need to look at the S-function :

$$S(r_T \leftarrow r_S) = \frac{1}{M_T} \sum_i E_i Y_i \phi_i (r_T \leftarrow r_S)$$

$\frac{1}{3}$: mean energy fraction for β^+/β^-

$$= \frac{1}{M_T} \left\{ \overset{\beta^+}{\tfrac{1}{3} \cdot 653\,\mathrm{keV} \cdot 0.175} + \overset{\beta^-}{\tfrac{1}{3} \cdot 579\,\mathrm{keV} \cdot 0.385} + (2 \cdot 511\,\mathrm{keV} \cdot 0.175 + 0.0048 \cdot 1341\,\mathrm{keV}) \cdot \frac{k M_T}{M_{body}} \right]$$

whole body abs fraction for γ:

$k = 0.5$

$$= \frac{1}{M_T} \left[38.1\,\mathrm{keV} + 74.3\,\mathrm{keV} + 185.3\,\mathrm{keV} \cdot \frac{k M_T}{M_{body}} \right]$$

$$= \frac{112.4\,\mathrm{keV}}{M_T} + 185.3\,\mathrm{keV} \cdot \frac{0.5}{M_{body}}$$

$$= \frac{112.4\,\mathrm{keV}}{M_T} + \frac{92.65\,\mathrm{keV}}{M_{body}}.$$

For absorbed dose,

$$D = \sum_{r_S} \tilde{A}_{body} \frac{M_S}{M_{body}} \left[\frac{112.4\,\mathrm{keV}}{M_T} + \frac{92.65\,\mathrm{keV}}{M_{body}} \right]$$

Since it is for whole body, $M_S = M_{body} = M_T$

$$\Rightarrow \quad D = \sum_{r_S} \tilde{A}_{body} \cdot 1 \cdot [1.50\,\mathrm{keV/kg} + 1.24\,\mathrm{keV/kg}]$$

$$= 2.44 \times 10^{13} \cdot 2.74 \frac{\mathrm{keV}}{\mathrm{kg}}$$

This calculation does not account for Auger emission dose.

$$= 2.44 \times 10^{13} \cdot 4.39 \times 10^{-16}\ \mathrm{J/kg}$$

$$= 0.011\ \mathrm{J/kg}$$

$$= 0.0107\ \mathrm{J/kg} = 10.7\ \mathrm{mGy}$$

Not required. $\left(\frac{10.7\,\mathrm{mGy}}{370\,\mathrm{MBq}} = 0.0289\ \mathrm{mGy/MBq} \right)$

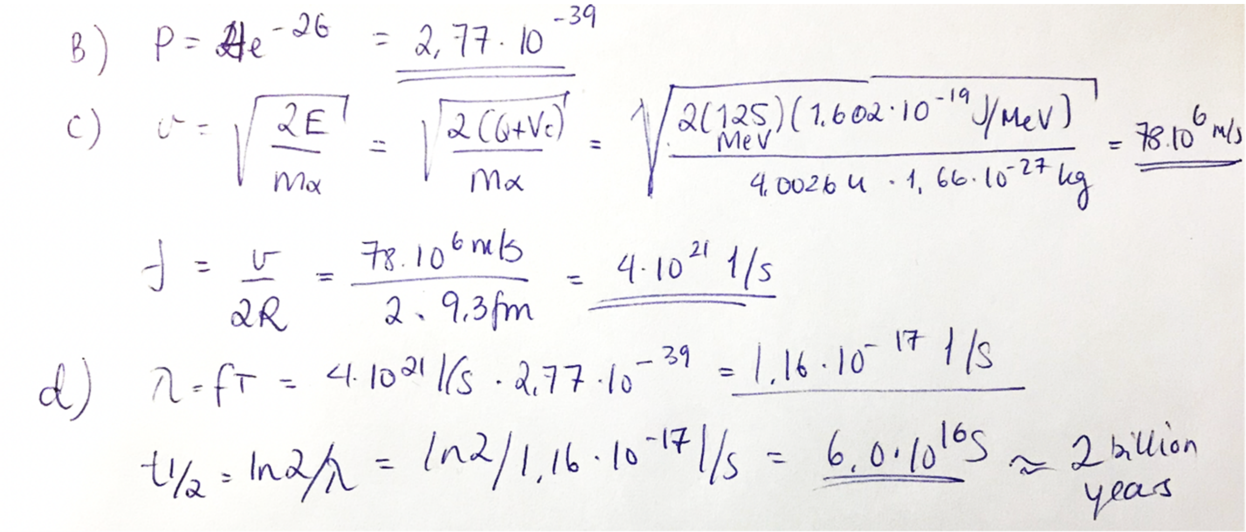
In lit: $\sim 0.03\ \mathrm{mGy/MBq}$

Problem 3: Alpha decay

A) Here we simply set up the Coulomb potential to equal to the Q value:

$$V(r) = \frac{zZe^2}{4\pi\varepsilon b} = 4.268\ MeV$$

$$b = \frac{2 * 90 * 1.43998\ MeV * fm}{4.268\ MeV} = 60.73\ fm$$



B) $P = \cancel{A}e^{-26} = \underline{\underline{2,77 \cdot 10^{-39}}}$

c) $v = \sqrt{\frac{2E}{m_\alpha}} = \sqrt{\frac{2(Q+V_c)}{m_\alpha}} = \sqrt{\frac{2(\underset{MeV}{125})(1.602 \cdot 10^{-19}\ J/MeV)}{4,0026\ u \cdot 1,66 \cdot 10^{-27}\ kg}} = \underline{\underline{78 \cdot 10^{6}\ m/s}}$

$f = \frac{v}{2R} = \frac{78 \cdot 10^{6}\ m/s}{2 \cdot 9,3\ fm} = \underline{\underline{4 \cdot 10^{21}\ 1/s}}$

d) $\lambda = fT = 4 \cdot 10^{21}\ 1/s \cdot 2,77 \cdot 10^{-39} = \underline{1,16 \cdot 10^{-17}\ 1/s}$

$t_{1/2} = \ln 2/\lambda = \ln 2/1,16 \cdot 10^{-17}\ 1/s = \underline{\underline{6,0 \cdot 10^{16} s}} \approx$ 2 billion years

Problem 4: Potential well

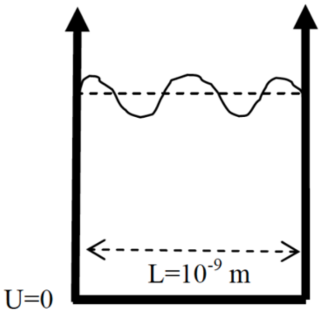
a) The wave functions are the usual free wave solutions, but they must go to zero at x=0 and x=L, so they are:

$\psi(x)=\sin(n\pi x/L)$

These waves have energy $E_n = \hbar^2\pi^2 n^2/2mL^2 = n^2*(0.37\ eV)$

b) The n=5 state has five peaks in it, and is shown on the drawing to the right.

c) The energy difference is $E_5 - E_4 = (25 - 16)*0.37\ eV = 3.36\ eV$



Problem 5 – Multiple choices

a. Two electrons, each with 100 keV of kinetic energy, pass close to a tungsten nucleus and produce x-ray photons via bremsstrahlung. The first electron passes close to the nucleus and emits a photon of energy E1, the second passes by at a greater distance from the nucleus and produces a photon of energy E2. The photon energies satisfy:

A. E1 <E2 <100keV C. E1 =E2 <100keV E. 100keV<E2 <E1
B. E2 <E1 <100keV D. 100keV<E1 <E2

b. Of the following radiations, which would be the most desirable for radionuclide imaging?

A. 15 keV gamma;
B. 150 keV beta;
C. 150 keV gamma;
D. 5 MeV alpha;
E. 1500 keV gamma

c. In SPECT, one way to improve the image resolution is to use a high-resolution collimator (such as pinholes). Which of the following represent a principle limitation (trade-off) when using this collimator?

A. limited field of view;
B. more distortion;
C. decreased scattering rejection;
D. reduced sensitivity;
E. there is no trade-offs.

(In selecting gamma camera collimators there is a general tradeoff between sensitivity and the blurring produced by the collimator. A so-called "high resolution" collimator which produces minimal blurring has a relatively low sensitivity.)

d. d). If m_H is the atomic mass of Hydrogen, m_n is the mass of a neutron, and M is the atomic mass of the atom, which of the following is the mass defect formula?

A. $\Delta m = Z \cdot m_H + N \cdot m_n - M$

B. $\Delta m = Z \cdot m_H + N \cdot m_n + M$

C. $\Delta m = Z \cdot m_H - N \cdot m_n - M$

D. $\Delta m = Z \cdot m_H - N \cdot m_n + M$

E. $\Delta m = M - Z \cdot m_H - N \cdot m_n$

e. The following reaction: ^{1}n + ^{235}U → ^{141}Ba + ^{92}Kr +^{31}n is called:

A. Fusion
B. Fission
C. Alpha decay
D. Beta decay
E. Gamma decay

f. Which of the following statement is correct?

A. Compton scattering between a gamma photon and an atom is between the photon and inner shell electrons of the atom.

b. Photoelectric effect is dominant with low energy gamma photons and is independent of Z.

c. Absorption cross section of a neutron with a certain energy by an atom is dependent of the atomic number Z; different isotopes of the same element will have the same absorption cross section.

d. The kinetic energy of an electron knocked out from the atom by a gamma photon via photoelectric interaction equals to the energy of the gamma photon.

e. **The interaction cross section of a gamma photon absorbed by an atom via photoelectric is dependent of Z but independent of which isotope of the same element.**

g. Which of the following statement is incorrect?

a. Linear energy transfer (LET) increases with the speed of a charged particle decreases.

b. Linear energy transfer (LET) increases with increasing charge.

c. **Auger electrons are typically considered high energy electrons and therefore they have high linear energy transfer (LET).**

d. linear energy transfer (LET) is an average quantity and can be calculated in different ways.

e. The linear energy transfer (LET) of low kinetic energy alpha particles is much higher than high energy electrons.

h. A nucleus with A=235 splits into two nuclei of mass numbers in the ratio 1 : 2.The ratio of the radii of the new nuclei are

a. 1:2
b**<u>. 1:1.26</u>**
c. 8:1
d. 1:1.41
e. 1:2.82

i. The Surface-energy term appears in semi-empirical mass formula as a result of

A. Repulsion between the charged particles, protons, in the nucleus
B. <u>Reduction of total binding energy due to nucleons on the surface of the nucleus</u>
C. Excess number of neutrons in the nucleus
D. Intrinsic nucleonic spin
E. Reduction of total binding energy due to nucleons not on the surface of the nucleus

j. j) It is Friday afternoon and you are still working on your project in the lab alone, which involves making a new PET tracer using F-18 ($t_{1/2}$ = 110

minutes). Accidentally a vial of 100 GBq of F-18 solution dropped to the floor and broke (luckily you are not contaminated with any radioactive materials). What is the best way to handle this situation?

A. <u>Leave the lab and put a warning sign on the door. Come back to clean up on Monday.</u>
B. Immediately use clean paper towels to absorb the spilled F-18 solution, then clean the floor with soap and water.
C. Ignore and continue the experiment with a new vial of F-18 solution.
D. Cover the part of the floor with the spilled F-18 solution with ~1 cm of sand, then continue experiment.
E. Use a pipet to collect the spilled F-18 solution on the floor and transfer into a new vial in a lead shield container.