1) Radon
   a. The decay constant, λ, signifies the likelihood of one decay to happen per atom per unit time. The activity, A, in unit Bq denotes the number of decays per second. The number of Rn-222 atoms, N, is thus given by A/λ.
$$\frac{A}{\lambda} = \frac{200Bq \cdot 3.82 \cdot 24 \cdot 3600s}{\ln(2)} = 9.52 \cdot 10^7$$

   b. At equilibrium, the daughter nucleus is produced at the same rate as it decays. Since we know that Po-218 is being produced at a rate of 200Bq, it also has to have an equal activity. Thus the number of Po-218 is:
$$\frac{A}{\lambda} = \frac{200Bq \cdot 3.1 \cdot 60s}{\ln(2)} = 5.37 \cdot 10^4$$

   c. Po-218: 12 μGy/year and 29 μSv/year
   Po-218 and further decay of daughter Po-214: 28 μGy/year and 67 μSv/year
   See handwritten solution below
   Note that the dose contribution from Po-214 present in the air will be negligible (due to its short half-life, very few atoms are present at equal activity in equilibrium). However, the Po-218 that disintegrate in the airways may further decay as Pb-214 → Bi-214 → Po-214. Assuming that the biological clearance is low, then one disintegration of Po-218 eventually leads to one disintegration of Po-214 in this way. Thus I have accepted both answers that only consider Po-218, and answers that assume an equal number of disintegrations of Po-218 and Po-214. Regarding the kinetic energy of the alpha-particles, either calculating it, or looking it up in a database is fine. Alternative disintegration pathways (β^- of Po-218) is small enough to be negligible.

   d. Ionizing radiation carries sufficient energy to damage complex biomolecules either directly or via free radicals. Most important in the context of cancer development is that it can damage the DNA molecules that carry the genetic material of the cell. Large damages to the DNA of a cell will kill the cell, but a smaller damage may lead to mutations in the DNA, while the cell still survives. Some of the mutations can facilitate cell division and disable inhibitory processes. Accumulation of such mutations can in the long run give rise to cancer, in a stochastic process.
   e. As a heavy charged particle, alpha radiation interacts strongly with matter and therefore only has a small penetration depth. Alpha radiation in the surrounding air will not be able to penetrate the outer layer of dead cells in the skin, and therefore will not cause any biological damage. Inside the body, however, the alpha particles can deposit their energy in living cells, which are then susceptible to the harmful effects of the radiation.
   f. Alpha particles have a very high linear energy transfer (LET), i.e. the amount of energy deposited per path length is very high. High LET is associated with increased biological damage, as the likelihood of several damages to a DNA molecule increases. The likelihood that the DNA damages can be repaired decrease when they are accumulated in one molecule.

2) *See handwritten solution below.*

3) Radiotherapy, beta- and gamma-decay

   a. ^{90}Y decays to almost 100% via beta decay to the ground state of ^{90}Zr, and thus almost all of the energy in the radiation is emitted via electrons that deposit their energy very locally in the tissue. This is desirable to be able to deliver a high dose to the tumour, while keeping the dose to healthy tissue low. ^{131}I on the other hand mostly decays to an excited state of ^{131}Xe, which then deexcites by emitting gamma radiation. The photons of the gamma radiation will penetrate further into the tissue than the electrons, and thus deposit a considerable fraction of the dose in surrounding healthy tissue. Other reasonable explanations based on for example difference in half-life are also acceptable.

   b. **^{131}I: 7/2+ → 5/2+, $\Delta\pi$=no, ΔI = 1,2,…,6**
   Because there is no change in parity, the emitted particles must have an even orbital angular momentum ($l = 0,2,\ldots$). $l = 0$ corresponds to an allowed transition, whereas $l = 2$ is second-level forbidden, and so on. In Gamow-Teller transitions, the spins of the electron and the neutrino couple so as to contribute S=1, allowing for ΔI = 1. Thus this decay is an allowed pure Gamow-Teller transition. Note that it is also compatible with a second-level forbidden mixed Fermi and Gamow-Teller transition, but this is so much less likely than an allowed transition, that it is negligible.

   ^{90}Y: 2- → 0+, $\Delta\pi$=yes, ΔI = 2
   The change in parity means that the orbital angular momentum must be odd. Thus this is a first-level forbidden pure Gamow-Teller transition.

   c. For even atomic mass numbers, there is a difference in binding energy depending on whether it is an even-even or odd-odd distribution between protons and neutrons in the nucleus (the 5th term in the SEMF). This yields two mass parabolas that are offset by 2*δ. Nuclides that are close to the minimum of the mass parabola, and have an odd-odd configuration, often have two neighbouring even-even isobars with lower mass (higher binding energy) that they can decay to. ^{127}I has an odd atomic mass number, and thus the δ term does not contribute to the binding energy, and there is consequently only one mass parabola.

   d. At low photon energies, the interaction of photons with matter will be dominated by the photoelectric effect, where the photons are fully absorbed and electrons are released. Thus the photon energy is very effectively converted into kinetic energy of charged particles, which contributes to the mass energy absorption coefficient and mass attenuation coefficient alike. At higher photon energies, Compton scattering will start to dominate, which will contribute to attenuation of the photon beam (thereby contribute to the mass attenuation coefficient) without converting the energy to kinetic energy of charged particles (mass energy absorption coefficient). Also, at higher energies the secondary electrons are more likely to emit Bremsstrahlung, which does not contribute to the mass energy absorption coefficient.

e. The peaks appear when the photons have sufficient energy to overcome the binding energy of a given shell of the atomic electrons, and thus have a larger pool of electrons they can interact with (thereby increasing the cross-section, and increasing the attenuation of the photons). The innermost shells have the highest binding energy, and thus the peak around 10^{-1} MeV corresponds to the K shell of lead. The peak before that corresponds to the L shell, and so on.
f. *0.12 Gy – see handwritten solution below*
g. *1.012 cm – see handwritten solution below*

4) Nuclear reactions
   a. Neutron moderation is the process of slowing down fast neutrons via elastic collisions down towards thermal energies. Elements with a low mass number are more effective at slowing down neutrons because a larger amount of energy is transferred in each elastic collision on average. It is also important that the cross-section for elastic scattering is high, so that the likelihood of a collision to occur is high for a given neutron. Finally, the absorption cross-section should ideally be low, so that the neutrons are not removed by absorption.
   b. Q-values
      i. *Q=-0.7637 MeV, endoergic reaction, Tth = 1.02MeV*
         See handwritten solution below
      ii. *Q = 0.63 MeV, exoergic reaction, T_{th} = 0*
         See handwritten solution below
5) Applications of radioactivity
   a. *309 million years – see handwritten solution below*
   b. See Lilley, Chapter 8
   c. Accelerated mass spectrometry to be able to accurately determine the amount of C-14 in the sample. AMS is highly sensitive and can separate isotopes as well as isobars.
   d. Neutron activation analysis, PIXE and RBS are all non-destructive analysis techniques that can be used to analyse the composition of materials in paint. They have different advantages and disadvantages (e.g. neutron activation analysis can penetrate deeper, but leaves the sample slightly radioactive) and may be used in different contexts. Each of them, with a reasonable motivation, has been accepted as answer.
6) MC1: If there is some probability of a nuclear reaction to occur, then there is also a probability of scattering
7) MC2: Photons emitted through acceleration of electrons
8) MC3: A Geiger-Mueller counter has a higher voltage between electrodes than a proportional counter
9) MC4: The maximum energy of a charged particle accelerated in a cyclotron depends on the strength of the magnetic field
   AND

Cyclotrons use an oscillating voltage between electrodes and a fixed magnetic field to accelerate charged particles

10) MC5: The shell model is based on solving the Schrödinger equation for a 3D harmonic oscillator
11) MC6: Nuclear density is approximately constant between elements

Handwritten solutions:

1c)

Volume air inhaled per year:

$$6\ \mathrm{l/min} \cdot 10^{-3}\ \frac{\mathrm{m}^3}{\mathrm{l}} \cdot 60\ \frac{\mathrm{s}}{\mathrm{min}} \cdot 24 \cdot 365 \cdot 0.6 = 1892.16\ \mathrm{m^3/year}$$

Total activity of Po-218 sticking to airways:

$$200\ \frac{\mathrm{Bq}}{\mathrm{m}^3} \cdot 1892.16\ \frac{\mathrm{m}^3}{\mathrm{year}} \cdot 0.1 = 3.78 \cdot 10^4\ \mathrm{Bq}$$

$$\hat{A}_{Po\text{-}218} = \frac{3.78 \cdot 10^4 \cdot 3.1 \cdot 60}{\ln(2)} = 1.01\cancel{8} \cdot 10^7$$ 2

$$\hat{A}_{Po\text{-}214} = \frac{3.78 \cdot 10^4 \cdot 164.3 \cdot 10^{-6}}{\ln(2)} = 9 \quad \text{negligible}$$

^{218}Po: yearly dose:

$$D = \frac{1.01 \cdot 10^7}{\mathrm{year}} \cdot \frac{6\ \mathrm{MeV} \cdot 1.6 \cdot 10^{-13}\ \mathrm{J/MeV}}{0.8\ \mathrm{kg}} = 1.2 \cdot 10^{-5}\ \frac{\mathrm{J}}{\mathrm{kg \cdot year}} = 12\ \frac{\mu\mathrm{Gy}}{\mathrm{year}}$$ 2

Equivalent dose: $20 \cdot 12\ \frac{\mu\mathrm{Gy}}{\mathrm{y}} = 0.24\ \frac{\mathrm{mSv}}{\mathrm{y}}$

Effective dose: $0.12 \cdot 0.24\ \frac{\mathrm{mSv}}{\mathrm{y}} = 28.8\ \frac{\mu\mathrm{Sv}}{\mathrm{y}}$ 2

1c)

Q_{Po-218} : $8.3569 + 0.1827 - 2.4249 = 6.1147$ MeV

T_α : $6.1147 \text{ MeV} \cdot \frac{1}{1+4/214} \approx 6$ MeV

NNDC $\Delta_{Po-218} = 8.3569$ MeV

$\Delta_{Pb-214} = -0.1827$ MeV

$\Delta_{He-4} = 2.4249$ MeV

$\Delta_{Po-214} = -4.47$ MeV

$\Delta_{Pb-210} = -14.7919$ MeV

Q_{Po-214} : $-4.47 + 14.7919 - 2.4249 \text{ MeV} = 7.897$ MeV

T_α : $7.897 \text{ MeV} \cdot \frac{1}{1+4/210} = 7.75$ MeV

Dose ^{218}Po & ^{214}Po :

$$12 \frac{\mu Gy}{y} \cdot \left(1 + \frac{7.75}{6}\right) \approx 28 \frac{\mu Gy}{y}$$

2

Equivalent dose: $20 \cdot 28 \frac{\mu Gy}{y} = 0.56 \frac{mSv}{y}$

Effective dose: $0.12 \cdot 0.56 \frac{mSv}{y} \approx 67 \frac{\mu Sv}{y}$

2)

α: $^{226}_{88}Ra \rightarrow ^{222}_{86}Rn + ^{4}_{2}He$

^{14}C: $^{226}_{88}Ra \rightarrow ^{212}_{82}Pb + ^{14}_{6}C$

NNDC

$\Delta_{Ra\text{-}226} = 23.6678$ MeV

$\Delta_{Rn\text{-}222} = 16.3722$ MeV

$\Delta_{He\text{-}4} = 2.4249$ MeV

$\Delta_{Pb\text{-}212} = -7.5488$ MeV

$\Delta_{C\text{-}14} = 3.0198$ MeV

$Q_\alpha = \Delta_{Ra\text{-}226} - \Delta_{Rn\text{-}222} - \Delta_{He\text{-}4} \approx 4.87$ MeV

$Q_{^{14}C} = \Delta_{Ra\text{-}226} - \Delta_{Pb\text{-}212} - \Delta_{C\text{-}14} \approx 28.2$ MeV

2

2)
$$\frac{\lambda_{14C}}{\lambda_\alpha} = \frac{f_{14C}}{f_\alpha} \cdot \frac{P_{14C}}{P_\alpha} = \frac{v_{14C}}{v_\alpha} \cdot \frac{a_\alpha}{a_{14C}} \cdot \frac{e^{-2G_{14C}}}{e^{-2G_\alpha}}$$

$$a_\alpha = 1.2\left(4^{1/3} + 222^{1/3}\right)\,\text{fm} = 9.17\,\text{fm}$$

$$a_{14C} = 1.2\left(14^{1/3} + 212^{1/3}\right)\,\text{fm} = 10.05\,\text{fm}$$

$$\frac{v_{14C}}{v_\alpha} = \sqrt{\frac{(V_0+Q_C)m_\alpha}{(V_0+Q_\alpha)m_{C14}}} = \sqrt{\frac{(35+28.2)\cdot 4}{(35+4.87)\cdot 14}} \approx 0.67$$ 2

$$G = \sqrt{\frac{2m}{\hbar^2 Q}}\,\frac{zZ'e^2}{4\pi\epsilon_0}\left(\cos^{-1}\sqrt{x} - \sqrt{x(1-x)}\right)$$

$x = a/b$ $\quad b = \frac{1}{4\pi\epsilon_0}\cdot\frac{zZ'e^2}{Q}$ $\quad \frac{e^2}{4\pi\epsilon_0} = 1.44\,\text{MeV}\cdot\text{fm}$

$$b_\alpha = 1.44\cdot\frac{2\cdot 86}{4.87}\,\text{fm} \approx 50.86\,\text{fm}$$

$$b_{14C} = 1.44\cdot\frac{6\cdot 82}{28.2}\,\text{fm} \approx 25.12\,\text{fm}$$

α: $\cos^{-1}\sqrt{x} - \sqrt{x(1-x)} \approx 0.7478$

^{14}C: ≈ 0.396

2) $\hbar c = 197\ \mathrm{MeV \cdot fm}$

$$G_\alpha = \sqrt{\frac{2 \cdot 4 \cdot 931.5}{4.87}} \cdot \frac{2 \cdot 86 \cdot 1.44}{197} \cdot 0.7478 \approx 36.78$$

$$G_{^{14}C} = \sqrt{\frac{2 \cdot 14 \cdot 931.5}{28.2}} \cdot \frac{6 \cdot 82 \cdot 1.44}{197} \cdot 0.396 \approx 43.31$$ 2

$$\frac{\lambda_{^{14}C}}{\lambda_\alpha} = 0.67 \cdot \frac{9.17}{10.05} \cdot e^{-2(G_{^{14}C} + G_\alpha)} \approx 1.3 \cdot 10^{-6}$$ 1

NNDC: $\alpha: 100\%$ $\quad \frac{\lambda_{^{14}C}}{\lambda_\alpha} \approx 3.2 \cdot 10^{-11}$ 1

$^{14}C: 3.2 \cdot 10^{-9}\ \%$

3 f) <u>NIST</u>

soft tissue, 300 keV: $\frac{\mu_{en}}{\rho} = 3.164 \cdot 10^{-2} \text{ cm}^2/\text{g}$

At CPE:

$$D = K_c = \Psi \cdot \frac{\mu_{en}}{\rho}$$

$$= 300 \text{ keV} \cdot 8 \cdot 10^{10} \text{ cm}^{-2} \cdot 3.164 \cdot 10^{-2} \frac{\text{cm}^2}{\text{g}} \cdot 1.6 \cdot 10^{-16} \text{ J/keV}$$

$$\approx 1.2 \cdot 10^{-4} \text{ J/g} = 1.2 \cdot 10^{-1} \text{ J/kg} = 0.12 \text{ Gy}$$

g) $I = I_0 e^{-\mu x}$ $\qquad \mu = \frac{\mu}{\rho} \cdot \rho$

$$x = -\frac{\ln(I/I_0)}{\mu}$$

NIST: Lead, 300 keV $\frac{\mu}{\rho} = 4.031 \cdot 10^{-1} \text{ cm}^2/\text{g}$

$\rho = 11.3 \text{ g/cm}^3$

$$x = -\frac{\ln(0.01)}{4.031 \cdot 10^{-1} \cdot 11.3} \text{ cm} \approx 1.01 \text{ cm}$$

4b) <u>NNDC</u>

$\Delta_n = 8.0713$ MeV

$\Delta_{^1H} = 7.2889$ MeV

$\Delta_p = \Delta_H - m_e = 7.2889 - 0.511$ MeV $= 6.7779$ MeV

$\Delta_{^3H} = 14.9498$ MeV

$\Delta_{^3He} = 14.9312$ MeV

$\Delta_{^{14}C} = 3.0199$ MeV

$\Delta_{^{14}N} = 2.8634$ MeV

i. $Q = \Delta_{^3H} - m_e + \overset{\Delta_{^1H} - m_e}{\cancel{}} - (\Delta_{^3He} - 2m_e) - \Delta_n$

$= \Delta_{^3H} + \Delta_H - \Delta_{^3He} - \Delta_n$

$= 14.9498 + 7.2889 - 14.9312 - 8.0713$ MeV

$= -0.7638$ MeV

$Q<0 \Rightarrow T_{th} = (-Q)\dfrac{m_{^3He} + m_n}{m_{^3He} + m_n - m_p}$

$\approx (-Q)\cdot\dfrac{4}{3} \approx 1.02$ MeV

4b)

ii. $Q = \Delta_{^{14}N} - 7m_e + \Delta_n - (\Delta_{^1H} - m_e) - (\Delta_{^{14}C} - 6m_e)$

$= \Delta_{^{14}N} + \Delta_n - \Delta_{^{14}C} - \Delta_{^1H}$

$= 2.8634 + 8.0713 - 3.0199 - 7.2889$ MeV

$= 0.6259$ MeV

$Q>0 \Rightarrow T_{th} = 0$

5a)

$$N_1 = N_0 e^{-\lambda t}$$

$$N_2 = f_{Ar} \cdot N_0 (1 - e^{-\lambda t}) \qquad f_{Ar} = \frac{\lambda_{Ar}}{\lambda} = 0.107$$

$$\frac{N_2}{N_1} = f_{Ar} \frac{1 - e^{-\lambda t}}{e^{-\lambda t}} = f_{Ar} (e^{\lambda t} - 1)$$

$$\frac{N_2}{f_{Ar} N_1} + 1 = e^{\lambda t}$$

$$t = \frac{1}{\lambda} \ln\left(1 + \frac{N_2}{f_{Ar} N_1}\right)$$

$$\frac{N_2}{N_1} = 0.02 \implies t = \frac{1.248 \cdot 10^9}{\ln(2)} \cdot \ln\left(1 + \frac{0.02}{0.107}\right) \text{ years}$$

$$\approx 0.31 \cdot 10^9 \text{ years}$$

($\approx 35.65 \cdot 10^6$ years
if f_{Ar} is not accounted for)