TFY4225 Fall 2022 - Exam solutions

Johanna Vannesjö

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Question 1

a) Let the starting activity be A_0. Use that $t_{1/2} = 0.218 \cdot 10^6\ y$.

$$\begin{aligned} A_0 e^{-\lambda t} &= 0.01 A_0 \\ -\lambda t &= \ln 0.01 \\ t &= -\frac{\ln 0.01}{\lambda} = -\frac{\ln 0.01 \cdot t_{1/2}}{\ln 2} \approx 1.45 \cdot 10^6\ y \end{aligned}$$

b) There are two possible solutions to this question, depending on the time scale one looks at (either solution is given full points). The half-life of the daughter is much shorter than the parent, giving that $\lambda_2 >> \lambda_1$. From the formula sheet, we have:

$$A_2 = A_0 \frac{\lambda_2}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t}).$$

Using $\lambda_2 >> \lambda_1$ yields:

$$\frac{\lambda_2}{\lambda_2 - \lambda_1} \approx 1.$$

Looking at short time scales we can use the approximation:

$$e^{-\lambda_1 t} \approx 1,$$

yielding the equation for the daughter activity approaching secular equilibrium:

$$A_2 \approx A_0(1 - e^{-\lambda_2 t}).$$

Alternatively, looking at long time scales, we can instead use the approximation:

$$e^{-\lambda_2 t} \approx 0,$$

yielding the equation for the daughter activity once it is in secular equilibrium (equaling the activity of the parent):

$$A_2 \approx A_0 e^{-\lambda_1 t}.$$

c) Here we need to use the equation for the daughter activity when approaching secular equilibrium. Use that $t_{1/2} = 12.35\ d$.

$$\begin{aligned} A_0(1 - e^{-\lambda_2 t}) &= 0.99 A_0 \\ 1 - 0.99 &= e^{-\lambda_2 t} \\ -\lambda_2 t &= \ln 0.01 \\ t &= -\frac{\ln 0.01}{\lambda} = -\frac{\ln 0.01 \cdot t_{1/2}}{\ln 2} \approx 82.05\ d \end{aligned}$$

Question 2

a) The Geiger-Müller (GM) detector is a type of gas detector, with a gas placed between two electrodes. A voltage is applied over the electrodes, such that when incoming radiation ionizes molecules in the gas, the electrons will migrate towards the positive anode and the positive ions will migrate towards the negative cathode. The detector usually has a cylindrical geometry to increase the density of electric field lines towards the central anode. The GM detector uses a higher voltage than other gas detectors. Due to the high voltage, the electrons will accelerate sufficiently to ionize further molecules, creating a cascade of ionization. The voltage is also sufficiently high that the accelerated electrons can excite inner electrons, which in turn emit UV photons when de-excited. The UV photons have sufficient energy to ionize further molecules in the gas. As a consequence, a single initial ionization event due to interaction with radiation will cause an ionization cascade that results in a discharge near the anode. This gives a very high pulse amplitude in the detected pulse, and high sensitivity to detect radiation. On the other hand, it is not possible to resolve the energy of the initial ionization event. This type of detector thus works as a radiation counter, but is not suitable to determine the type of radiation. To avoid secondary discharges as the ions are neutralized, a quenching gas is usually used. The quenching process is quite slow, so the detector is not effective at very high count rates.

b) As the neutrally charged neutrons do not interact with electrons in matter, they do not cause direct ionization of atoms/molecules. Instead neutron detectors detect secondary radiation emitted from nuclear reactions between the neutrons and the nuclei of the material. Fast neutron detectors differ from slow neutron detectors because the interaction cross-section for most reactions have a 1/v dependence, and is thus very low at high neutron energies. Fast neutron detectors therefore use moderating materials with a high scattering cross-section at a large range of energies. One type of fast neutron detectors uses a scintillating material (e.g. plastics or organic liquids) with a high content of hydrogen, which is a good neutron moderator. The fast neutrons can transfer sufficient energy to the hydrogen nuclei (protons) that the recoil protons are released and can be detected as secondary radiation by the scintillation detector. An alternative design is to have a gas detector surrounded by a moderating material, where the secondary radiation from neutron interactions is detected by the central gas detector.

Question 3

a) The SEMF estimates the mass of a nuclide as a function of Z and A. For constant A, the SEMF takes the form of a parabola (or two offset parabolas for even A). The most stable nuclide for constant A can thus be found by taking the derivative with respect to Z, and setting it to zero.

$$M(Z,A) = Zm(^1H) + (A-Z)m_n - \left[a_V A - a_S A^{2/3} - a_C A^{-1/3} Z(Z-1) - a_{sym} A^{-1}(A-2Z)^2 + \delta\right]/c^2$$

$$\frac{dM}{dZ} = m(^1H) - m_n - \left[-a_C A^{-1/3}(2Z-1) - a_{sym}A^{-1}\cdot 2(A-2Z)\cdot(-2)\right]/c^2 = 0$$

$$(m(^1H) - m_n)c^2 - a_C A^{-1/3} - 4a_{sym} = -2a_C A^{-1/3} Z - 8a_{sym}A^{-1}Z$$

$$Z_{min} = \frac{(m_n - m(^1H))c^2 + a_C A^{-1/3} + 4a_{sym}}{2a_C A^{-1/3} + 8a_{sym}A^{-1}}$$

b) The terms that are relevant to determine the most stable isobar are:

- **Coulomb term:** The Coulomb term accounts for the repulsive Coulomb force between the positively charged protons in the nucleus. As it is a repulsive force it decreases the binding energy (increases the mass). Each proton sees the effect of the other protons in the nucleus, hence the dependence on Z(Z-1).
- **Symmetry term:** The symmetry term describes the tendency of nuclei to have equal numbers of protons and neutrons. This depends on the quantum mechanical shell model of the nucleus, where there are discrete energy levels for the nucleons. As protons and neutrons are fermions,

two equal particles cannot be in identical quantum mechanical states. But protons are not equal to neutrons, so the protons and the neutrons fill up the energy levels independently. If there are more of one than the other, it means that the excessive protons/neutrons will need to reside at higher energy levels, thereby decreasing the total binding energy of the nucleus.

- (**Pairing term:** The pairing term also depends on quantum mechanical properties of the nucleons. As fermions, it is energetically favorable for two protons/neutrons to pair up. For odd A, there will always be one unpaired proton or neutron, and the pairing term does not contribute to the SEMF. However for even A, there can either be odd numbers of neutrons and protons, or even numbers of neutrons and protons, where the latter case has higher binding energy (lower mass). The pairing term does not appear in the equation derived in a), but is also relevant to determine the most stable nuclide. It is however not necessary to include the pairing term to get full points on this question.)

c) Inserting numerical values in the equation derived in a) yields:

$$Z_{min} = \frac{(8.071 - 7.289) + 0.72A^{-1/3} + 92}{1.44A^{-1/3} + 184A^{-1}}$$

$$A = 121 \Rightarrow Z_{min} \approx 51.3 \;\Rightarrow\; {}^{121}_{51}Sb$$
$$A = 74 \Rightarrow Z_{min} \approx 32.9 \;\Rightarrow\; {}^{74}_{33}As$$

d) In the derivation of Z_{min} we did not take the pairing term, δ, into account. For even A nuclei, this gives two offset parabolas, where nuclei with odd numbers of protons and neutrons have higher mass (and are thereby less stable) than nuclei with even numbers of protons and neutrons. The difference between the parabolas is $2\delta = 2 \cdot 34A^{-3/4}\ MeV$. As the calculated Z_{min} is close to the center between ${}^{74}_{32}Ge$ and ${}^{74}_{34}Se$ it is reasonable that both nuclides are stable. ${}^{74}_{33}As$ is however not stable, due to the pairing term.

Question 4

a) The Coulomb barrier is given by the Coulomb potential between the two nuclei at their closest possible separation distance before fusing to become one nucleus.

$$R = R_0(2^{1/3} + 3^{1/3}) = 1.4 \cdot (2^{1/3} + 3^{1/3}) \approx 3.78 fm$$
$$B = \frac{zZe^2}{4\pi\epsilon_0 R} = \frac{1 \cdot 2}{3.78} \cdot 1.44\ MeV \approx 0.76\ MeV$$

b) For the particles to overcome the Coulomb barrier classically, the temperature must be high enough that the kinetic energy of the particles equals, or exceeds, the Coulomb barrier.

$$T = \frac{T_p}{k_B} = \frac{0.76\ MeV}{8.6 \cdot 10^{-11}\ MeV/K} \approx 8.8 \cdot 10^9 K.$$

This calculation assumes one particle at rest, and the other with the most probable kinetic energy at the temperature. Assuming two particles in a head-on collision where each has the most probable kinetic energy yields half the temperature, $4.4 \cdot 10^9 K$. Either assumption is fine - it's the order of magnitude that matters.

c) At a temperature of $10^8 K$, the most probable kinetic energy of a particle is:

$$T_p = k_B \cdot T = 8.6 \cdot 10^{-11} \cdot 10^8\ MeV = 8.6\ keV.$$

The closest the particles can get in the classical case is when the Coulomb potential corresponds to the kinetic energy of the particle. The distance can be estimated by:

$$d = \frac{zZe^2}{4\pi\epsilon_0 T_p} = \frac{2}{8.6 \cdot 10^{-3}\ MeV} 1.44\ MeV \cdot fm \approx 335\ fm$$

Again, this calculation assumes one particle at rest, the other at T_p. If we instead assume both particles at T_p in a head-on collision, the closest they can get classically is 167 fm.

d) The probability of tunneling through the Coulomb barrier is given by:

$$\begin{aligned} P &= e^{-2G} \\ G &= \sqrt{\frac{2m}{\hbar^2 Q}} \frac{zZe^2}{4\pi\epsilon_0} \left[\arccos\sqrt{x} - \sqrt{x(1-x)}\right] \\ x &= Q/B \end{aligned}$$

In this case, Q corresponds to the kinetic energy of the free particles. B is the maximum height of the Coulomb barrier, calculated in a). Assuming one particle of ^2H at kinetic energy T_p colliding with one particle of ^3He at rest yields:

$$\begin{aligned} x = \frac{T_p}{B} = \frac{8.6\cdot 10^{-3}}{0.76} &\approx 0.0113 \\ \arccos\sqrt{x} - \sqrt{x(1-x)} &\approx 1.36 \\ \frac{zZe^2}{4\pi\epsilon_0} = 2\cdot 1.44\ MeV\cdot fm &= 2.88\ MeV\cdot fm \\ \sqrt{\frac{2m}{\hbar^2 Q}} = \sqrt{\frac{2\cdot 2\ u\cdot 931.5 MeV/u}{8.6\cdot 10^{-3}\ MeV}}\frac{1}{197\ MeV\cdot fm} &\approx 3.34\ MeV^{-1}\cdot fm^{-1} \end{aligned}$$

$$\begin{aligned} G = 3.34\cdot 2.88\cdot 1.36 \approx 13 \\ P = e^{-2G} \approx 5\cdot 10^{-12} \end{aligned}$$

Other reasonable assumptions (e.g. kinetic energy of two particles, or particle with mass of ^3He) are also accepted.

Question 5

a) The Q-value is given by

$$Q = \left(\sum m_{initial} - \sum m_{final}\right) c^2.$$

For β^+ decay we start with the parent nucleus, and end up with the daughter nucleus plus one positron (the neutrino is practically massless and can be ignored in the calculation).

$$\begin{aligned} m_N(X) - m_N(X') - m_e &= m(X) - Zm_e - [m(X') - (Z-1)m_e] - m_e \\ &= m(X) - m(X') - 2m_e \end{aligned}$$

Here we can directly use mass excesses in the equation, yielding:

$$Q = \Delta_{^{13}N} - \Delta_{^{13}C} - 2m_e c^2 = 5346 - 3125 - 2\cdot 511\ keV = 1199\ keV \approx 1.2\ MeV$$

b) The accumulated activity is given by:

$$\begin{aligned} \tilde{A} &= A_0 \int_0^\infty e^{-\lambda t} dt = A_0 \left[\frac{e^{-\lambda t}}{-\lambda}\right]_0^\infty = \frac{A_0}{\lambda} = \frac{A_0 t_{1/2}}{\ln 2} \\ &= \frac{600\cdot 10^6\ Bq\cdot 600\ s}{\ln 2} \approx 5.2\cdot 10^{11}\ disintegrations \end{aligned}$$

Each disintegration gives rise to one positron and one neutrino. The latter will not interact with the body, and can be ignored in the dose calculation. The positron will share the kinetic energy with the neutrino, so the average kinetic energy of the positron will be lower than the Q-value of the decay by some factor, which we here assume to be 1/3. Note that the Q-value in a) was already calculated for β^+ decay and does not need to be corrected by $2m_e$ again. However, if you would instead read out the Q^+ value from a decay chart, it only gives the mass difference between parent and daughter nuclei and needs to be corrected by $2m_e$ for β^+ decay. All of the kinetic energy of the positron can be assumed to be deposited in the tissue, as it has a small penetration depth. After depositing its kinetic energy, the positron will recombine with one electron and release two photons with $E_\gamma = 511\ keV$. The photons will also interact with the tissue and deposit energy, but as they have a much larger penetration depth, some of the photons will escape from the body. We can here assume that half of the photon energy is deposited in tissue. Assuming a body weight of 70 kg, we get:

$$\begin{aligned} S_{\beta+} = \frac{1}{M_T}\sum E_i Y_I \phi_i &= \frac{1199}{3}\cdot 1\cdot 1\ keV \cdot 1.6\cdot 10^{-16}\ J/keV \cdot \frac{1}{70\ kg} = 0.91\cdot 10^{-15} J/kg \\ S_\gamma &= 511\cdot 2\cdot 0.5\ keV\cdot 1.6\cdot 10^{-16}\ J/keV\cdot \frac{1}{70\ kg} = 1.17\cdot 10^{-15} J/kg \end{aligned}$$

The radiation weighting factor for electrons and photons are both $w_R = 1$, and the tissue weighting factor for the whole body is $w_T = 1$ assuming uniform distribution. This yields the effective dose:

$$D = w_T \tilde{A}(w_{\beta+}S_{\beta+} + w_\gamma S_\gamma) = 1\cdot 5.2\cdot 10^{11}(1\cdot 0.91\cdot 10^{-15} + 1\cdot 1.17\cdot 10^{-15})\ J/kg \approx 1.1\ mSv$$

Note that any reasonable assumptions for necessary quantities that were not given in the question (e.g. whole body mass) are accepted.

c) In the case where we don't have uniform distribution of the radionuclide, we can calculate the dose to the different organs separately. Here we need to take into account the different tissue weighting factors. As the positron has low penetration depth, we can assume that all of the kinetic energy of the positron is deposited locally in the respective tissue. On the other hand, the photons have large penetration depth and will deposit their energy also outside of the source tissue. We can therefore assume a uniform distribution in the body for the photons. This gives:

$$\begin{aligned} D_{\beta+,liver} &= w_{liver}\cdot 0.2\tilde{A}S_{\beta+,liver} = 0.04\cdot 0.2\cdot 5.2\cdot 10^{11}\cdot \frac{400\cdot 1.6\cdot 10^{-16}}{1.5}\ Sv = 0.18\ mSv \\ D_{\beta+,bladder} &= w_{bladder}\cdot 0.2\tilde{A}S_{\beta+,bladder} = 0.04\cdot 0.2\cdot 5.2\cdot 10^{11}\cdot \frac{400\cdot 1.6\cdot 10^{-16}}{0.04}\ Sv = 6.62\ mSv \\ D_{\beta+,brain} &= w_{brain}\cdot 0.2\tilde{A}S_{\beta+,brain} = 0.01\cdot 0.2\cdot 5.2\cdot 10^{11}\cdot \frac{400\cdot 1.6\cdot 10^{-16}}{1.3}\ Sv = 0.05\ mSv \\ D_{\beta+,rest} &= w_{rest}\cdot 0.4\tilde{A}S_{\beta+,rest} \\ &= (1-0.04-0.04-0.01)\cdot 0.4\cdot 5.2\cdot 10^{11}\cdot \frac{400\cdot 1.6\cdot 10^{-16}}{70-1.5-0.04-1.3}\ Sv = 0.18\ mSv \\ D_\gamma &= w_{body}\tilde{A}S_\gamma = 1\cdot 5.2\cdot 10^{11}\cdot 1.17\cdot 10^{-15}\ Sv = 0.61\ mSv \\ D_{tot} &= D_{\beta+,liver} + D_{\beta+,bladder} + D_{\beta+,brain} + D_{\beta+,rest} + D_\gamma \\ &= 0.18 + 6.62 + 0.05 + 0.18 + 0.61\ mSv = 7.64\ mSv \end{aligned}$$

Multiple choice questions

1) "The energy levels in the nuclear shell model are equal to the energy levels of electrons in the atomic model" is **false**.
2) "Alpha or beta decay is often followed by gamma decay" is **true**.
3) "Heavy charged particles deposit their kinetic energy through many interactions with electrons in materia" is **true**.
4) "Cells are less sensitive to ionizing radiation in a high-oxygen environment" is **false**.
5) "In the charged particle equilibrium (CPE) the collision KERMA directly corresponds to the dose for a photon beam" is **true**.