NORGES TEKNISK- NATURVITENSKAPELIGE UNIVERSITET
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Eksamen, SIF 4060: Elektromagnetisk teori,

Onsdag 20. desember, 2000, tid: 09.00-13.00

Exam, SIF 4060: Electromagnetic theory,

Wednesday, December 20, 2000, time: 09.00-13.00

Innhold / Content:

I. Norsk oppgavetekst:

Tillatte hjelpemidler: B2 - Typegodkjent kalkulator, med tomt minne, i henhold til utarbeidet liste.
Rottmann: Matematische Formelsammlung
Barnett & Cronin: Mathematical Formulae
Øgrim: Størrelser og enheter i fysikken
Se også oppgitte formeler side 8-11.

Oppgave 1

a) Vis at ladningsbevarelsen følger direkte av Maxwells ligninger.

I ledende materialer er strømtettheten gitt som: $\mathbf{J} = \sigma\mathbf{E}$, hvor σ er materialets konduktivitet (ledningsevne). Anta at vi i et ledende materiale har en gitt ladningsfordeling $\rho_0(\mathbf{r})$ ved tid $t = 0$. Beregn $\rho(\mathbf{r},t)$ for $t \geq 0$.

Forklar hvorfor man ikke finner frie ladninger i det indre av ledende materialer.

b) I resten av oppgaven ser vi på det generelle tilfellet.

Vis at når feltene uttrykkes ved hjelp av potensialene V og $\mathbf{A}$ (se oppgitte formeler), så er to av Maxwells ligninger automatisk oppfylt.

De siste to av Maxwells ligninger gir to koplede ligninger for V og $\mathbf{A}$. Finn disse.

c) Forklar hva som menes med en justeringstransformasjon (gauge-transform).

Vis at ved Lorentz-justering, hvor vi har $\nabla \cdot \mathbf{A} = -\mu_0\varepsilon_0 \dfrac{\partial V}{\partial t}$, reduseres de to koplede ligningene for V og $\mathbf{A}$ i b) til de to bølgeligningene:

$$\nabla^2 V - \varepsilon_0\mu_0 \frac{\partial^2 V}{\partial t^2} = -\rho/\varepsilon_0 \quad \text{og} \quad \nabla^2 \mathbf{A} - \varepsilon_0\mu_0 \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu_0 \mathbf{J}.$$

d) I c) er de to bølgeligningene for V og $\mathbf{A}$ dekoplet. Har de to ligningene løsninger som er uavhengige av hverandre? Svaret skal begrunnes.

Oppgave 2

a) I elektrostatikk er det elektriske feltet bare gitt av skalarpotensialet V. For en gitt statisk ladningsfordeling i fritt rom er løsningen for potensialet gitt som:

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{1}{\imath}\rho(\mathbf{r}')d\tau' \text{; hvor } \imath = |\mathbf{r}-\mathbf{r}'|.$$

Hvilken differensialligning er dette løsning av? Skriv den ned.

Bestem den ladningsfordelingen som gir opphav til følgende potensial:

$$V(\mathbf{r}) = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 r} & \text{; for } r > R \\ \dfrac{Q}{4\pi\varepsilon_0}\dfrac{1}{2R}\left[3-(r/R)^2\right] & \text{; for } r \le R. \end{cases}$$

b) Det forutsettes kjent at

$$\frac{1}{\imath} = \frac{1}{|\mathbf{r}-\mathbf{r}'|} = \frac{1}{r}\sum_{m=0}^{\infty}\left(\frac{r'}{r}\right)^m P_m(\cos\theta'),$$

hvor de første Legendrepolynomene er: $P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = (3x^2-1)/2$, ..osv., og θ' er vinkelen mellom vektorene $\mathbf{r}$ og $\mathbf{r}'$.

Vis hvordan potensialet fra en gitt statisk ladningsfordeling $\rho(\mathbf{r})$ kan representeres ved en multipolutvikling.

Skriv opp uttrykk for monopol- og dipolleddene.

c) Vis at dipolleddet kan uttrykkes som:

$$V_{dip}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^2}, \text{ hvor } \mathbf{p} \text{ er dipolmomentet.}$$

Anta at dipolmomentet **p** er rettet langs z aksen. Innfør kulekoordinater og beregn komponentene E_r, E_θ og E_ϕ av det elektriske dipolfeltet.

d) Vis at dipolfeltet kan skrives i koordinatfri form som:

$$\mathbf{E}_{dip}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^3}\left[3(\mathbf{p}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{p}\right].$$

Oppgave 3

a) Ta utgangspunkt i Maxwells ligninger på differensiell form (se oppgitte formeler) og skriv ned, eller utled, de tilhørende ligningene på integralform.

Hvilke grensebetingelser tilfredsstiller **E**, **B**, **D** og **H** ved grenseflaten mellom to materialer uten frie ladninger?

b) I stor avstand fra en dipol med tidsavhengig dipolmoment $\mathbf{p}(t)$ er potensialene gitt ved:

$$V(\mathbf{r},t) = \frac{\mathbf{p}(t-r/c)\cdot\hat{\mathbf{r}}}{4\pi\varepsilon_0 r^2} + \frac{\dot{\mathbf{p}}(t-r/c)\cdot\hat{\mathbf{r}}}{4\pi\varepsilon_0 cr} \quad \text{og} \quad \mathbf{A}(\mathbf{r},t) = \frac{\mu_0\dot{\mathbf{p}}(t-r/c)}{4\pi r}, \text{ hvor } \dot{\mathbf{p}}(t) = \frac{d\mathbf{p}(t)}{dt}.$$

Beregn elektrisk felt i bølgesonen, dvs. for så store r at vi kan se bort fra bidrag som går mot null raskere enn $1/r$ når $r\to\infty$.

c) Vis at resultatet i b) kan skrives som: $\mathbf{E}(\mathbf{r},t) = \frac{\mu_0}{4\pi r}\left[\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\ddot{\mathbf{p}}(t-r/c)\right)\right].$

Finn tilsvarende uttrykk for $\mathbf{B}(\mathbf{r},t)$.

d) Bruk resultatet fra c) til å beregne Poyntings vektor $\mathbf{S}(\mathbf{r},t)$ og totalt utstrålt effekt fra dipolen.

II. English text

The following tools are allowed:

B2 - Calculator with empty memory, according to approved list.

Rottmann: Matematische Formelsammlung

Barnett & Cronin: Mathematical Formulae

Øgrim: Størrelser og enheter i fysikken

See also the formulas given on pages 8-11.

Problem 1

a) Show that charge conservation follows directly from Maxwell's equations.

In conductive materials the current density is given by: $\mathbf{J} = \sigma\mathbf{E}$, where σ is the conductivity of the material. Assume that, in a conductive material, we have a given charge distribution $\rho_0(\mathbf{r})$ at time $t = 0$. Compute $\rho(\mathbf{r},t)$ for $t \geq 0$.

Explain why free charges are not found in the interior of conducting materials.

b) In the rest of this problem we consider the general case.

Show that, when the fields are expressed by the potentials V and $\mathbf{A}$ (see the given formulas), two of Maxwell's equations are automatically satisfied.

The last two of Maxwell's equations yield two coupled equations for V and $\mathbf{A}$. Find these equations.

c) Explain what is meant by a gauge transform.

Show that, in the Lorentz-gauge, where $\nabla \cdot \mathbf{A} = -\mu_0\varepsilon_0 \frac{\partial V}{\partial t}$, the two coupled equations for V and $\mathbf{A}$ in b) are reduced to the two wave equations:

$$\nabla^2 V - \varepsilon_0\mu_0 \frac{\partial^2 V}{\partial t^2} = -\rho/\varepsilon_0 \quad \text{and} \quad \nabla^2 \mathbf{A} - \varepsilon_0\mu_0 \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu_0 \mathbf{J}.$$

d) In c) the two wave equations for V and $\mathbf{A}$ are de-coupled. Do the two equations have independent solutions? State the reason for the answer.

Problem 2

a) In electrostatics, the electric field is fully determined by the scalar potential V. For a given static charge distribution in free space, the solution for the potential is given by:

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{1}{\mathscr{r}}\rho(\mathbf{r}')d\tau' ; \text{ where } \mathscr{r} = |\mathbf{r} - \mathbf{r}'| .$$

For which differential equation is this the solution? Write it down.

Determine the charge distribution that gives rise to the following potential:

$$V(\mathbf{r}) = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 r} & ; \text{for } r > R \\ \dfrac{Q}{4\pi\varepsilon_0}\dfrac{1}{2R}\left[3 - (r/R)^2\right] & ; \text{for } r \le R. \end{cases}$$

b) The following formula is assumed known:

$$\frac{1}{\mathscr{r}} = \frac{1}{|\mathbf{r} - \mathbf{r}'|} = \frac{1}{r}\sum_{m=0}^{\infty}\left(\frac{r'}{r}\right)^m P_m(\cos\theta'),$$

where the first Legendre polynomials are: $P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = (3x^2-1)/2$, ..etc, and θ' is the angle between the vectors $\mathbf{r}$ and $\mathbf{r}'$.

Show how the potential of a static charge distribution $\rho(\mathbf{r})$ can be represented by a multi-pole development.

Write down the expressions for the monopole- and the dipole-terms.

c) Show that the dipole-term can be expressed as:

$$V_{dip}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^2}, \text{ where } \mathbf{p} \text{ is the dipole moment.}$$

Assume that the dipole moment **p** is directed along the z axis. Use spherical coordinates and compute the components E_r, E_θ, and E_ϕ of the electric dipole field.

d) Show that the dipole field can be expressed in coordinate-free form as:

$$\mathbf{E}_{dip}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^3}\left[3(\mathbf{p}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{p}\right].$$

Problem 3

a) Use Maxwell's equations in differential form as the starting point (see the given formulas) and write down, or derive, the corresponding equations in integral form.

What are the boundary conditions satisfied by **E**, **B**, **D**, and **H** at an interface between two materials without free charges?

b) At a large distance from a dipole with a time-dependent dipole moment $\mathbf{p}(t)$, the potentials are given by:

$$V(\mathbf{r},t) = \frac{\mathbf{p}(t-r/c)\cdot\hat{\mathbf{r}}}{4\pi\varepsilon_0 r^2} + \frac{\dot{\mathbf{p}}(t-r/c)\cdot\hat{\mathbf{r}}}{4\pi\varepsilon_0 cr} \quad \text{and} \quad \mathbf{A}(\mathbf{r},t) = \frac{\mu_0\dot{\mathbf{p}}(t-r/c)}{4\pi r}, \quad \text{where } \dot{\mathbf{p}}(t) = \frac{d\mathbf{p}(t)}{dt}.$$

Compute the electrical field in the wave-zone, i.e., when r is so large that we can neglect all terms that approach zero faster than $1/r$ for $r\to\infty$.

c) Show that the result in b) can be written as. $\mathbf{E}(\mathbf{r},t) = \frac{\mu_0}{4\pi r}\left[\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\ddot{\mathbf{p}}(t-r/c)\right)\right]$.

Find the corresponding expression for $\mathbf{B}(\mathbf{r},t)$.

d) Use the results from c) to compute Poynting's vector $\mathbf{S}(\mathbf{r},t)$ and the totalt radiated power of the dipole.

III. Oppgitte formeler / Given formulas

VECTOR DERIVATIVES

Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$; $d\tau = dx\,dy\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial x}\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}$; $d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient : $$\nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

Curl : $$\nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian : $$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}$; $d\tau = s\,ds\,d\phi\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial t}{\partial\phi}\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial\phi}\right]\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2}$$

VECTOR IDENTITIES

Triple Products

(1) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$

(2) $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$

Product Rules

(3) $\nabla(fg) = f(\nabla g) + g(\nabla f)$

(4) $\nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A}$

(5) $\nabla \cdot (f\mathbf{A}) = f(\nabla \cdot \mathbf{A}) + \mathbf{A} \cdot (\nabla f)$

(6) $\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$

(7) $\nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)$

(8) $\nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B} + \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A})$

Second Derivatives

(9) $\nabla \cdot (\nabla \times \mathbf{A}) = 0$

(10) $\nabla \times (\nabla f) = 0$

(11) $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$

FUNDAMENTAL THEOREMS

Gradient Theorem : $\int_{\mathbf{a}}^{\mathbf{b}} (\nabla f) \cdot d\mathbf{l} = f(\mathbf{b}) - f(\mathbf{a})$

Divergence Theorem : $\int (\nabla \cdot \mathbf{A})\, d\tau = \oint \mathbf{A} \cdot d\mathbf{a}$

Curl Theorem : $\int (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \oint \mathbf{A} \cdot d\mathbf{l}$

BASIC EQUATIONS OF ELECTRODYNAMICS

Maxwell's Equations

In general :

$$\begin{cases} \nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \end{cases}$$

In matter :

$$\begin{cases} \nabla \cdot \mathbf{D} = \rho_f \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t} \end{cases}$$

Auxiliary Fields

Definitions :

$$\begin{cases} \mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \\ \mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M} \end{cases}$$

Linear media :

$$\begin{cases} \mathbf{P} = \epsilon_0 \chi_e \mathbf{E}, \quad \mathbf{D} = \epsilon \mathbf{E} \\ \mathbf{M} = \chi_m \mathbf{H}, \quad \mathbf{H} = \frac{1}{\mu}\mathbf{B} \end{cases}$$

Potentials

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}, \qquad \mathbf{B} = \nabla \times \mathbf{A}$$

Lorentz force law

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Energy, Momentum, and Power

Energy : $U = \frac{1}{2}\int \left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) d\tau$

Momentum : $\mathbf{P} = \epsilon_0 \int (\mathbf{E} \times \mathbf{B})\, d\tau$

Poynting vector : $\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B})$

Larmor formula : $P = \frac{\mu_0}{6\pi c} q^2 a^2$

FUNDAMENTAL CONSTANTS

ϵ_0	$= 8.85 \times 10^{-12}\, \mathrm{C^2/Nm^2}$	(permittivity of free space)
μ_0	$= 4\pi \times 10^{-7}\, \mathrm{N/A^2}$	(permeability of free space)
c	$= 3.00 \times 10^{8}\, \mathrm{m/s}$	(speed of light)
e	$= 1.60 \times 10^{-19}\, \mathrm{C}$	(charge of the electron)
m	$= 9.11 \times 10^{-31}\, \mathrm{kg}$	(mass of the electron)

SPHERICAL AND CYLINDRICAL COORDINATES

Spherical

$$\begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases} \qquad \begin{cases} \hat{\mathbf{x}} = \sin\theta\cos\phi\,\hat{\mathbf{r}} + \cos\theta\cos\phi\,\hat{\boldsymbol{\theta}} - \sin\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} = \sin\theta\sin\phi\,\hat{\mathbf{r}} + \cos\theta\sin\phi\,\hat{\boldsymbol{\theta}} + \cos\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}} \end{cases}$$

$$\begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \theta = \tan^{-1}(\sqrt{x^2+y^2}/z) \\ \phi = \tan^{-1}(y/x) \end{cases} \qquad \begin{cases} \hat{\mathbf{r}} = \sin\theta\cos\phi\,\hat{\mathbf{x}} + \sin\theta\sin\phi\,\hat{\mathbf{y}} + \cos\theta\,\hat{\mathbf{z}} \\ \hat{\boldsymbol{\theta}} = \cos\theta\cos\phi\,\hat{\mathbf{x}} + \cos\theta\sin\phi\,\hat{\mathbf{y}} - \sin\theta\,\hat{\mathbf{z}} \\ \hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}} \end{cases}$$

Cylindrical

$$\begin{cases} x = s\cos\phi \\ y = s\sin\phi \\ z = z \end{cases} \qquad \begin{cases} \hat{\mathbf{x}} = \cos\phi\,\hat{\mathbf{s}} - \sin\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} = \sin\phi\,\hat{\mathbf{s}} + \cos\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} = \hat{\mathbf{z}} \end{cases}$$

$$\begin{cases} s = \sqrt{x^2+y^2} \\ \phi = \tan^{-1}(y/x) \\ z = z \end{cases} \qquad \begin{cases} \hat{\mathbf{s}} = \cos\phi\,\hat{\mathbf{x}} + \sin\phi\,\hat{\mathbf{y}} \\ \hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}} \\ \hat{\mathbf{z}} = \hat{\mathbf{z}} \end{cases}$$