TFY 4240

SOLUTION EXAM DEC 2012

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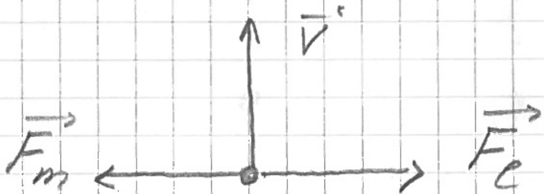
Problem 1

a) The wire will set up both an electric and magnetic field. Since the charge particle is moving both electric and magnetic forces will act on it



$$\vec{F}_e = q\vec{E} \tag{1.1a}$$

$$\vec{F}_m = q\vec{v} \times \vec{B} \tag{1.1b}$$

We note at a certain velocity there will be balance between $\vec{F}_m$ and $\vec{F}_e$ since they have oposite directions.

b) The charge density of the wire is

$$\rho(\vec{r}) = \lambda\, \delta(x)\delta(y) \tag{1.2a}$$

since it is located on the z-axis.
The current density is:

$$\vec{J}(\vec{r}) = I\,\delta(x)\delta(y)\,\hat{x}_3 \qquad (1.2b)$$

We will now calculate the electric field.
Consider a volum V consisting of a cylinder of length L and radius r with the axis along the wire. Gauss law applied to V gives:

$$\nabla\cdot\vec{D} = \rho(\vec{r})$$
$$\nabla\cdot\vec{E} = \frac{\rho(\vec{r})}{\varepsilon_0}$$

Integrating over V,

$$\int_V d^3r\,\nabla\cdot\vec{E} = \int d^3r\,\frac{\rho(\vec{r})}{\varepsilon_0}$$
$$\int_{\partial V} d\vec{S}\cdot\vec{E} = \frac{\lambda}{\varepsilon_0}\int dz = \frac{\lambda L}{\varepsilon}$$

Here we have used the divergense theorem. Due to symmetry the electric field most be radial, i.e.

$$\vec{E}(\vec{r}) = E(r)\,\hat{r}$$

Therefor

$$E(r)\cdot 2\pi r\cdot L = \frac{\lambda L}{\varepsilon_0}$$

so

$$\vec{E}(\vec{r}) = \frac{\lambda}{2\pi\varepsilon_0 r}\,\hat{r} \tag{1.3}$$

The magnetic field follows from Amperes law applied to a circular disc of radius r and oriented perpendicular to the wire

$$\nabla\times\vec{H} = \vec{J}$$

$$\int_A d\vec{S}\;\nabla\times\vec{H} = \int_A d\vec{S}\; I\,\delta(x)\delta(y)\,\hat{x}_3$$

$$\int_{\partial A} d\vec{\ell}\cdot\vec{H}(\vec{r}) = I$$

$$H(r) = \frac{I}{2\pi r}$$

Due to symmetry $\vec{H}(\vec{r}) = H(r)\hat{\theta}$ so

$$\vec{H}(\vec{r}) = \frac{I}{2\pi r}\,\hat{\theta} \tag{1.4}$$

Since we have axial symmetry we have used cylindrical coordinates (r, θ, z).

c) The forces acting on the particle when $r=d$ and $\vec{v} = v\hat{z}$ are

$$\vec{F}_e(d) = q\vec{E}(d)$$

$$= \frac{q\lambda}{2\pi\varepsilon_0 d}\,\hat{r}$$

$$\vec{F}_m(\vec{v}, d) = q\vec{v} \times \vec{B}(d)$$

$$= \mu_0 q v \frac{I}{2\pi d} \underbrace{\hat{z} \times \hat{\theta}}_{-\hat{r}}$$

so that the total force becomes

$$\left[\frac{q\lambda}{2\pi\varepsilon_0 d} - \mu_0 q v_c \frac{I}{2\pi d}\right]\hat{r} = 0$$

The critical velocity is when there is no net force acting on the particle, i.e.

$$\underline{\underline{v_c = \frac{\lambda}{\varepsilon_0\mu_0 I} = c^2\frac{\lambda}{I}}}$$

Problem 2

a) From Gauss law it follows:

$$\begin{aligned}
\rho(\vec{r}) &= \nabla\cdot\vec{D}(\vec{r}) \\
&= \varepsilon_0 \nabla\cdot\vec{E}(\vec{r}) \\
&= \varepsilon_0 A\ \nabla\cdot\left(\frac{e^{-br}}{r^2}\hat{r}\right) \\
&= \varepsilon_0 A\left[\nabla(e^{-br})\cdot\frac{\hat{r}}{r^2} + e^{-br}\nabla\left(\frac{\hat{r}}{r^2}\right)\right] \\
&= \varepsilon_0 A\left[-b e^{-br}\,\hat{r}\cdot\frac{\hat{r}}{r^2} + e^{-br}4\pi\delta(\vec{r})\right] \\
&= -\frac{\varepsilon_0 A b}{r^2}e^{-br} + 4\pi\varepsilon_0 A\,\delta(\vec{r})
\end{aligned} \tag{2.1}$$

In obtaing this result we have used that the electric field around a point charge q placed at $r=0$, is

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

For a point charge at the origin the charge density reads

$$\rho(\vec{r}) = q\,\delta(\vec{r})$$

so that from Gauss law we get

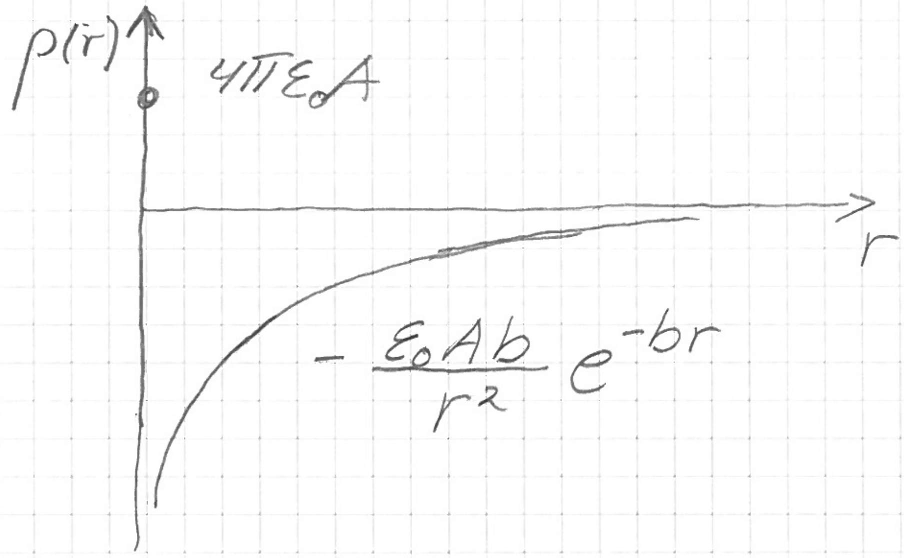
$$\nabla\cdot\vec{E} = \frac{\rho(\vec{r})}{\varepsilon_0}$$

$$\nabla\cdot\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}\right) = \frac{q}{\varepsilon_0}\delta(\vec{r})$$

or

$$\nabla \left(\frac{\hat{r}}{r^2} \right) = 4\pi \delta(\vec{r}) \qquad (2.2)$$

From (2.1) it follows that the charge distribution consists of a positive charge $4\pi\varepsilon_0 A$ at the origin and a spherically symmetric negative charge distribution in the surrounding space.



b) The total charge can for instance be obtained by integrating the charge distribution over all of space

$$Q = \int d^3r \, \rho(\vec{r})$$

$$= -\varepsilon_0 A b \int d^3r \, \frac{e^{-br}}{r^2} + 4\pi\varepsilon_0 A$$

$$= -\varepsilon_0 A b \int \underbrace{dr\, d\Omega\, r^2}_{d^3r} \frac{e^{-br}}{r^2} + 4\pi\varepsilon_0 A$$

$$= -4\pi\varepsilon_0 A b \int_0^\infty dr\, e^{-br} + 4\pi\varepsilon_0 A$$

$$= -4\pi\varepsilon_0 A b \left[-\frac{1}{b} e^{-br}\right]_0^\infty + 4\pi\varepsilon_0 A$$

$$= -4\pi\varepsilon_0 A b \left[+\frac{1}{b}\right] + 4\pi\varepsilon_0 A$$

$$= \underline{\underline{0}} \tag{2.3}$$

Hence the total charge is zero!

Alternatively, the same result can be obtained from Gauss law applied to a Gauss spherical surface of infinite radius $(r \to \infty)$

$$Q = \lim_{r\to\infty} \int_V d^3r\, \rho(\vec{r}) = \lim_{r\to\infty} \int_V d^3r\, \varepsilon_0 \nabla \cdot \vec{E}(\vec{r})$$

$$= \lim_{r\to\infty} \int_{\partial V} d\vec{S}\, \varepsilon_0 \vec{E}(\vec{r})$$

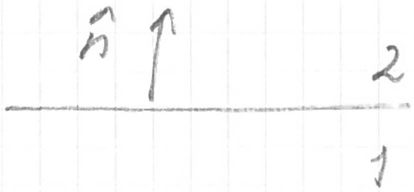
$$= \lim_{r\to\infty} 4\pi r^2\, \varepsilon_0 A \frac{e^{-br}}{r^2}$$

$$= \underline{\underline{0}}$$

Problem 3

a) If there are no sources at the surface the boundary conditions are

$$\hat{n} \times (\vec{E}_2 - \vec{E}_1) = 0 \qquad (3.1a)$$
$$\hat{n} \times (\vec{H}_2 - \vec{H}_1) = 0 \qquad (3.1b)$$
$$\hat{n} \cdot (\vec{D}_2 - \vec{D}_1) = 0 \qquad (3.1c)$$
$$\hat{n} \cdot (\vec{B}_2 - \vec{B}_1) = 0 \qquad (3.1d)$$



Here $\vec{E}_1$ is the electric field in medium 1 etc.

b) The magnetic field in the region $x_3 > 0$ consists of an incident and a scattered wave. Since the incident field is p polarized and the plane of incidence is the $x_1 x_3$ plane, it follows that $\vec{H}(\vec{x},t)$ is directed along $\hat{x}_2$. If the amplitude of the incident wave is H_0 it follows that the incident magnetic field is

$$\vec{H}(\vec{x},t) = H_0 \hat{x}_2 \, e^{i\vec{k}\cdot\vec{x}} \, e^{-i\omega t} \qquad (3.2)$$

where $\vec{k}$ is the wave vector. If we write

$$\vec{k} = \vec{k}_{\parallel} - \alpha_1(k_{\parallel})\,\hat{x}_3$$

where $\vec{k}_{\parallel} = k_{\parallel}\hat{x}_1$ is the projection of $\vec{k}$ onto the x_1x_2 plane and $-\alpha_1(k_{\parallel})$ is the 3rd component of $\vec{k}$. We have here explicitly used that $\vec{k}$ is pointing towards the surface.
To determine $\alpha_1(k_{\parallel})$ we use the dispersion relation

$$k^2 = \varepsilon_1\mu_1\frac{\omega^2}{c^2}$$

or

$$k_{\parallel}^2 + \alpha_1^2(k_{\parallel}) = \varepsilon_1\mu_1\frac{\omega^2}{c^2}$$

$$\Rightarrow \quad \alpha_1(k_{\parallel}) = \sqrt{\varepsilon_1\mu_1\frac{\omega^2}{c^2} - k_{\parallel}^2} \qquad (3.3)$$

The dispersion relation has to be satisfied in order for the plane wave (3.2) to be a solution of the Maxwells eq. (Helmholtz or wave eq).

Hence, we conclude that the incident magnetic field of the p polarized incident plane EM wave reads

$$H_0\,\hat{x}_2\,\exp(i\vec{k}_{\parallel}\cdot\vec{x}_{\parallel} - i\alpha_1(k_{\parallel})x_3) \qquad (3.4)$$

This incident magnetic field is scattered by the surface into a magnetic field characterized by the wave vector

$$\vec{q} = \vec{q}_{\parallel} + \alpha_1(q_{\parallel})\hat{x}_3 \qquad (3.5)$$

where $\vec{q}_{\parallel} = q_{\parallel}\hat{x}_1$ and $\alpha_1(q_{\parallel})$ given by Eq. (3.3). In Eq. (3.5) the coefficient in the front of $\hat{x}_3$ is positive since the scattered field is propagating away from the surface $x_3=0$. The amplitude of the scattered magnetic field relative the incident magnetic field is $r(\vec{q}_{\parallel}|\vec{k}_{\parallel})$ for incident and scattered lateral wave vectors $\vec{k}_{\parallel}$ and $\vec{q}_{\parallel}$, respectively.

In total we get the following form of the magnetic field above the surface

$$H^{>}(\vec{x}|\omega) = H_0\hat{x}_2\Big[e^{i\vec{k}_{\parallel}\cdot\vec{x}_{\parallel} - i\alpha_1(k_{\parallel})x_3} + r(\vec{q}_{\parallel}|\vec{k}_{\parallel})\,e^{i\vec{q}_{\parallel}\cdot\vec{x}_{\parallel} + i\alpha_1(q_{\parallel})x_3}\Big] \qquad (3.6)$$

where

$$\vec{k}_{\parallel} = k_{\parallel}\hat{x}_1 \qquad \vec{q}_{\parallel} = q_{\parallel}\hat{x}_1 .$$

c] The function $\alpha_2(p_{\|})$ is determined in the same way as $\alpha_1(k_{\|})$. The only difference is that we now have to use ε_2 and μ_2, i.e.

$$\alpha_2(p_{\|}) = \sqrt{\varepsilon_2 \mu_2 \frac{\omega^2}{c^2} - p_{\|}^2}$$

If ε_2 and/or μ_2 are complex, the Riemann sheet of the sqrt-functic has to be choosen so that $\operatorname{Im}\alpha_2 > 0$ since otherwise the plane wave will "explode" when $x_3 \to -\infty$.

The corresponding electric fields follow from Amperes law without sources ($\vec{J}=0$)

$$\nabla \times \vec{H}(\vec{x},t) = \vec{J} + \partial_t \vec{D}(x,t)$$

$$-i\omega \vec{D}(\vec{x}|\omega) = \nabla \times \vec{H}(\vec{x}|\omega)$$

By using the constitutive relation one gets

$$\vec{E}(\vec{x}|\omega) = \frac{1}{-i\omega\varepsilon_0\varepsilon_m} \nabla \times \vec{H}(\vec{x}|\omega) \qquad (3.7)$$

Assuming the generic form

$$\vec{H}(\vec{x}|\omega) = \hat{x}_2 H(\vec{x}|\omega)$$

with

$$H(\vec{x}|\omega) = H_0\, e^{ik_1x_1 \pm i\alpha_m(k_1)x_3},$$

Eq. (3.7) gives

$$\vec{E}(\vec{x}|\omega) = \frac{1}{-i\omega\varepsilon_0\varepsilon_m}\begin{vmatrix} \hat{x}_1 & \hat{x}_2 & \hat{x}_3 \\ \partial_1 & \partial_2 & \partial_3 \\ 0 & H(\vec{x}|\omega) & 0 \end{vmatrix}$$

$$= \frac{-1}{i\omega\varepsilon_0\varepsilon_m}\left\{\hat{x}_1\left[-\partial_3 H(\vec{x}|\omega)\right] + \hat{x}_3\left[\partial_1 H(\vec{x}|\omega)\right]\right\}$$

$$= \frac{-1}{i\omega\varepsilon_0\varepsilon_m}\left\{\hat{x}_1\left[-(\pm i\alpha_m(k_1)\right] + \hat{x}_3\left[ik_1\right]\right\}H(\vec{x}|\omega)$$

$$= \underline{\frac{1}{\omega\varepsilon_0\varepsilon_m}\left\{\pm\alpha_m(k_1)\,\hat{x}_1 - k_1\,\hat{x}_3\right\}H(\vec{x}|\omega)}$$

Alternatively you may also show that

$$\vec{E}(\vec{x}|\omega) = \frac{1}{-i\omega\varepsilon_0\varepsilon_m}\, i\vec{k}\times\vec{H}(\vec{x}|\omega)$$

$$= -\frac{\omega\sqrt{\varepsilon_0\varepsilon_m\mu_0\mu_m}}{\omega\varepsilon_0\varepsilon_m}\,\hat{k}\times\vec{H}(\vec{x}|\omega)$$

$$= -\sqrt{\frac{\mu_0}{\varepsilon_0}\frac{\mu_m}{\varepsilon_m}}\;\hat{k}\times\vec{H}(\vec{x}|\omega)$$

$$= -\;Z_m\;\hat{k}\times\vec{H}(\vec{x}|\omega)$$

where we in the last line have introduced the impedance Z_m.

Thus the electric fields are

$$\vec{E}^{>}(\vec{x}|\omega)$$

$$= \frac{1}{\omega\varepsilon_0\varepsilon_1}\left\{-\alpha_1(k_\parallel)\hat{x}_1 - k_\parallel\hat{x}_3\right\} \times H_0 e^{i\vec{k}_\parallel\cdot\vec{x}_\parallel - i\alpha_1(k_\parallel)x_3}$$

$$+ \frac{r(\vec{q}_\parallel|\vec{k}_\parallel)}{\omega\varepsilon_0\varepsilon_1}\left\{+\alpha_1(q_\parallel)\hat{x}_1 - q_\parallel\hat{x}_3\right\} \times H_0 e^{i\vec{q}_\parallel\cdot\vec{x}_\parallel + i\alpha_1(q_\parallel)x_3} \qquad (3.8a)$$

$$\vec{E}^{<}(\vec{x}|\omega) =$$

$$= \frac{t(\vec{p}_\parallel|\vec{k}_\parallel)}{\omega\varepsilon_0\varepsilon_2}\left\{-\alpha_2(p_\parallel)\hat{x}_1 - p_\parallel\hat{x}_3\right\} H_0 e^{i\vec{p}_\parallel\cdot\vec{x}_\parallel - i\alpha_2(p_\parallel)x_3} \qquad (3.8b)$$

d) We start by satisfying BC (3.1b)

$$\hat{x}_3 \times \left[\vec{H}^{>}(\vec{x}|\omega) - \vec{H}^{<}(\vec{x}|\omega)\right]\Big|_{x_3=0} = 0$$

$$e^{i\vec{k}_\parallel\cdot\vec{x}_\parallel} + r(\vec{q}_\parallel|\vec{k}_\parallel)e^{i\vec{q}_\parallel\cdot\vec{x}_\parallel} = t(\vec{p}_\parallel|\vec{k}_\parallel)e^{i\vec{p}_\parallel\cdot\vec{x}_\parallel}$$

Since this eq. should hold for any $\vec{x}_\parallel$ it follows that one must have

$$\vec{k}_\parallel = \vec{q}_\parallel = \vec{p}_\parallel \qquad (3.9)$$

Hence it follows that:

$$1 + r_p(k_{\parallel}) = t_p(k_{\parallel}) \tag{3.10}$$

where we have used $r_p(\vec{k}_{\parallel}) = r(\vec{k}_{\parallel}|\vec{k}_{\parallel})$ and the same for t_p.

The BC (3.1a) gives from Eqs. (3.8) and (3.9)

$$-\frac{\alpha_1(k_{\parallel})}{\varepsilon_1} + r_p(k_{\parallel})\frac{\alpha_1(k_{\parallel})}{\varepsilon_1} = -t_p(k_{\parallel})\frac{\alpha_2(k_{\parallel})}{\varepsilon_2(\omega)} \tag{3.11}$$

Hence the system of eq. for r_p and t_p becomes from Eqs. (3.10) and (3.11)

$$\begin{aligned} -r_p \quad &+ \quad t_p &= 1 \\ +\frac{\alpha_1}{\varepsilon_1} r_p \quad &+ \quad \frac{\alpha_2}{\varepsilon_2} t_p &= +\frac{\alpha_1}{\varepsilon_1} \end{aligned} \tag{3.12}$$

Equation (3.12) has solution

$$r_p(k_{\parallel}) = \frac{\dfrac{\alpha_1(k_{\parallel})}{\varepsilon_1} - \dfrac{\alpha_2(k_{\parallel})}{\varepsilon_2(\omega)}}{\dfrac{\alpha_1(k_{\parallel})}{\varepsilon_1} + \dfrac{\alpha_2(k_{\parallel})}{\varepsilon_2(\omega)}} \tag{3.13a}$$

$$t_p(k_\parallel) = \frac{2\,\frac{\alpha_1(k_\parallel)}{\varepsilon_1}}{\frac{\alpha_1(k_\parallel)}{\varepsilon_1} + \frac{\alpha_2(k_\parallel)}{\varepsilon_2}} \tag{3.13b}$$

e) The intensity of a EM wave being incident, scatterered of or transmitted through a surface is defined by

$$\underline{I} = |\langle \vec{S} \rangle \cdot \hat{x}_3|$$

where

$$\langle \vec{S} \rangle = \frac{1}{2} \vec{E} \times \vec{H}^*$$

is the time-averaged Poynting's vector. The SI-unit of this quantity is

$$[I] = \underline{\frac{W}{m^2}}.$$

For a plane wave propagating in direction $\hat{k}$ one finds

$$\begin{aligned}
\langle \vec{S} \rangle &= \frac{1}{2} \vec{E} \times \vec{H}^* \\
&= \frac{1}{2} (-Z_m\, \hat{k} \times \vec{H}) \times \vec{H}^*, \quad Z_m = \sqrt{\frac{\mu_0 \mu_m}{\varepsilon_0 \varepsilon_m}} \\
&= \frac{Z_m}{2} \vec{H}^* \times (\hat{k} \times \vec{H}), \\
&= \frac{Z_m}{2} \left[\hat{k}\, |\vec{H}|^2 - \vec{H} (\hat{k} \cdot \vec{H}) \right]
\end{aligned}$$

$$= \frac{Z_m}{2} \hat{k} |\vec{H}|^2$$

The intensity therefore becomes

$$I = |\langle \vec{S} \rangle \cdot \hat{x}_3|$$

$$= \frac{Z_m}{2} \frac{\alpha_m(k_{\parallel})}{k} |\vec{H}|^2$$

$$= \frac{1}{2} \sqrt{\frac{\mu_0 \mu_m}{\varepsilon_0 \varepsilon_m}} \frac{\alpha_m(k_{\parallel})}{\omega \sqrt{\varepsilon_0 \varepsilon_m \mu_0 \mu_m}} |\vec{H}|^2$$

$$= \frac{1}{2} \frac{\alpha_m(k_{\parallel})}{\omega \; \varepsilon_0 \varepsilon_m} |\vec{H}|^2$$

This result can also be obtained directly from Eq (3.8).

In this way we obtain the intenseties

$$I_r = \frac{1}{2} \frac{\alpha_1(k_{\parallel})}{\omega \varepsilon_0 \varepsilon_1} |r_p(k_{\parallel})|^2 |H_0|^2 \qquad (3.14a)$$

$$I_t = \frac{1}{2} \frac{\alpha_2(k_{\parallel})}{\omega \varepsilon_0 \varepsilon_2} |t_p(k_{\parallel})|^2 |H_0|^2 \qquad (3.14b)$$

For later convenience we also give the incident intensity

$$I_0 = \frac{1}{2} \frac{\alpha_1(k_{\parallel})}{\omega \varepsilon_0 \varepsilon_1} |H_0|^2 \qquad (3.14c)$$

f)
$$R = \frac{I_r}{I_o} = |r_p(k_{\parallel})|^2 \qquad (3.15a)$$

$$T = \frac{\frac{\kappa_2(k_{\parallel})}{\varepsilon_2}}{\frac{\kappa_1(k_{\parallel})}{\varepsilon_1}} |t_p(k_{\parallel})|^2 \qquad (3.15b)$$

Under the assumption that there is no absorption we should have $I_o = I_r + I_t$ or $R + T = 1$.

$$|r_p|^2 + \frac{\kappa_2/\varepsilon_2}{\kappa_1/\varepsilon_1} |t_p|^2$$

$$= \frac{\left(\frac{\kappa_1}{\varepsilon_1} - \frac{\kappa_2}{\varepsilon_2}\right)^2 + \frac{\kappa_2/\varepsilon_2}{\kappa_1/\varepsilon_1}\left(2\frac{\kappa_1}{\varepsilon_1}\right)^2}{\left(\frac{\kappa_1}{\varepsilon_1} + \frac{\kappa_2}{\varepsilon_2}\right)^2}$$

$$= \frac{\left(\frac{\kappa_1}{\varepsilon_1}\right)^2 + \left(\frac{\kappa_2}{\varepsilon_2}\right)^2 - 2\frac{\kappa_1}{\varepsilon_1}\frac{\kappa_2}{\varepsilon_2} + 4\frac{\kappa_1}{\varepsilon_1}\frac{\kappa_2}{\varepsilon_2}}{(\qquad)^2}$$

$$= \underline{\underline{1}}$$

g) Now $d \to 0$ and the film is therefore present. We note that the form of $\varepsilon(x_3, \omega)$ is not known.

However, the clue to the solution is to note that the dielectric function of the film only depends spatially on x_3. Hence we can slice the film medium into a stack of very thin slices where each interface is parallel with the x_1x_2-plane.

Inside each slice the dielectric function can be considered independent of space. Since the latteral wave vector is conserved over every interface, so it is for medium 1 and 2, i.e. also with the film present one has

$$k_{\parallel} = p_{\parallel}$$

or equivalently Snells law:

$$\sqrt{\varepsilon_1 \mu_1} \sin\theta_0 = \sqrt{\varepsilon_2 \mu_2} \sin\theta_t$$

Hence, the precence of the film will not affect the direction of the transmitted wave.

b) To calculate the intensity of the reflected/transmitted wave is much more challenging than the direction when the film is present. The reason for this is that now multiple scattering may take place within each of the thin layers described in the previous subproblem.

To formally obtain the intenseties, one has to write the H-field inside each layer in the form

$$\vec{H}(\vec{x}|\omega) = \hat{x}_2 \, A_-(\vec{q}_{\parallel}|\vec{k}_{\parallel}) \exp(i\vec{q}_{\parallel}\vec{x}_{\parallel} - i\alpha_m(\vec{q}_{\parallel})x_3)$$
$$+ \hat{x}_2 \, A_+(\vec{q}_{\parallel}|\vec{k}_{\parallel}) \exp(i\vec{q}_{\parallel}\vec{x}_{\parallel} + i\alpha_m(q_{\parallel})x_3)$$

where

$$\alpha_m(q_{\parallel}) = \sqrt{\varepsilon_m \frac{\omega^2}{c^2} - q_{\parallel}^2}$$

is the 3rd component of the wave vector of layer m where the dielectric function is $\varepsilon_m(\omega)$ as obtained from $\varepsilon(x_3, \omega)$ for the value of x_3 for that layer.

Now the BC for each interface imposed and the amplitudes $A_{\pm}(\vec{q}_{\parallel}|\vec{k}_{\parallel})$ for each layer are elliminated.

This will leave us with an expression for $R(\vec{q}_{\parallel}|\vec{k}_{\parallel})$ and $T(\vec{p}_{\parallel}|\vec{k}_{\parallel})$.

As a side comment one may mention that this process is simplified by the so-call transfer matrix formalis. This formalism gives the total effect of a stack of layers given the ε_m, μ_m and the thickness d_m of each layer.