Studentnumber:______________________

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NORGES TEKNISK-
NATURVITENSKAPELIGE UNIVERSITET
INSTITUTT FOR FYSIKK

Contact during the exam:
Department of Physics
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EXAM: TFY4245 FASTSTOFF-FYSIKK VK

Friday 2. june 2006
Tid: kl 09.00-13.00

Allowed exam material: Alternative C
Standard pocket calculator
Rottman: Mathematical Formula (all language editions)
Barnett og Cronin: Mathematical Formula

The exam consists of:

1. The first page (the present page) which must be delivered with answers to the multiple choice questions.
2. 3 "normal" Problems 1, 2 and 3 (Appendix A)
3. One set of multiple choice questions, Problem 4 (Appendix B)

The three "normal" problems count altogether 50%, and the multiple choice questions count altogether 50%. Only ONE of the alternatives A-D must be marked for each of the 20 multiple choice questions. Correct answer gives one point, wrong answer gives zero points.

Answers to the multiple choice questions in Appendix B:

Question	1	2	3	4	5	6	7	8	9	10	11	12
Answer												

Question	13	14	15	16	17	18	19	20
Answer								

Problem 1. Phase transitions:

a) Discuss the terms "first order" and "second order" phase transitions.
b) What is meaning of an order parameter in the context of phase transitions? Give examples of order parameters for real systems.

Landau theory is a thermodynamic theory describing phase transitions, and this theory considers the Helmholtz free energy $f(\eta,T)$, where η is the order parameter, and T is the temperature: Landau theory assumes that near the phase transition temperature T_C, it is possible to expand the free energy in powers of η, thus

$$f(\eta,T) = f_0 + c_2\eta^2 + c_3\eta^3 + c_4\eta^4 + c_5\eta^5 + c_6\eta^6 + ...+ c_n\eta^n + ...$$

Assume that $c_2(T) = b(T - T_C)$, where b is a positive constant, and also assume that $c_3, c_4, c_5, c_6, \ldots\ldots, c_n, \ldots\ldots\ldots$ are constants independent of T.
Consider the following cases:

c) $c_3 = 0$, c_4 is positive, and $c_n = 0$ for $n > 4$. Does this case describe a first order, or a second order phase transition? First sketch and discuss $\eta(T)$, and sketch and discuss $f(\eta,T)$ for various T. Then derive the temperature dependence of the order parameter for this case. Do you know of any real systems that belong to this case? What difference does it make if c_4 is negative?
d) c_3 is negative, c_4 is positive, and $c_n = 0$ for $n > 4$. Does this case describe a first order, or a second order phase transition? First sketch and discuss $\eta(T)$, and sketch and discuss $f(\eta,T)$ for various T. Then derive the temperature dependence of the order parameter for this case. Do you know of any real systems that belong to this case?
e) Assume that there exists an external field, X, which couples linearly to the order parameter, η, i.e. an extra term $-\eta X$ must be added in the free energy expression (i.e. use Gibbs free energy rather than Helmholtz free energy given above). Derive an expression the susceptibility for case c).
f) Discuss the validity of the Landau theory for phase transitions.

Problem 2. Fields, response functions and relaxation processes:

Consider a material for which one of the molecules may be in one of two states separated by an energy barrier, E. At finite temperatures, the molecule in question changes back and forth between the two situations (states) with a "jump" frequency between states given by a so-called Arrhenius expression $f = 1/\tau = \tau_0^{-1}\exp(-E/k_BT)$, where τ_0 is a local time characteristic for oscillations <u>within</u> one of the states, and k_B is Boltzmann's constant. Assume further that by application of an external field, σ, a local preference for one of the states is introduced, thus resulting in a redistribution of the population of the two states, and that this may be described by a simple Debye type relaxation process.

a) Use common sense to write down a simple linear differential equation describing the simple relaxation process.
A creep experiment means application of constant external field σ_0 at and for all times after some time t_0, and $\sigma_0 = 0$ for $t < t_0$.

We define the response field as $\gamma = \alpha(\omega)\ \sigma$, where $\alpha(\omega)$ is the frequency (ω) - dependent response function.

Show that the response field, γ, for the case when a "creep experiment" is performed on a sample of the material, may be written as

$$\gamma(t > t_0) = \alpha(0)\ \sigma_0\ (1\text{-}\exp(-t/\tau))$$

Sketch and discuss the result as a function of time for various temperatures.

b) Perform a dynamic experiment as a function of external applied frequency (assuming constant force amplitude) on the same material. Derive the expression for the frequency dependent complex compliance response function. Sketch and discuss the result as a function of frequency and temperature.

The real and imaginary parts of the compliance are often referred to as the storage part and the loss part respectively. Discuss the physics behind this distinction.

Problem 3. Microscopic dynamics:

a) Discuss the physics contained in the Einstein relation for the "self-diffusion coefficient"

$$D_S = k_B T \nu$$

,where k_B is Boltzmanns constant, T is the temperature, and ν is the mobility. In the discussion, consider that it can be shown that $6D_s = <|\mathbf{r}(t)|^2>/t$ which is the mean squared displacement achieved by a diffusing particle per unit time, t.

Consider a suspension of non-interacting colloidal particles, and assume the following:

- a particle density gradient $\nabla\rho$ resulting in Fick's law for the diffusive particle current $\mathbf{j}_D = -D_s\nabla\rho$.
- an external force (for example gravity) $\mathbf{f} = \nabla u_{pot}$, where u_{pot} is the potential energy difference set by the external force that drives a particle current $\mathbf{j}_f = \rho\nu\mathbf{f}$.
- at equilibrium, the particle density $\rho = \rho_{eq} = \exp(-u_{pot}/k_B T)$ according to Boltzmann statistics

Using these assumptions, show how the Einstein relation can be derived.

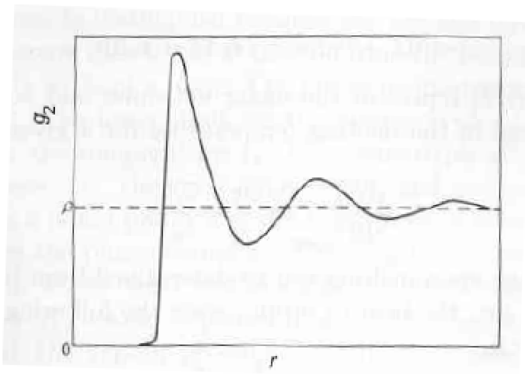
Discuss the difference between the "self-diffusion coefficient", D_S, and the "diffusion coefficient", D.

b) The probability of finding a diffusing particle at distance $\mathbf{r}(t)$ at time t, assuming $\mathbf{r}(t=0) = 0$, is given by the time-dependent auto-correlation function $g_1(\mathbf{r}(t),t) = (1/(4\pi D_s t))^{3/2}\exp(-|\mathbf{r}(t)|^2/(4D_s t))$ which follows from a Greens function solution of the diffusion equation for $g_1(\mathbf{r}(t),t)$.

For such non-interacting particles the time dependent structure function $S(\mathbf{q},t)$ equals the Fourier transform of $g_1(\mathbf{r}(t),t)$, where the scattering vector $\mathbf{q}$ is the difference between the outgoing and incoming wave-vectors respectively.

Discuss (in terms of words and equations) a light-scattering experiment which may be used to measure D_S for non-interacting colloidal particles in solution.

Problem 4. Multiple choice questions:



1. The figure above shows a typical pair-distribution function $g_2(r)$ for:
- a) An isotropic atomic amorphous solid?
- b) A crystalline solid?
- c) A liquid in an electric field?
- d) None of the above?

2. Smectic order is a term used for:
- a) Positional order, but no orientational order
- b) Isotropic order
- c) Orientational order, but no positional order
- d) None of the above

3. The order parameter for nematic order of quadrupoles is (< > denotes an average using the appropriate orientational distribution function, and $\theta = 0$ is the director orientation):
- a) $<(3\cos^2\theta-1)/2>$?
- b) $<(3\sin^2\theta-1)/2>$?
- c) $<\cos\theta>$?
- d) None of the above?

4. The order parameter for nematic order of dipoles is (< > denotes an average using the appropriate orientational distribution function, and $\theta = 0$ is the director orientation):
- a) $<(3\cos^2\theta-1)/2>$?
- b) $<(3\sin^2\theta-1)/2>$?
- c) $<\cos\theta>$?
- d) None of the above?

5. A self-avoiding random walk in 3 dimensions will create an open object with a fractal dimension D of:
- a) $2<D<3$
- b) $D = 2$
- c) $D = 1$
- d) None of the above?

6. The double differential cross section measured in dynamic scattering experiments is proportional to the dynamic structure factor S(**q**,ω), which is the Fourier transform of a function that describes

a) Probabilities for single particle movements within the time t?
b) Space-time correlations between different particles?
c) Both a) and b)?
d) None of the above?

7. The finite width of experimentally measured Bragg scattering peaks from a periodic structure may be due to:

a) Finite size, i.e. finite number of scattering planes?
b) Disorder in the periodicity?
c) Finite instrument resolution?
d) All of the above?

8. A pure elastic solid is described by:

a) The Newtonian flow law: stress = viscosity*shear-rate
b) Hooke's law: stress = modulus*strain
c) Time-delayed relaxational behavior
d) None of the above?

9. In general, the imaginary part of the frequency dependent dielectric susceptibility describes:

a) The relaxation of elastic dipoles in the sample?
b) Dipolar interactions?
c) The power provided to the sample by the applied electric field?
d) None of the above?

10. Piezoelectricity in a material means that:

a) There is a quadratic relationship between elastic and electric fields in a material?
b) There is a linear relationship between the elastic and electric fields in a material?
c) There is no coupling between elastic and electric fields in a material?
d) None of the above?

11. Diamagnetism:

a) Diamagnetism is the term used for magnetism in materials with permanent magnetic dipoles
b) The atomic diamagnetic susceptibility is independent of the number of electrons in the atom
c) Diamagnetism is present in only a few materials
d) None of the above?

12. Paramagnetism:

a) Paramagnetism is present in all materials
b) The paramagnetic susceptibility for a sample with localized magnetic moments is proportional to the temperature
c) The paramagnetic susceptibility is in general very small (compared to the diamagnetic susceptibility for example)
d) None of the above?

13. So-called "adiabatic demagnetization" means that the temperature of a sample with paramagnetic ions can be lowered by:

a) application of a static magnetic field which is much larger than the saturation magnetization of the sample?
b) application of a static magnetic field opposite to the direction of the magnetization?
c) application af magnetic field at high temperature, followed by thermal insulation of the sample in zero external field?
d) None of the above?

14. The Kramers-Kronig (K-K) formulas give relations between the real and imaginary parts of generalized linear response functions, i.e. between the storage and the loss moduli respectively. The physics behind the K-K relations is:

a) That the system obeys simple Debye relaxation behavior.
b) The causality principle, i.e. no response until after the applied force?
c) The superposition principle, i.e the system behavior may be described as a linear combination of fundamental modes?
d) None of the above?

15. The superconducting Cooper pairs:

a) Are fermions?
b) Are formed by pairs of electrons bound together by exchanging photons?
c) Are not localized, but extend over typically 1000 lattice spacings?
d) None of the above?

16. The isotope effect in superconductors demonstrates that:

a) The superconducting transition temperature is large for materials with heavy atoms?
b) The superconducting transition temperature is small for materials with heavy atoms?
c) The superconducting transition temperature is proportional to changes in lattice frequencies caused by exchanging isotopes in a material?
d) None of the above?

17. Applied magnetic fields are displaced from inside type-I superconducting samples (Meissner effect). The exponential decay length (The London penetration depth) of an applied magnetic field into a type-I superconductor beneath the surface, also describes (assume no applied voltage):

a) the decay length for superconducting currents of Cooper pairs in the superconductor?
b) the length over which Cooper pairs are correlated?
c) the decay length for the density of Cooper pairs ?
d) None of the above?

18. The lower critical field H_{c1} in type-II superconductors describes:

a) the field for which there is complete penetration of the externally applied field in all the sample?
b) the field for which the first single flux-line is created?
c) the field for which the number of flux-lines equals the number of Cooper-pairs?
d) None of the above?

19. The upper critical field H_{c2} in type-II superconductors describes:

a) the field for which there is complete penetration of the externally applied field in all the sample?
b) the field for which the first single flux-line is created?
c) the field for which the number of flux-lines equals the number of Cooper-pairs?
d) None of the above?

20. Rayleigh - Brillouin experiments provide information about:

a) Lifetimes of sound waves?
b) Frequencies of sound waves?
c) Propagation velocities of heat waves?
d) None of the above?