

Norges teknisk-naturvitenskapelige universitet
 Institutt for fysikk

Candidate number: _____

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EXAM TFY4245 – FASTSTOFF FYSIKK VK

Monday 21. May 2012
 Time: 0900-1300

Allowed exam material - Alternative C:
 Standard pocket calculator
 K. Rottmann: Mathemaical Formulae (all language editions)
 S. Barrett and T.M. Cronin: Mathematical Formulae

The exam consists of 8 pages:

1. The first page (page 1) which must be delivered with answers to the multiple choice questions.
2. 3 "normal" Problems 1, 2 and 3 (pages 2-4)
3. One set of multiple choice questions, Problem 4 (pages 4-8)

The three "normal" problems count altogether 60%, and the multiple choice questions count altogether 40%. Only ONE of the alternatives a)-d) must be marked for each of the 16 multiple-choice questions. Correct answer gives one point, no answer gives 0 points, while wrong answer gives -0.25 point. Place your answers to the questions of problem 4 in the table below

Table – Answers to Problem 4 questions

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Maxwell equations:

$$\begin{aligned} \nabla \times \vec{H} &= \vec{j} + \frac{\partial \vec{D}}{\partial t} & \nabla \cdot \vec{B} &= 0 \\ \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} & \nabla \cdot \vec{D} &= \rho \end{aligned}$$

Problem 1.

In a generalized thermodynamic approach to order-disorder phase transitions, the free energy of the system, $G(\eta)$, may be expanded as a power series of the general ordering parameter η ,

$$G(\eta) = g_0 + g_1\eta + \frac{1}{2}g_2\eta^2 + \frac{1}{3}g_3\eta^3 + \frac{1}{4}g_4\eta^4 + \dots \quad (1)$$

which holds in the absence of external fields that may act on the ordering parameter.

In the power series, $g_0 = G_I(T_C)$, where $G_I(T_C)$ represents the free energy at the phase transition temperature T_C , calculated for the phase which is stable above T_C . The remaining terms are general functions collecting all expansion terms of the same order in η , with all g_1, \dots, g_n functions assumed to vary smoothly as functions of T .

a) Let $g_2(T) = \gamma(T - T_C)$, where $\gamma > 0$, and constant, and assume eqn. (1) applied to a second order phase transition with an ordered state at $T < T_C$, and a disordered state at $T > T_C$.

Why may we, in the given situation, and for temperatures close to T_C , justify a termination of the series in eqn. (1) after 4th order?

Find eventual restrictions that apply to g_1 , g_3 and g_4 of eqn. (1) for it to be consistent with the current situation. Provide explicit reasons for the restrictions you impose, and show that these lead to well defined free energy minima at both sides of T_C .

b) We introduce a static external field, $\vec{\theta}_{ext}$, which couples linearly to the system field response variable symbolised by the order parameter in eqn. (1). Furthermore, assume the sample to be small and the spatial variation of the field to be in the long-wavelength limit, leaving $\vec{\theta}_{ext} \sim \text{constant}$ throughout the sample volume.

How does the presence of an external field modify the free energy in eqn. (1) ?

Assuming the system to be in thermal equilibrium at any T , find expressions for a linear system susceptibility to the external field above and below T_C .

c) In some materials exhibiting second order phase transitions, e.g. in ferroelectrics, ferroelastics or ferromagnetics, a common response of the system to the external field is hysteresis, as illustrated in Fig. 1.

Explain why eqn. (1), in its current form, is inadequate to account for such behaviour, and describe briefly the changes occurring in the system throughout a hysteresis loop, by accounting for the system state at the characteristic points labelled 1-8 in the graph.

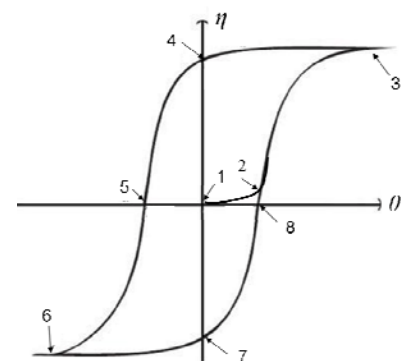


Figure 1. Hysteresis loop

Problem 2

a) Show that the London-equation, $\nabla \times \vec{j} + \frac{e^2 n}{m} \vec{B} = 0$, can be derived from a classical equation of motion of a single electron under the influence of a DC electric field inside a superconductor, under the assumption that the superconductor resistivity, $\rho = 0$.

b) Under the assumptions that the superconductor supports no magnetisation currents (Amperian currents) or time-dependent displacement currents, show that the London-equation can be transformed to

$$\nabla^2 \vec{B} = \frac{ne^2}{m\epsilon_0 c^2} \vec{B} = \frac{1}{\lambda_L^2} \vec{B} \quad \text{and} \quad \nabla^2 \vec{j} = \frac{1}{\lambda_L^2} \vec{j}$$

c) We assume the superconductor to be placed as illustrated in Fig. 2, and $B_{ext}(x < 0) = B_{ext} = \text{const}$ to be a uniform and static magnetic field surrounding the superconductor.

Use the London-equation to find a solution for $B(x > 0)$ inside the superconductor.

Comment on the result with respect to well-known characteristics of superconductivity.

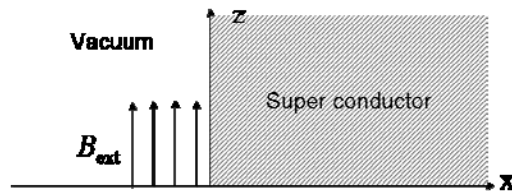


Figure 2

How do you interpret the result concerning the superconducting current density?

Problem 3

Consider a solid consisting of N identical atoms with angular momentum quantum numbers $J=S=1/2$. At low temperatures, the system stabilizes in an anti-ferromagnetic ordered structure.

The interaction between nearest neighbour magnetic moments is accounted for by a Heisenberg spin-exchange on the form

$$U = -2\mathfrak{J} \sum_{p=1}^N \vec{S}_p \cdot \vec{S}_{p+1}$$

where U is the internal magnetic energy of the system, \mathfrak{J} is a term that accounts for the spin exchange energy, \vec{S}_p and \vec{S}_{p+1} are the spins of nearest neighbour atoms p and $p+1$, respectively, and the sum is to be taken over all atoms of the system.

a) What restriction applies to \mathfrak{J} if the spins are to stabilise in an anti-ferromagnetic order?

Call the energy of the perfect anti-ferromagnetic ordering U_0 . Determine the energy, relative to U_0 , required to reverse one spin into a parallel alignment with its nearest neighbours.

Show that this energy is significantly reduced if the flip of a single spin can be distributed as slight misalignments between all the N spins of the system.

The final situation in a) provides the energetic motivation for the formation of spin waves. Deep into the anti-ferromagnetic phase, i.e. far from T_N , we may employ a relatively simple model to arrive at the following differential equations for the spin wave:

$$\begin{aligned}\frac{dS_{2p}^\dagger}{dt} &= \frac{2i\mathfrak{I}S}{\hbar} \left[2S_{2p}^\dagger + S_{2p-1}^\dagger + S_{2p+1}^\dagger \right] \\ \frac{dS_{2p+1}^\dagger}{dt} &= \frac{-2i\mathfrak{I}S}{\hbar} \left[2S_{2p+1}^\dagger + S_{2p}^\dagger + S_{2p+2}^\dagger \right]\end{aligned}\quad (2)$$

where $S_{2p}^\dagger = S_{2p}^x + iS_{2p}^y$ represents the x - and y coordinates of spin number $2p$. The $2p$ even numbered spins are assumed to be up, $S_{2p}^z = S$, while the $2p+1$ spins are down, $S_{2p+1}^z = -S$.

b) A possible solution to eqn. (2) are spin waves on the form $ue^{i(kpa-\omega t)}$, where u is an amplitude and a is the lattice spacing between nearest neighbour atoms. In general the amplitudes that apply to the spin-up and spin-down parts are different. Use such solutions for the nearest neighbour pairs contained in eqn. (2) to derive a dispersion relation for the spin wave.

c) A part of the curriculum covered the main mechanisms decisive for domain formation in ferromagnets. What would you expect with respect to domain formation in anti-ferromagnets? Justify your answer.

Problem 4. Multiple-choice questions.

1. In the long-wavelength limit, the response of a metal to an external electric field, $E = E_0 e^{-i\omega t}$, is characterised by so-called plasma optics, for which the dielectric function may be expressed

$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$. Of the following statements, which one gives the most accurate description of the plasma frequency, ω_p ?

- A. ω_p represents the low frequency cut-off for photon-phonon coupling.
- B. ω_p represents the collective transversal displacement of free electrons relative to a background of fixed positive ions.
- C. ω_p represents the collective longitudinal displacement of free electrons relative to a background of fixed positive ions.
- D. ω_p represents the collective transversal displacement of free electrons *and* positive ions.

2. For a metal operating at room temperature, which of the following dissipative mechanisms normally represents the most dominant loss to its electric conductivity?

- A. Interactions between the free electrons and lattice defects.

- B. Interactions between the free electrons and phonons.
- C. Interactions between the free electrons themselves.
- D. Interactions between the free electrons and the magnetic field induced by the current.

3. The so-called Lyddane-Sachs-Teller (LST) relation can be expressed as $\frac{\omega_L^2}{\omega_T^2} = \frac{\epsilon(0)}{\epsilon(\infty)}$.

Generally, it couples the longitudinal and transverse optical phonons with the static dielectric constant and the high-frequency dielectric response of ionic or dielectric crystalline solids.

Assume a dielectric exhibiting a second-order ferroelectric phase transition. Which of the following scenarios would best characterise the typical behaviour of the involved quantities as $T \rightarrow T_C$?

- A. $\omega_T \rightarrow 0$ and $\epsilon(0) \rightarrow \infty$, corresponding to a softening of the lattice restoring forces transverse to the wave vector of the local electric field.
- B. $\omega_L \rightarrow 0$ and $\epsilon(\infty) \rightarrow \infty$, corresponding to a softening of the lattice restoring forces along the wave vector of the local electric field.
- C. $\omega_L \rightarrow \infty$ and $\epsilon(\infty) \rightarrow 0$, corresponding to a dielectric breakdown of the paraelectric phase.
- D. The question cannot be answered, since critical fluctuations in polarisation, which is the ordering parameter in second-order ferroelectric phase transitions, invalidates the LST-relationship as T_C is approached.

4. Consider Raman scattering processes in a crystal. Of the following, which statement is false ?

- A. In general terms, the Raman effect refers to inelastic photon scattering by one or more phonons.
- B. In the second-order Raman effect, phonon creation and annihilation may be involved in the same scattering process.
- C. The Raman effect is made possible by the strain-dependence of the electronic polarisability.
- D. At high temperatures, the high density of phonons excited leaves the total intensity of first-order Raman scattering to be dominated mainly by phonon annihilation processes.

5. Of the following, which statement is generally valid concerning piezoelectricity?

- A. The direct and converse piezoelectric coefficients are second rank tensors.
- B. Due to their thermal expansion and contraction, pyroelectric crystals are also simultaneously piezoelectric.
- C. At constant temperature, the converse piezoelectric coefficients are reciprocally equivalent to the direct piezoelectric coefficients.
- D. At constant temperature, the converse piezoelectric coefficients and the direct piezoelectric coefficients are the same.

6. In a dielectric, electronic polarisation is caused by the local electric field acting at any given position corresponding to an atomic charge distribution inside the material. The local field may be expressed in terms of macroscopic and microscopic field contributions, i.e.

$$\vec{E}_{local}(\vec{r}) = \vec{E}_{macro}(\vec{r}) + \vec{E}_{micro}(\vec{r}) = \vec{E}_{external}(\vec{r}) + \vec{E}_1(\vec{r}) + \vec{E}_2(\vec{r}) + \vec{E}_3(\vec{r})$$

Generally, derivation of the local field from this equation is computationally intensive, but under particular circumstances the local field becomes identical to the external field, i.e.

$\vec{E}_{local}(\vec{r}) = \vec{E}_{external}(\vec{r})$. Under which circumstances does this apply?

- A. In spherically shaped single crystals, but only in special positions in the unit cell where the field contributions from nearby dipoles cancel by symmetry.
- B. In single crystals with cubic point group symmetry.
- C. Only with very strong external fields.
- D. It may never happen in any dielectric solid which is polarised in an external field.

7. What distinguishes a first-order from a second-order phase transition?

- A. Only first-order transitions can involve a change of the crystal symmetry.
- B. Only second-order transitions can involve a discontinuous change in heat capacity.
- C. Only first-order transitions can involve a discontinuous change in entropy.
- D. Only second-order transitions can involve fluctuations in the vicinity of the transition temperature.

8. The transition from a normal to a superconducting state can be described as an ordering of electrons into so-called Cooper-pairs. Cooper-pairs form the quantum-mechanical basis of the so-called BCS theory of superconductivity. Which, of the following, give the most proper description of ordering into the BCS ground state.

- A. Cooper-pairs are formed by any pair of free electrons with opposite spins.
- B. Cooper-pairs are formed by any pair of free electrons with opposite spins and momentums.
- C. Cooper-pairs are formed only by free electrons close to maximum kinetic energy, with opposite spins and momentums.
- D. Cooper-pairs are formed only by free electrons close to maximum kinetic energy and with opposite spins, but inelastic momentum exchange may occur between the electrons and the crystal lattice.

9. Pb-8.3%wtIn is a type II superconductor. At $T=4.2$ K, $H_{C2}=0.24$ T, while $H_{C1}=0.02$ T. If the penetration depth of the magnetic field in the transition region between vortices and the surrounding superconducting regions is known to be 40 nm, what is the associated coherence length in Pb-8.3%wtIn at 4.2 K ?

- A. 3.4 nm
- B. 8.3 nm
- C. 11.4 nm
- D. 20.1 nm

10. Curie's law, $\chi = C/T$, describes the paramagnetic response of atomically bound electrons in many solids. Of the following, are there any restrictions which do not hold for a general case?

- A. It applies only to systems where there is no exchange between the individual paramagnetic moments, and only when the atoms involved exhibit a non-zero electronic magnetic moment in the ground state.
- B. It applies only to situations where the local magnetic field is limited in strength.
- C. It applies only to describe the low-temperature paramagnetic response.
- D. All restrictions above apply.

11. One of the following mechanisms is considered to be of little relevance to the ordering of magnetic moments in ferro-, anti-ferro and ferrimagnetic crystal lattices. Which one ?

- A. Direct exchange, i.e. spin-exchange between electrons of nearest neighbour ions
- B. Itinerant exchange, i.e. spin-exchange between conduction electrons.
- C. Indirect exchange, i.e. spin-exchange between electrons in deep-in shells, e.g. 4f, via the conduction electrons.
- D. Super exchange, i.e. spin-exchange between magnetic ions separated by non-magnetic ions.

12. What is contained in the so-called Bloch $T^{3/2}$ law ?

- A. Temperature-dependence for the net magnetisation in a single ferromagnetic domain.
- B. Temperature-dependence for the net magnetisation in a single anti-ferromagnetic domain.
- C. Temperature-dependence for the net magnetisation in a single ferrimagnetic domain.
- D. All of the above.

13. The non-zero paramagnetic susceptibility in anti-ferromagnetic crystals at $T=0$ K is ascribed to:

- A. Unbalance in the alignment of magnetic moments parallel and anti-parallel with the applied magnetic field.
- B. Slight deviations in the anti-parallel alignment of magnetic moments orthogonal to the applied magnetic field.
- C. Higher-order effects, such as second nearest-neighbour interactions.
- D. None of the above

14. Of the following quantum, which is the least likely to be present inside a metal region exhibiting normal electronic conductivity?

- A. Plasmons
- B. Magnons
- C. Polarons
- D. Fluxons

15. The Kramers-Kronig relations are based on the following physical principle:

- A. Causality of generalized linear response functions of the system
- B. Superposition of all contributions from excited modes of the system.

- C. Time-dependent relaxation of excited modes of the system.
- D. None. The Kramers-Kronig relations are based exclusively on mathematical principles that apply to the integration of complex functions.

16. SQUID may refer to?

- A. A web cache daemon and proxy server released under GNU GPL
- B. A class of ten-armed marine animals belonging to the Cephalopods family.
- C. A super-conducting quantum interference device that can be used for very precise measurements of magnetic fields
- D. All of the above.