NTNU – Trondheim
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Department of Physics

Examination paper for TFY4245 Faststoff-fysikk, videregående kurs

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Examination date: 27.05.2013

Examination time (from-to): 09:00-13:00

Permitted examination support material:

Alternative C:

Standard pocket calculator

K. Rottmann: Mathemaical Formulae (all language editions)

S. Barrett and T.M. Cronin: Mathematical Formulae

Other information:

The exam paper consists of four problems: 3 ”normal” Problems 1, 2 and 3, and one set of multiple choice questions, Problem 4.

The ”normal” problems count altogether 70%, and the multiple choice questions count 30%. Only ONE of the alternatives a)-d) must be marked for each of the 10 multiple-choice questions. Correct answer gives 1 point, no answer or wrong answer give 0 points.

Language: English

Number of pages: 8

Number of pages enclosed:

Checked by:

Date Signature

Table – Answers to Problem 4 questions

1	2	3	4	5	6	7	8	9	10

Some relationships that may be found useful:

Maxwell equations:

$$\nabla \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t} \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \cdot \vec{D} = \rho$$

Curie-Brillouin relation: $M = \frac{N}{V} g(JLS)\mu_B J \cdot B_J(\frac{g(JLS)\mu_B \mu_0 H}{k_B T} J)$

where the Brillouin function: $B_J(x) = \frac{2J+1}{2J}\coth(\frac{2J+1}{2J}x) - \frac{1}{2J}\coth(\frac{x}{2J})$

For small x: $\coth(x) \simeq \frac{1}{x} + \frac{1}{3}x$

Problem 1.

The interaction between external electromagnetic fields and a metal can be handled by regarding the metal response to the field similar to that of plasma, i.e. a highly ionized gas where the charges of the free electrons are balanced by the positive charges of the ion cores.

Let $\vec{r}$ and $\vec{v}_r$ be the position and velocity, respectively, for a single free electron of the plasma.

a) Set up a classical equation of motion for the electron due to the forces exerted on it by the external electromagnetic field, taking into account also resistive losses. Which mechanism would tend to be most decisive to the resistance?

Assume that both the system response and the external fields can be represented by solutions on the form $\vec{r} = \vec{r}_0(\omega,\vec{k})e^{i(\vec{k}\cdot\vec{r}-\omega t)}$, where the wave vectors and frequencies are the same for all fields and response variables.

In the so-called long wavelenght limit, the *collective* response of the free electron system may be accounted for by the polarisation, $\vec{P} = -ne\vec{r}$, with n as the free electron number density and e the fundamental charge.

b) Transfer the single-electron equation of motion in a) to an equation for the collective motion of the electron gas. Show that in the long wavelenght limit, and with the assumption of negligible resitive losses, the following relation for the dielectric response function of the electron gas may be derived

$$\varepsilon_{el}(\omega) = 1 - \frac{\omega_p^2}{\omega^2}, \qquad \omega_p = \sqrt{\frac{ne^2}{\varepsilon_0 m_e}} \qquad (1)$$

where m_e is the electron mass and ε_0 the vacuum permittivity.

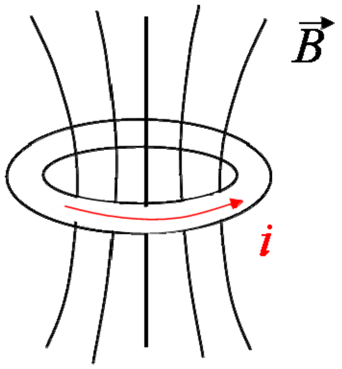
The solution above can be transferred to a relation that accounts for the whole plasma by including the background of the positive ion cores. It can be shown that $\varepsilon_{plasma}(\omega) = \varepsilon(\infty)\cdot\varepsilon_{el}(\omega)$, with $\varepsilon(\infty)=$ constant holds up to very high frequencies.

Assume the plasma to be isotropic and non-magnetic, i.e. $\mu_{ij}=\mu=\mu_0$, so that the wave equation for the plasma media may be expressed

$$\frac{\partial^2\vec{D}}{\partial t^2} = \frac{1}{\mu_0}\nabla^2\vec{E} \qquad (2)$$

c) Derive the dispersion relation for the long-λ plasma oscillations, and sketch the result in a ω(k) - graph. Give explicit accounts for the behaviour of the plasma and the field in the ranges i) $\omega < \omega_p$, ii) $\omega = \omega_p$ and iii) $\omega > \omega_p$.

Problem 2.



The current loop shown in the figure is made from a metal which is a type I superconductor at T < T_c. In the superconducting state current is carried by Cooper-pairs.

Assume the loop to be operating at a fixed temperature T well below T_C, just sligthly above 0 K, and in the absence of any external magnetic fields.

The Cooper-pairs can be associated with wave functions on the form, $\psi = n^{1/2}e^{i\theta(r)}$, leaving the concentration of pairs $n = \psi^*\psi =$ constant. The generalised momentum operator for a charge q moving in an electromagnetic field is $p_{op} = -i\hbar\nabla - q\vec{A}$, where $\vec{A}$ is the vector potential, i.e. $\vec{B} = \nabla \times \vec{A}$

a) Find an expression for the superconducting current density, and verify that the result is consistent with the London equation

$$\nabla \times \vec{j} + \frac{e^2 n}{m}\vec{B} = 0$$

b) Recall the Meissner-effect which follows directly from the London equation, and show that this leads to a quantisation of the magnetic flux through the superconducting loop

$$\Phi_m = \Phi_0 \cdot s = \frac{\hbar\pi}{e} \cdot s \qquad s = 1, 2, ...$$

c) Find an expression for the stabilisation energy of the superconducting state, ΔU.

In practice there is an upper restriction on the flux in b), which may be indicated by s_{max}. If the loop area is adequately small, the flux density may to a first approximation be regarded as uniform. Use the approximation to find a restricton on s_{max} expressed in terms of ΔU.

How would the upper restriction on flux be effected

i. by changes in temperature within the range 0 K < T < T_C ?
ii. by the presence of an external magnetic field ?
iii. if the loop was made from a type II superconducting material ?

Justify your answers.

Problem 3

Consider a solid consisting of N identical atoms with partially filled 3d shells, so that their angular momentum quantum numbers $J \neq 0$. The solid yields a paramagnetic response for $T > T_C$, and at $T < T_C$, it orders into a ferromagnetic structure.

In a mean field approach the magnetic field experienced by each atom can be expressed $\vec{H} = \vec{H}_{ext} + \vec{H}_{exch} = \vec{H}_{ext} + \lambda\vec{M}$, where $\vec{H}_{ext}$ is the external field, and the so-called exchange field, $\vec{H}_{exch} = \lambda\vec{M}$, is assumed to be proportional to the magnetisation of the system.

a) Express the magnetisation of the system both in the paramagnetic and ferromagnetic phase as functions of the variables external to the system (field and temperature), and verify that the expressions seems reasonable both for $\vec{H}_{ext} = 0$ and $\vec{H}_{ext} \neq 0$.

b) Show that the suceptibility well inside the paramagnetic phase follows the so-called Curie-Weiss law

$$\chi = \frac{C}{T - T_C}$$

The ferromagnetic ordering is a second-order phase transition, and can be handled by Landau theory. In the vicinty of T_C, we may expand the Gibbs free energy of the system in terms of M as the ordering parameter.

For T close to T_C it suffices to expand to 4^{th} order in M.

$$G(M) = g_0 + \tfrac{1}{2}g_2M^2 + \tfrac{1}{4}g_4M^4 - \vec{H}_{ext}\vec{M}$$

where $g_0 = G_{para}$ ($T \geq T_C$, $H_{ext} = 0$) ; $g_2 = \gamma(T - T_C)$ where γ is a constant, while g_4 varies slowly with temperature and changes sign from negative above to positive below T_C.

c) Assume that our system is in thermal equilibrium at any T close to T_C and find expressions for the suceptibilities both for the paramagnetic and ferromagnetic phase.

How does the constant γ compare with the Curie constant C in b) ?

Problem 4. Multiple-choice questions.

1. Piezoelectricity:
a) is present in all dielectric materials
b) refers to all materials exhibiting a dielectric response to an applied mechancial force.
c) refers to all materials exhibiting a dielectric response, either to an applied mechanical force or to a change in temperature.
d) None of the above

2. Paramagnetism:
a) Paramagnetism is present in all solids containing atoms with partially filled shells
b) The paramagnetic susceptibility for a sample with localized magnetic moments is proportional to the temperature
c) The paramagnetic susceptibility is generally positive and smaller in magnitude than the diamagnetic susceptibility
d) None of the above

3. Phase transitions may be described by Landau theory starting from a free energy expansion of the form $G= - \theta\eta + g_0 + \gamma(T-T_C)\eta^2 + g_4\eta^4 + g_6\eta^6 +\ldots + g_{2n}\eta^{2n}$, where T is the temperature, θ is the external field, η is the order parameter, γ and g_0 are constants, T_C is the transition temperature, while $g_4, g_{6, \ldots,} g_{2n}$ are smooth functions of T that can be considered constant over a small temperature range near T_C.
a) Landau theory in this form is a general field theory valid for both first and second order phase transitions
b) Landau theory in this form describes first order phase transitions, with $g_4 < 0$ and $g_n = 0$ for all n>2
c) Landau theory in this form applies only to second order ferroelectric phase transitions, with $g_4 > 0$ and $g_n = 0$ for all n>2
d) None of the above

4. You are asked to build a very accurate positioning device with resolution < 1nm. What kind of property would generally be most relevant in selection of a material suited for this purpose:
a) Ferromagnetism
b) Piezoelectricity.
c) Antiferromagnetism
d) Ferroelectricity

5. In general, the real part of a frequency-dependent linear susceptibility describes:
a) Heat loss from the sample as response to an applied external force.
b) Reversible storage of potential energy in the sample as an external force is applied and released.
c) The power taken up by the system from the applied force, which in general may be redistributed either reversibly or irreversibly (as heat loss).
d) None of the above.

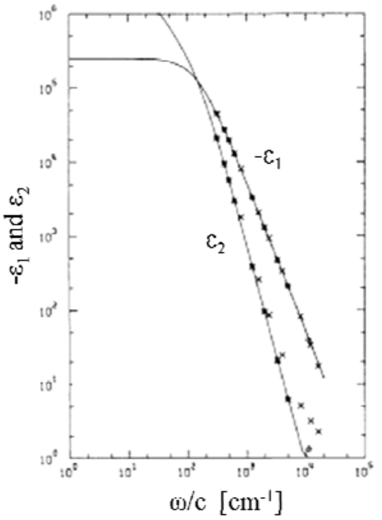
6. The Clausius-Mosotti relation relates:
a) The dielectric susceptibility to the individual molecular/atomic/electronic polarisibility?
b) The dielectric constant to the macroscopic dielectric polarisability?
c) The dielectric constant to the dielectric susceptibility?
d) None of the above?

7. The stabilization energy of a superconducting state is given by:
a) The energy associated with the formation of a Cooper pair?
b) The vortex formation energy?
c) The energy associated with the critical magnetic field intensity ?

d) None of the above?

8. In zero applied magnetic field, the size and shape distribution of Weiss domains in a ferromagnet are in general given as a result of:
a) A competition between the the zero-field magnetic energy of the bulk structure and the domain wall energy?
b) A competition between the diamagnetic and paramagnetic energies of the bulk structure ?
c) A competition between the zero-field magnetic energy of the bulk structure and the earth magnetic field energy?
d) None of the above?

9.



The figure abow shows - ε_1 and ε_2, the real and imaginary parts, respectivly, of the long-wavelenght dielectric response function $\varepsilon(\omega) = \varepsilon_1(\omega)+i\ \varepsilon_2(\omega)$ of a "unknown" material. The points represent values converted from optical reflectivity measurements, whereas the lines are calculated from theoretical models. Following a convention used in optical measurements, ω/c is scaled in cm^{-1}, where c is the vacuum speed of light.

Judging from the figure, the unknown material is most likely to be a
a) dielectric
b) semiconductor
c) metal
d) superconductor

10.
In NMR experiments the external field applied, $\vec{H}$, is normally
a) a static magnetic field
b) an oscillating/rotating magnetic field

c) a static magnetic field directed orthogonally to an oscillating field with a similar amplitude
d) a strong static magnetic field directed orthongonally to a substantially weaker oscillating field