Problem 4. Multiple-choice questions.

1. Pyroelectricity:

a) is present in all dielectric materials.
b) is principally identical to ferroelectricity, apart from being irreversible upon reversal of an external field.
c) refers to materials exhibiting a dielectric response to a change in temperature.
d) refers to materials exhibiting a dielectric response, either to an applied mechanical force or to a change in temperature.

2. The electronic paramagnetic susceptibility is:

a) non-zero in all matter containing atoms with partially filled shells
b) proportional to the temperature in samples with localized magnetic moments.
c) generally positive and smaller in magnitude than the diamagnetic susceptibility.
d) predominantly non-zero only at cryogenic temperatures

3. The electronic diamagnetic susceptibility is:

a) non-zero only in systems with filled shells
b) non-zero in systems with filled shells and in systems where the partially filled shells are one electron short of being half-filled (J=0).
c) non-zero in the systems accounted for by answer b) plus in all metals (free electron systems).
d) non-zero in all matter

4. In general, the imaginary part of a frequency-dependent linear susceptibility describes:
a) Energy from the applied force which cannot be taken up by the response system.
b) Reversible storage of potential energy in the sample as an external force is applied and released.
c) Energy transferred from the applied force which is dissipated into the response system in terms of disordered thermal motion (heat loss).
d) None of the above

5. The stabilization energy of a superconducting state is given by:
a) The energy associated with the formation of a Cooper pair?
b) The energy associated with the critical magnetic field intensity at 0 K?
c) The magnetic field energy produced by the superconducting current?
d) The vortex formation energy?

6. In zero applied magnetic field, the size and shape distribution of Weiss domains in a ferromagnet are in general given as a result of:
a) a competition between the zero-field magnetic energy of the bulk structure and the domain wall energy?
b) a competition between the diamagnetic and paramagnetic energies of the bulk structure ?
c) a competition between the zero-field magnetic energy of the bulk structure and the earth magnetic field energy?

d) a competition between exited spin-waves?

7. Polarons are quasi particles referring to the interactions between:
a) electromagnetic fields and phonons in all crystalline solids.
b) electromagnetic fields and free electrons in all crystalline solids.
c) free electrons and phonons in ionic and covalently bound crystals.
d) electromagnetic fields and free electrons in metals and semiconductors.

8. Sm^{3+}-ions have ground state electron configuration $4f^5 5s^2 p^6$. What is the effective number of Bohr-magnetons for such ions?

a) 0.84
b) 2.68
c) 4.54
d) 9.72

9.
In NMR experiments the external field applied, $\vec{H}$, is normally
a) a static magnetic field
b) a rotating magnetic field
c) a static magnetic field directed orthogonally to an oscillating field with a similar amplitude
d) a strong static magnetic field directed orthogonally to a substantially weaker oscillating field

10. Which one of the fundamental interaction mechanisms is predominantly responsible for the ordering of independent magnetic moments in paramagnetic-to-ferro/ferri/antiferromagnetic phase transitions?

a) Magnetic dipole-dipole interactions
b) Spin-orbit interactions
c) Electrostatic electron-electron interactions
d) Spin-wave-lattice interactions

11. The current density of Cooper-pairs in a superconductor can be expressed as $\vec{J}_{cp}(\vec{r}) = -\frac{2en}{m_e}\left[\hbar\nabla\theta(\vec{r}) + 2e\vec{A}\right]$, with $\vec{A}$ as the vector potential, and with the Cooper-pairs described by a wave-function on the form $\psi(\vec{r}) = \sqrt{n}e^{i\theta(\vec{r})}$, such that their concentration $n = \psi(\vec{r})\psi^*(\vec{r})$. Due to the Meissner effect, the flux through a closed loop of superconducting current is quantized, $\Phi = \Phi_0 \cdot s$, s=1,2… What is the minimum flux quantity (minimum fluxon) Φ_0?

a) $2.0678 \cdot 10^{-16}$ Tm^2
b) $2.0678 \cdot 10^{-15}$ Tm^2
c) $2.0678 \cdot 10^{-14}$ Tm^2
d) $2.0678 \cdot 10^{-13}$ Tm^2

12. For superconductors one operates with two characteristic lengths, the London penetration depth, λ, and the coherence length, ξ. In a classical type II superconductor, typically

a) λ and ξ are fully independent, accordingly no restrictions apply.
b) $\lambda > \xi$
c) $\lambda \approx \xi$
d) $\lambda < \xi$

13. The temperature dependence of the critical field, $H_c(T)$, in a type I superconductor is

a) $H_C(T) \propto (T_C - T)^{-1}$
b) $H_C(T) \propto (T_C - T)^{1/2}$
c) $H_C(T) \propto (T_C - T)$
d) $H_C(T) \propto (T_C - T)^2$

14. In crystalline solids Raman scattering may be used to probe

a) The molecular structure (bonding nature) of the compound
b) Optical phonon modes of the crystal lattice
c) Magnon excitation modes of spin-waves in ordered magnetic lattices
d) All of the above

15. Let ε(ω) be the dielectric response function of a system to a time-varying electromagnetic field. If $\varepsilon(\omega) < 0$, then:

a) The electromagnetic wave is phase shifted by π/2 upon entrance from vacuum to the response system.
b) The system contains free charges which counteracts the polarisation of the system by a depolarisation field.
c) The system may polarise even in the absence of external fields
d) The electromagnetic wave does not propagate in the response system.

Short answer questions:

List the characteristic differences between 1^{st} and 2^{nd} order phase transitions

What is an exciton?

The magnetic susceptibility in a paramagnetic salt in which magnetisation arises from ions with half-filled 4f shells is measured at a given temperature T to be $\chi = 2.78 \cdot 10^{-3}$. Estimate the saturation magnetisation, $M_S = NgJ\mu_B$, for this salt.

CaF_2 crystallizes in a fluorite structure (fcc) containing 4 Ca^{2+} ions and 8 F^- ions per unit cell. The lattice parameter is a = 5.46 Å. The polarisabilities of Ca^{2+} and F^- are $5.22 \cdot 10^{-41}$ and $1.16 \cdot 10^{-40}$ C m^2

V^{-1}, respectively. What is the refractive index of CaF_2 if the magnetic permeability of the compound is assumed to be as that of vacuum?

The Landau free energy model describes the nature of second order phase transitions in which the order parameter of the system is assigned to a physical quantity that undergoes a disordered-to-ordered phase transition. The model may also include terms that couple this physical quantity to an external field. The Ginzburg-Landau (GL) free-energy model is an extension to the original Landau model, and was introduced in the course to describe superconductivity.

Close to the transition temperature, T_C, the GL free energy density may be expressed generically as

$$G_{GL}(T, \psi(\vec{r}), \theta) = G_0(T) + \alpha(T)\psi^2\overrightarrow{(r)} + \beta\psi^4\overrightarrow{(r)} + \gamma\left(\nabla\psi\overrightarrow{(r)}\right)^2 - \theta\nabla\psi\overrightarrow{(r)}$$

With T as temperature, $\psi(\vec{r})$ as the order parameter, which may now vary non-uniformly in space, and with θas an external field which couples to the ordering parameter. G_0 refers to the free energy density of the disordered phase (stable at $T > T_C$), while the next two terms are standard Landau expansion terms for 2nd order phase transitions with $\alpha(T) = \alpha_0(T - T_C)$, while β is assumed constant. The third term on the right hand side accounts for spatial variations in the ordering parameter, while the last term is a field term which we may neglect here. In the course, we have applied the GL model exclusively to the case of superconductivity, with the order parameter describing the ordering of free electrons in to Cooper pairs.

In current research, however, the GL model is employed more generally in the description of other types of second order phase transitions, including e.g. ferroelectricity and ferromagnetism. Compared to a standard Landau model approach to describe e.g. ferroelectricty, what could be gained by including the additional $\gamma\left(\nabla\psi\overrightarrow{(r)}\right)^2$ -term?

A given material exhibits ferromagnetic properties at temperatures $T < T_C$, and paramagnetic response for $T > T_C$. The magnetic moments arise from ions with partially filled 3d shells, that occupy nearest neighbour lattice sites of an equidistant spacing, such that their magnetic moments interact with one another through a direct exchange. In the ferromagnetic phase, the magnetic moments align parallel or anti-parallel with the c-axis.

In a generalised microscopic model, containing both the Heissenberg exchange terms and the spin interaction with and external magnetic field, $\vec{H}_{ext}$, the energy associated with magnetic response may be expressed

$$U = -\sum_{i \neq j} \Im(\Delta\vec{r}_{ij})\vec{S}_i \cdot \vec{S}_j + \sum_i g(JLS)\mu_B\mu_0\vec{H}_{ext} \cdot \vec{S}_i = \sum_i g(JLS)\mu_B\mu_0\vec{H}_{eff} \cdot \vec{S}_i \qquad (1)$$

The first sum is taken over all exchanging spin pairs *ij*. $\Im$ represents the exchange energy and its value depends on the interatomic distance Δr_{ij}. In simple systems, where all exchange pair distances are identical, Δr_{ij} and thus $\Im$ is constant (at fixed temperatures).

a) In direct exchange $\Im$ in eqn. (1) is non-zero only for nearest neighbour spins. Employ a mean field approach to the microscopic model presented in eqn. (1) to show that this yields the standard Weiss molecular mean field $\vec{H}_{eff} = \vec{H}_{ext} + \lambda\vec{M}$, and express the constant λ by quantities given in eqn. (1). (Hint: Find an expression for the microscopic $\vec{H}_{eff}$, and replace microscopic quantities by their thermal average values, assuming that the net magnetisation of the system is accounted for by N equivalent spins).

b) Use the mean Weiss molecular field representation to find an expression for the paramagnetic susceptibility for $T > T_C$ in the presence of an external magnetic field $H_{ext,}$ and show that $T_C = \frac{\mathbf{2S(S+1)<\Im>_{T_C}}}{\mathbf{3k_B}}$

c) Find expressions for the magnetisation, $M(T)$, in the ferromagnetic region in the absence of external magnetic fields, based both on microscopic and mean field quantities. Compare the expressions and discuss possible sources for discrepancies between the two approaches.

a) Based on Maxwells equations, show that the dielectric response function of can be expressed as $\varepsilon(\omega,\vec{k}) = 1 - \frac{\rho_{ind}(\omega,\vec{k})}{\rho_{tot}(\omega,\vec{k})}$, where ρ_{ind} refers to the changes induced in the system charge density in response to an external field, whereas ρ_{tot} refers to the total charge density.

b) From the continuity relation, $\frac{\partial\rho_{ind}}{\partial t} + \nabla\vec{j}_{ind} = 0$, show that the response in a system of mobile charges can be expressed as $\varepsilon(\omega) = 1 + \frac{i\sigma(\omega)}{\varepsilon_0\omega}$ in the long wave-length limit, where $\sigma(\omega)$ denotes the electrical conductivity of the system, $\vec{j} = \sigma\vec{E}$.

c) Let the system in b) be a plasma where the contribution to the dielectric response from the ion cores is negligible, i.e $\varepsilon_{ic}(\infty) = 1$. In an experiment, the AC electrical conductivity of the free electron gas is mapped out, and one finds $\sigma(\omega) = \frac{\sigma_0}{1-i\omega\tau}$, where $\sigma_0 = 9.4 \cdot 10^{-8}\ \Omega^{-1}\ \mathrm{m}^{-1}$ is the DC conductivity, and $\tau = 10^{-13}$ s is the characteristic relaxation time associated with resistive losses. What is the plasma frequency of the system?