NTNU

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Contact during the exam:
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**Exam in TFY4275 CLASSICAL TRANSPORT THEORY**

Tuesday March 4th, 2008

14:15–16:00

Allowed help: Alternativ **D**

No written material allowed

This problem set consists of 2 pages.

**Problem 1. Various topics**

**a)** Define the characteristic function (moment generating function), $G_\xi(k)$, of a random variable $\xi$ defined by the probability distribution $p(\xi)$.

**b)** How is the average $\langle e^{\xi} \rangle$ ($\xi$ is a random variable) defined in terms of cummulants? Given the characteristic function, $G_\xi(k)$, what is then the cummulant generating function?

**c)** Argue why the characteristic function of a *Gaussian* random variable, $\xi_G$, is given by

$$G_{\xi_G}(k) = \exp\left(i\mu k - \frac{\sigma^2 k^2}{2}\right),$$

where $\mu$ and $\sigma$ denote the mean and standard deviation, respectively. (Note: No full mathematical derivation is asked for).

**d)** What is the *main* difference between a random variable and a stochastic process? Give a real-life example of both of them.

**e)** Given a general stochastic process, what is needed to fully define it?

**Problem 2. Random Walk**

We consider the sum of $N$ random variables $\xi_i$

$$x_N = \sum_{i=1}^{N} \xi_i,$$

where *all* the $\xi_i$'s are *independent* and *identically* distributed (*i.e.* a random walk with steps $\xi_i$ and position $x_N$ at "time" $t = N$). The standard deviation of the (identical) steps (the random variables $\xi_i$) we denote by $\sigma_\xi$.

**a)** Formulate with words the Central Limit Theorem (CLT).

**b)** Use this to describe what the CLT predicts for the distribution of $x_N$, $p(x_N)$ when

*i)* $\sigma_\xi < \infty$?

*ii)* $\sigma_\xi = \infty$?

No mathematical details needed (if you do not like to do so....).

**c)** For the two cases mentioned above (*i* and *ii*), write down an expression for $\sigma_{x_N}$. When $\sigma_\xi < \infty$ express your answer in terms of $\sigma_\xi$ and $N$.

**d)** What is the characteristic function of the random variable $x_N$ given the $G_\xi(k)$? What is the distribution of $x_2$, denoted $p_2(x_2)$, expressed in terms of the distribution of $\xi_i$, $p(\xi)$.

**e)** Now let us set $\xi_i = N(0, \sigma)$ for all $i$'s, *i.e.*, is Normally (Gaussian) distributed with zero mean and standard deviation $\sigma < \infty$. Show *mathematically* that your above statement for $\sigma_{x_N}$ is correct.

**Problem 3. Markov processes**

**a)** Define in terms of mathematics what a Markov process is, and explain with words what this definition means. What "order" of the joint probability distribution is *sufficient* to fully define a Markov process?

**b)** What are the two *necessary* and *sufficient* conditions that have to be satisfied by *any* Markov process?

**c)** Give three (different!) examples of Markov processes.

**That is all for now guys. Good luck!**